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An introduction to Neutrino physics and oscillations

Standard Model Neutrino Interactions

- Lagrangian for electroweak interactions:

$$L_{\text{int}} = i\frac{g}{\sqrt{2}}\left[j_\mu^{(+)}W^\mu + j_\mu^{(-)}W^{\mu+}\right] + i\left[g\cos\theta_W j_\mu^{(3)} - g'\sin\theta_W j_\mu^{(Y/2)}\right]Z^\mu +$$

$$+ i\left[g\sin\theta_W j_\mu^{(3)} + g'\cos\theta_W j_\mu^{(Y/2)}\right]A^\mu$$

  - 1st term: charged current interactions (W^+, W^- exchange)
  - 2nd term: neutral current interactions (Z^0 exchange)
  - 3rd term:electromagnetic interactions (photon exchange)

  - Electron charge: $e = g\sin\theta_W = g'\cos\theta_W$

  - 3rd term: $e j_\mu^{e.m.} = e(j_\mu^{(3)} + j_\mu^{(Y/2)})$

    (neutrinos only couple to $W^\pm$ and Z^0)

S.M. interactions (cont)

- The vector boson masses are then predicted:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W{}^2} = \frac{e^2}{8m_W{}^2 \sin^2\theta_W} = \frac{4\pi\alpha}{8m_W{}^2 \sin^2\theta_W} \qquad \alpha = 1/137.036$$

- Masses:

$$m_W{}^2 = \left(\frac{37.2805}{\sin\theta_W}\right)^2$$

$$m_W = 80.390 \pm 0.018\, GeV$$
$$m_Z = 91.1874 \pm 0.0021\, GeV$$
$$\sin^2\theta_W = 0.22280 \pm 0.00035$$

- Need radiative corrections:

$$m_W = \frac{37.2805}{\sin\theta_W (1-\Delta r)^{1/2}}$$

$$with \quad \Delta r \approx 0.0376 \pm 0.0025 \; for \; m_t = 180.0\, GeV \; m_H = 300\, GeV$$

$$yields: \quad m_W = 80.51 \pm 0.11\, GeV$$

- Dirac equation for spin-1/2 particle: $$i\gamma^\mu \partial_\mu \psi - m\psi = 0$$
- define the "chiral" components: $$\psi_{L,R} = P_{L,R}\psi$$
- where $P_{L,R}$ are the chiral projectors: $$P_{R,L} = \frac{1 \pm \gamma^5}{2}$$
- where: $$\gamma^5 = \gamma^0 \cdot \gamma^1 \cdot \gamma^2 \cdot \gamma^3$$
- using: $$\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5 \Rightarrow P_{L,R}\gamma^\mu = \gamma^\mu P_{R,L}$$
- for massive particles the chiral components are coupled::

$$i\gamma^\mu \partial_\mu \psi_{R,L} - m\psi_{R,L} = 0$$

- for massless fermions the $\psi_{L,R}$ are independent and are also elicity eigenstates:

$$i\gamma^\mu \partial_\mu \psi_{R,L} = 0 \Rightarrow \frac{\mathbf{p} \cdot \mathbf{s}}{|\mathbf{p}|}\,\tilde{\psi}_{L,R}(\mathbf{p}) = \mp\frac{1}{2}\tilde{\psi}_{L,R}(\mathbf{p})$$

S.M. interactions (cont)

- A) Neutrino electron interaction

$$L_{\text{int}} = i\frac{g}{\sqrt{2}}\left[j_\mu^{(+)}W^\mu + j_\mu^{(-)}W^{\mu+}\right] + i\frac{g}{2\cos\theta_W}j_\mu^{(Z)}Z^\mu + iej_\mu^{e.m}$$

