$$\langle jj|\vec{\mu}\cdot\vec{j}|jj\rangle = \mu_N\left\{g^{(l)}\langle jj|\vec{j}\cdot\vec{l}|jj\rangle + g^{(s)}\langle jj|\vec{j}\cdot\vec{s}|jj\rangle\right\} \qquad 11)$$

$$\langle jj|\vec{j}\cdot\vec{l}|jj\rangle = \frac{1}{2}\langle jj|j^2+l^2-s^2|jj\rangle = \frac{1}{2}\left[j(j+1)+l(l+1)-\frac{3}{4}\right] \qquad 12a)$$

$$\langle jj|\vec{j}\cdot\vec{s}|jj\rangle = \frac{1}{2}\langle jj|j^2-l^2+s^2|jj\rangle = \frac{1}{2}\left[j(j+1)-l(l+1)+\frac{3}{4}\right] \qquad 12b)$$

$$\langle\mu\rangle = \frac{j}{(j+1)j}\left\{\frac{g^{(l)}}{2}\left(j(j+1)+l(l+1)-\frac{3}{4}\right)+\frac{g^{(s)}}{2}\left(j(j+1)-l(l+1)+\frac{3}{4}\right)\right\}\mu_N$$

Esercizio:

Dimostrare che:

$$\langle\mu\rangle = \mu_N\left[g^{(l)}\left(j-\frac{1}{2}\right)+\frac{1}{2}g^{(s)}\right] \quad \text{per } j = l+\frac{1}{2} \qquad 13a)$$

$$\langle\mu\rangle = \mu_N\left[g^{(l)}\left(j+\frac{3}{2}\right)-\frac{1}{2}g^{(s)}\right]\frac{j}{j+1} \quad \text{per } j = l-\frac{1}{2} \qquad 13b)$$

$l = j-\frac{1}{2}$

$$\frac{1}{2(j+1)}\left[j^2+j+\left(j-\tfrac{1}{2}\right)\left(j+\tfrac{1}{2}\right)-\frac{3}{4}\right] = \frac{1}{2(j+1)}\left[j^2+j+j^2-\frac{1}{4}-\frac{3}{4}\right] = \frac{2(j+1)(j-\frac{1}{2})}{2(j+1)}$$

$$\frac{1}{2(j+1)}\left[j^2+j-\left(j^2-\frac{1}{4}\right)+\frac{3}{4}\right] = \frac{1}{2(j+1)}\left[j+1\right] = \frac{1}{2}$$

$l = j+\frac{1}{2}$

$$\frac{1}{2(j+1)}\left[j^2+j+\left(j+\tfrac{1}{2}\right)\left(j+\tfrac{3}{2}\right)-\frac{3}{4}\right] = \frac{1}{2(j+1)}\left[j^2+j+j^2+\frac{3}{4}+2j-\frac{3}{4}\right] = \frac{j}{j+1}\left(j+\frac{3}{2}\right)$$

$$\frac{1}{2(j+1)}\left[j^2+j-\left(j^2+2j+\frac{3}{4}\right)+\frac{3}{4}\right] = \frac{1}{2(j+1)}\left[-j\right]$$