Caso a) <u>Buca di potenziale infinita</u>

Eq. di Schrödinger

$$\hat{H} u = \left(-\frac{\hbar^2}{2m} \nabla^2 + V(r) \right) u(\vec{r}) = E\, u(\vec{r})$$

con
$$V(r) = -V_0 \quad \text{per } r \leq R$$
$$V(r) = \infty \quad \text{per } r > R$$

$$\hat{H} u = \left(\frac{p_r^2}{2m} + \frac{\vec{\ell}^{\,2}}{2mr^2} + V(r) \right) u(\vec{r}) = E\, u(\vec{r})$$

$$p_r^2 = -\frac{\hbar^2}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right)$$

$$\vec{\ell}^{\,2} = -\frac{\hbar^2}{\sin^2\vartheta} \left(\sin\vartheta \frac{\partial}{\partial\vartheta} \left(\sin\vartheta \frac{\partial}{\partial\vartheta} \right) + \frac{\partial^2}{\partial\varphi^2} \right)$$

$$\vec{\ell}^{\,2}\, Y_{\ell\mu}(\vartheta,\varphi) \equiv \hat{\ell}^2\, Y_{\ell\mu}(\hat{r}) = \ell(\ell+1)\hbar^2\, Y_{\ell\mu}(\hat{r}) \qquad 6$$

$$\ell_z\, Y_{\ell\mu}(\hat{r}) = \mu\hbar\, Y_{\ell\mu}(\hat{r}) \qquad 6$$

$$Y^*_{\ell\mu}(\hat{r}) = (-)^\mu\, Y_{\ell-\mu}(\hat{r}) \qquad 6$$

$$\int d\hat{r}\, Y^*_{\ell\mu}(\hat{r})\, Y_{\ell'\mu'}(\hat{r}) = \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \sin\vartheta\, Y^*_{\ell\mu}(\vartheta,\varphi)\, Y_{\ell'\mu'}(\vartheta,\varphi) = \delta_{\ell\ell'}\,\delta_{\mu\mu'} \qquad 6$$

$$\sum_{\ell=0}^{\infty} \sum_{\mu=-\ell}^{\ell} Y^*_{\ell\mu}(\hat{r}_1)\, Y_{\ell\mu}(\hat{r}_2) = \delta(\hat{r}_1 - \hat{r}_2) \equiv \frac{\delta(\vartheta_1-\vartheta_2)\,\delta(\varphi_1-\varphi_2)}{\sin\vartheta_1} \qquad 6$$