- Where: $j_\mu^{(+)} = \overline{\nu}_{e,L}\gamma_\mu e_L = \frac{1}{2}\overline{\nu}_e\gamma_\mu(1-\gamma_5)e$

$$j_\mu^{(-)} = \overline{e}_L\gamma_\mu\nu_{e,L} = \frac{1}{2}\overline{e}\gamma_\mu(1-\gamma_5)\nu_e$$

$$j_\mu^{(Z)} = 2(j_\mu^{(3)} - \sin^2\theta_W j_\mu^{e.m}) =$$

$$= \overline{\nu}_{e,L}\gamma_\mu\nu_{e,L} - \overline{e}_L\gamma_\mu e_L + 2\sin^2\theta_W\overline{e}\gamma_\mu e =$$

$$= \frac{1}{2}\overline{\nu}_e\gamma_\mu(1-\gamma_5)\nu_e - \frac{1}{2}\overline{e}\gamma_\mu(1-\gamma_5)e + 2\sin^2\theta_W\overline{e}\gamma_\mu e$$

$$\Rightarrow j_\mu^{(Z)} = \frac{1}{2}\overline{\nu}_e\gamma_\mu(1-\gamma_5)\nu_e + \overline{e}\gamma_\mu(g_V - g_A\gamma_5)e$$

- With: $g_V = -\frac{1}{2} + 2\sin^2\theta_W \qquad g_A = -\frac{1}{2}$

S.M. interactions (cont)

- B) Quark weak interactions

$$L_{\text{int}} = i\frac{g}{\sqrt{2}}\left[j_\mu^{(+)}W^{\mu} + j_\mu^{(-)}W^{\mu+}\right] + i\frac{g}{2\cos\theta_W}\, j_\mu^{(Z)}Z^{\mu} + iej_\mu^{e.m}$$

- Where:

$$j_\mu^{(+)} = \frac{1}{2}\bar{u}\gamma_\mu(1-\gamma_5)d$$

$$j_\mu^{(-)} = \frac{1}{2}\bar{d}\gamma_\mu(1-\gamma_5)u$$

$$j_\mu^{(Z)} = \bar{u}\gamma_\mu(A_u - B_u\gamma_5)u + \bar{d}\gamma_\mu(A_d - B_d\gamma_5)d$$

- With:

$$A_u = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W \qquad B_u = \frac{1}{2}$$

$$A_d = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W \qquad B_d = -\frac{1}{2}$$

Three known families of leptons in nature…

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

L=1
(−1) for antiparticles

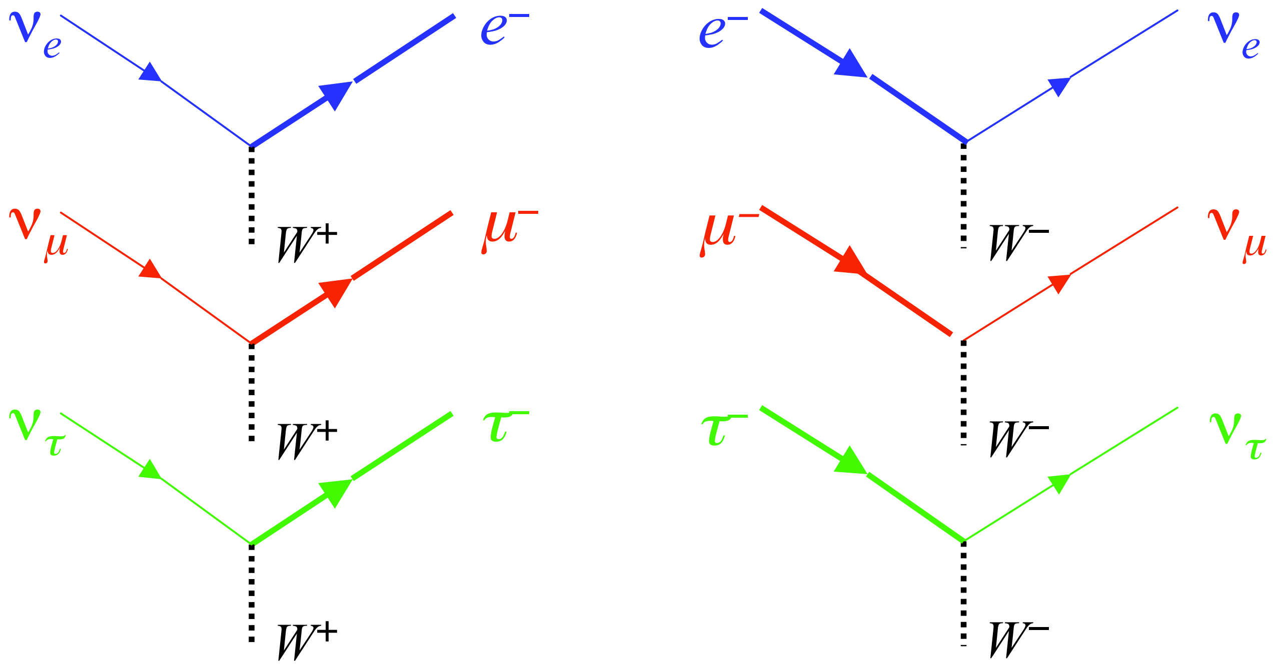
in the SM with zero netrino mass each family lepton number is exactly conserved
Charge current interactions change “up” leptons in “down” leptons

