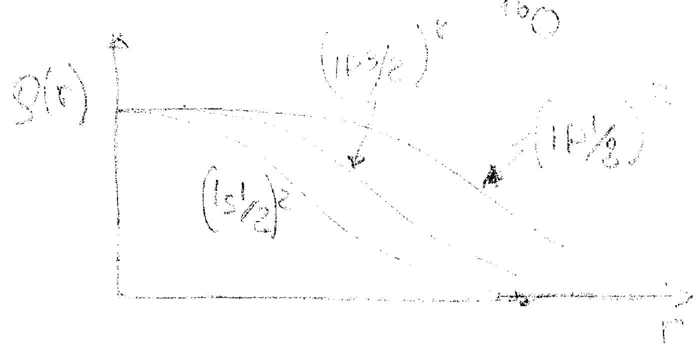
$$\rho(\vec r) = \sum_{n,j} R^2_{nl}(r) \sum_{\mu,s,m} \left[(-)^{j+l+s} \left(\frac{2j+1}{2l+1}\right)^{1/2} \langle j m \tfrac{1}{2} -s | l \mu \rangle \right]^2 Y^*_{l\mu}(\hat r) Y_{l\mu}(\hat r) =$$

$$\sum_{m,s} \langle j m \tfrac{1}{2} -s | l \mu \rangle^2 = 1$$

$$= \sum_{n,j} R^2_{nl}(r) \frac{(2j+1)}{(2l+1)} \sum_{\mu} Y^*_{l\mu}(\hat r) Y_{l\mu}(\hat r) =$$

$$= \sum_{n,j} R^2_{nl}(r) \frac{2j+1}{2l+1} \frac{2l+1}{4\pi} \underbrace{P_l(\cos\theta_{12})}_{1} = \sum_{n,j} (2j+1) R^2_{nl}(r) \frac{1}{4\pi}$$

Figura. Prepara la figura con Woods-Saxon e oscillatore armonico tipo ^{16}O



<u>Esercizio</u>: Dimostrare che nel modello a shell:

$$\langle r^2 \rangle = \int \rho(\vec r)\, r^2\, d\vec r = \sum_{j=1}^{A} \int dr\, r^4 R_j(r) R_j(r)$$

Usa: $\int d\hat r\, Y^*_{l\mu}(\hat r) Y_{l\mu}(\hat r) = 1$