…and also three families of quarks

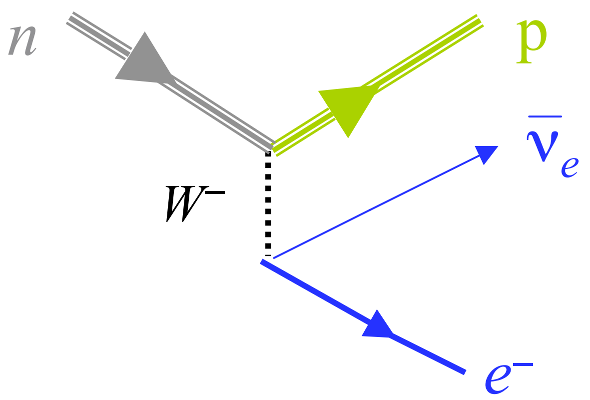
$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

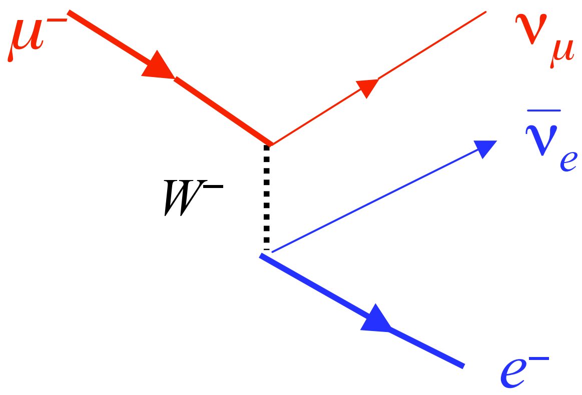
B=1
(−1) for antiparticles

Charge current interactions

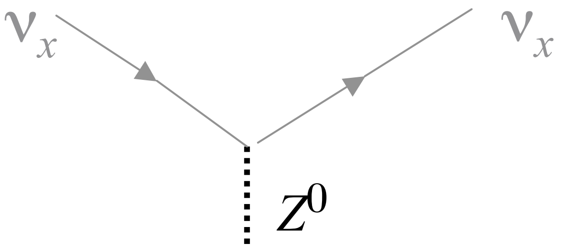


examples:

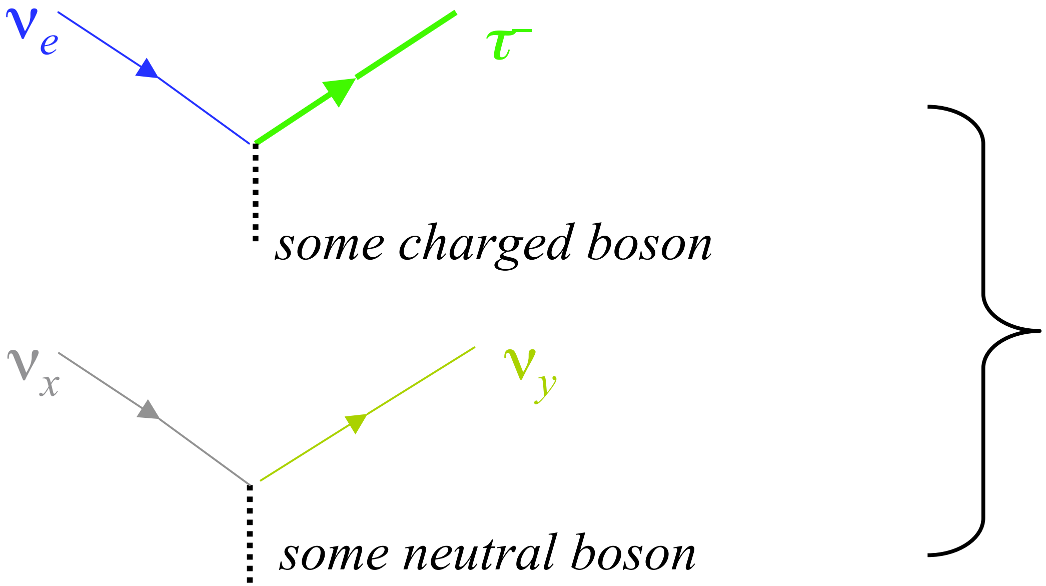




Neutral current interactions



notice that in the SM flavor changing interactions are not allowed



Forbidden

S.M. interactions (cont)

- After introducing Higgs field and spontaneous symmetry breaking:

$$L_{Higgs} = -\left|D_\mu \phi\right| - \mu^2 \left|\phi\right|^2 - \lambda \left|\phi\right|^4$$

- Masses:

$$m_H = \sqrt{2\lambda} v$$

$$m_{W^\pm} = \frac{gv}{2}$$

$$m_{Z^0} = \frac{\sqrt{g^2 + g'^2}}{2} v$$

$$\left(\frac{m_{W^\pm}}{m_{Z^0}}\right)^2 = \frac{g^2}{g^2 + g'^2} = \cos^2\theta_W$$

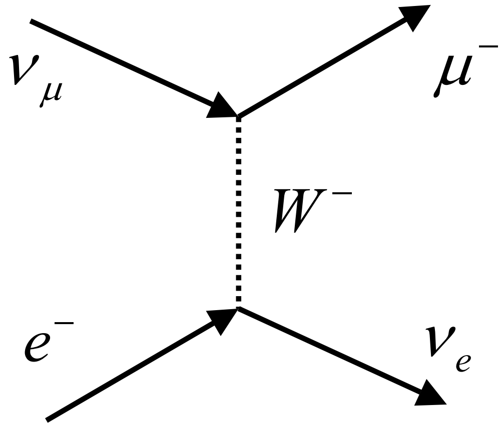
- Vacuum expectation value: $v = \frac{1}{\sqrt{\sqrt{2}G_F}} \approx 246\ GeV$
- Effective Hamiltonian:

$$H_{eff} = \frac{g^2}{4m_W^2}\left[j^{(+)\mu} j_\mu^{(-)} + h.c.\right] + \frac{g^2}{8m_Z^2 \cos^2\theta_W} j^{(Z)\mu} j_\mu^{(Z)} =$$

$$= \frac{G_F}{\sqrt{2}}\left[2 j^{(+)\mu} j_\mu^{(-)} + h.c. + j^{(Z)\mu} j_\mu^{(Z)}\right]$$

Neutrino-electron scattering

- Only charged current: $\nu_\mu + e^- \to \nu_e + \mu^-$



$$H_{eff} = \frac{G_F}{\sqrt{2}}\left[\bar{\nu}_e \gamma^\mu (1-\gamma_5) e\right]\left[\bar{e}\gamma_\mu (1-\gamma_5)\nu_e\right]$$

$$s = \left(p(\nu_\mu) + p(e)\right)^2 = 2m_e E(\nu_\mu)\ (in\ LAB)$$

$$t = q^2 = \left(p(\nu_\mu) - p(\mu)\right)^2$$

$$y = \frac{p(e)\cdot\left(p(\nu_\mu) - p(\mu)\right)}{p(e)\cdot p(\nu_\mu)} = \frac{E(\nu_\mu) - E(\mu)}{E(\nu_\mu)}\ (in\ LAB)$$

Inelasticity variable (0<y<1)

$$\frac{d\sigma_{CC}(\nu_\mu e^-)}{dy} = \frac{G_F{}^2 s}{\pi}\frac{m_W{}^2}{q^2 - m_W{}^2} \approx \frac{2G_F{}^2 m_e}{\pi} E(\nu_\mu)\ (in\ LAB)$$

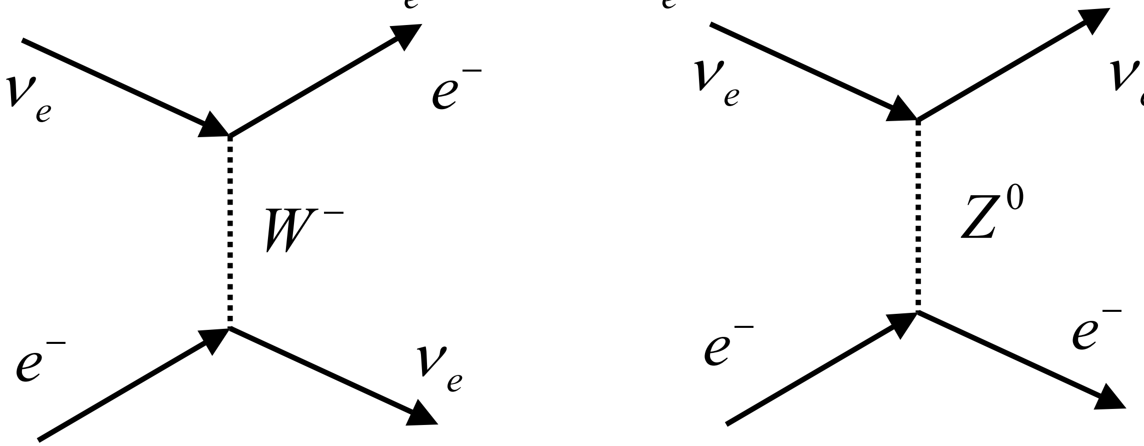
Total cross-section: $\sigma_{CC}(\nu_\mu e^-) = \frac{G_F{}^2 s}{\pi} = 1.7\times 10^{-43}\left(\frac{E}{10\ MeV}\right) cm^2$

(cross-section proportional to energy!)

Neutrino-electron scattering (cont)

- Tree level Feynman diagrams: both neutral and charged currents

$$\nu_e + e^- \rightarrow \nu_e + e^-$$



- Effective Hamiltonian:

$$H_{eff} = \frac{G_F}{\sqrt{2}}\left\{ \left[\bar{\nu}_e \gamma^\mu (1-\gamma_5) e\right]\left[\bar{e}\gamma_\mu (1-\gamma_5)\nu_e\right] + \left[\bar{\nu}_e \gamma^\mu (1-\gamma_5)\nu_e\right]\left[\bar{e}\gamma_\mu (g_V - g_A\gamma_5) e\right]\right\}$$

$$= \frac{G_F}{\sqrt{2}}\left\{ \left[\bar{\nu}_e \gamma^\mu (1-\gamma_5)\nu_e\right]\left[\bar{e}\gamma_\mu (1+g_V-(1+g_A)\gamma_5) e\right]\right\}$$

(through a Fierz transformation)

Neutrino-electron scattering (cont)

- Rearranging terms in charged and neutral current contributions for:

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

$$g_L = \frac{1}{2}(1 + g_V + 1 + g_A) = -\frac{1}{2} + \sin^2\theta_W + 1 = \frac{1}{2} + \sin^2\theta_W$$

$$g_R = \frac{1}{2}(1 + g_V - (1 + g_A)) = \sin^2\theta_W$$

Then: $$\frac{d\sigma(\nu_e e^-)}{dy} = \frac{G_F{}^2 s}{\pi}\left[\left(\frac{1}{2} + \sin^2\theta_W\right)^2 + \sin^4\theta_W (1-y)^2\right]$$

$$\Rightarrow \sigma(\nu_e e^-) = \frac{G_F{}^2 s}{\pi}\left[\left(\frac{1}{2} + \sin^2\theta_W\right)^2 + \frac{1}{3}\sin^4\theta_W\right] = 0.96\times 10^{-43}\left(\frac{E_\nu}{10\ MeV}\right) cm^2$$

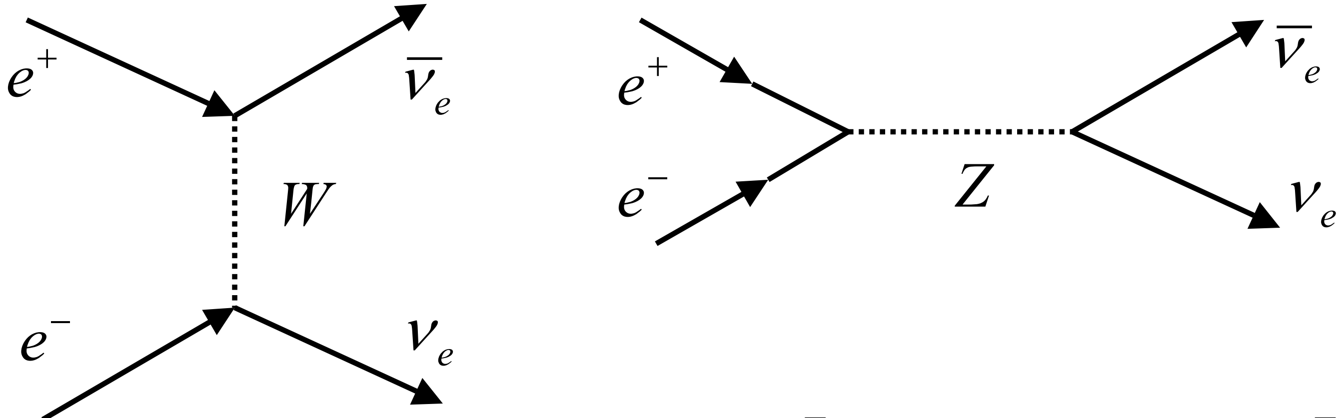
$$\text{Also}: \sigma(\bar{\nu}_e e^-) = \frac{G_F{}^2 s}{\pi}\left[\frac{1}{3}\left(\frac{1}{2} + \sin^2\theta_W\right)^2 + \sin^4\theta_W\right] = 0.40\times 10^{-43}\left(\frac{E_\nu}{10\ MeV}\right) cm^2$$

These cross-sections are a consequence of the interference of the charged and neutral current diagrams.

Neutrino-electron scattering (cont)

- Neutrino pair production: $e^+ + e^- \to \nu_e + \overline{\nu}_e$

  Contribution from both W and Z graphs.



Then: $$\sigma(e^+e^- \to \nu_e\overline{\nu}_e) = \frac{G_F{}^2 s}{12\pi}\left[\left(\frac{1}{2} + 2\sin^2\theta_W\right)^2 + \frac{1}{4}\right]$$

- Only neutral current contribution to: $e^+ + e^- \to \nu_\mu + \overline{\nu}_\mu$

$$\sigma(e^+e^- \to \nu_\mu\overline{\nu}_\mu) = \frac{G_F{}^2 s}{12\pi}\left[\left(\frac{1}{2} - 2\sin^2\theta_W\right)^2 + \frac{1}{4}\right]$$

Neutrino-electron scattering (cont)

- Summary neutrino electron scattering processes:

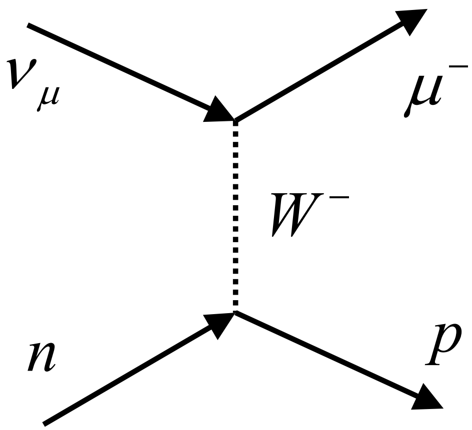
Process	Total cross-section
$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$	$\frac{G_F^2 s}{\pi}$
$\nu_e + e^- \rightarrow \nu_e + e^-$	$\frac{G_F^2 s}{4\pi}\left[\left(2\sin^2\theta_W - 1\right)^2 + \frac{4}{3}\sin^4\theta_W\right]$
$\overline{\nu}_e + e^- \rightarrow \overline{\nu}_e + e^-$	$\frac{G_F^2 s}{4\pi}\left[\frac{1}{3}\left(2\sin^2\theta_W + 1\right)^2 + 4\sin^4\theta_W\right]$
$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$	$\frac{G_F^2 s}{4\pi}\left[\left(2\sin^2\theta_W - 1\right)^2 + \frac{4}{3}\sin^4\theta_W\right]$
$\overline{\nu}_\mu + e^- \rightarrow \overline{\nu}_\mu + e^-$	$\frac{G_F^2 s}{4\pi}\left[\frac{1}{3}\left(2\sin^2\theta_W - 1\right)^2 + 4\sin^4\theta_W\right]$
$e^+ + e^- \rightarrow \nu_e + \overline{\nu}_e$	$\frac{G_F^2 s}{12\pi}\left[\frac{1}{2} + 2\sin^2\theta_W + 4\sin^4\theta_W\right]$
$e^+ + e^- \rightarrow \nu_\mu + \overline{\nu}_\mu$	$\frac{G_F^2 s}{12\pi}\left[\frac{1}{2} - 2\sin^2\theta_W + 4\sin^4\theta_W\right]$

$$s = 2m_e E(\nu_\mu) \text{ (in the LAB frame)}$$

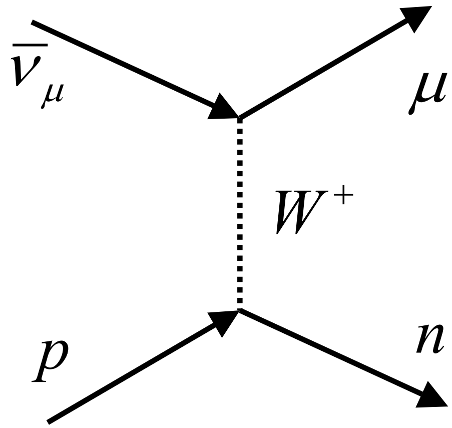
Neutrino-nucleon quasi-elastic scattering

- Quasi-elastic neutrino-nucleon scattering reactions (small q^2): affects nucleon as a whole

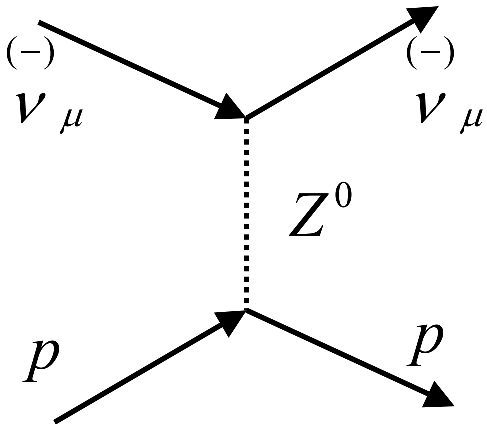
$\nu_\mu + n \to \mu^- + p$



$\bar{\nu}_\mu + p \to \mu^+ + n$



$\overset{(-)}{\nu}_\mu + p \to \overset{(-)}{\nu}_\mu + p$



$$M = \left\langle \mu^-, p \middle| H_{eff} \middle| \nu_\mu, n \right\rangle =$$

$$\frac{G_F \cos\theta_c}{\sqrt{2}} \left[\bar{\mu}\gamma^\mu (1-\gamma_5)\nu_\mu \right] \left[\bar{p}\gamma_\mu \left(F_V(q^2) + F_A(q^2)\gamma_5 \right) n \right]$$

$F_V(q^2) = \text{vector form factor}$

$\cos\theta_C = 0.975$ (Cabbibo angle)

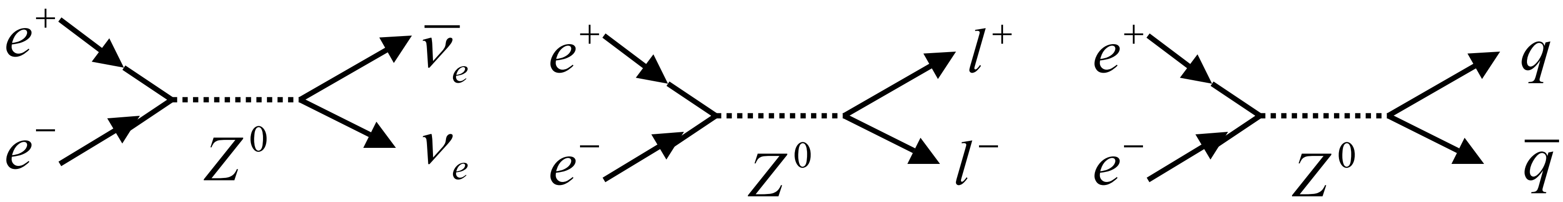
$F_A(q^2) = \text{axial} - \text{vector form factor}$

Number of neutrinos

- Width of the Z-pole resonance: Breit-Wigner distribution

$$\sigma(e^+e^- \to f) = \frac{12\pi(\hbar c)^2}{M_Z} \frac{s\Gamma_e \Gamma_f}{(s - M_Z{}^2)^2 + s^2\Gamma_Z^2 / M_Z}$$

$$\sigma_{peak}(e^+e^- \to f) = \frac{12\pi(\hbar c)^2}{M_Z} \frac{\Gamma_e \Gamma_f}{\Gamma_Z^2} = \frac{12\pi(\hbar c)^2}{M_Z} B(Z^0 \to e^+e^-) B(Z^0 \to f\bar{f})$$



$$\Gamma_Z = \Gamma_{had} + 3\Gamma_{l^+l^-} + N_\nu \Gamma_{\nu\bar{\nu}} = 2490 Mev$$

$$\Gamma_{had} = \Gamma_u + \Gamma_d + \Gamma_c + \Gamma_s + \Gamma_b = 1741 MeV$$

$$\Gamma_{l^+l^-} = 83.9 MeV$$

$$\Gamma_{\nu\bar{\nu}} = 167.1 MeV$$

$$\Rightarrow N_\nu = 2.9841 \pm 0.0083$$

- Only 3 neutrinos with mass less than the Z mass

$\sigma(Z \to \text{hadrons})$