

Il modulo quadrato della (26) dà la probabilità di evoluzione dei tre sapori al tempo  $t$ . Per il passaggio dal sapore  $\alpha$  a quello  $\beta$  abbiamo

$$\begin{aligned}
 P(\nu_\alpha \rightarrow \nu_\beta; t) &= \left| \sum_{j=1}^3 U_{\alpha j} U_{\beta j}^* e^{-iE_j t} \right|^2 \\
 &= \sum_{j=1}^3 U_{\alpha j} U_{\beta j} \left[ \cos\left(\frac{E_j}{2}t\right) - i \sin\left(\frac{E_j}{2}t\right) \right] \sum_{k=1}^3 U_{\alpha k} U_{\beta k}^* \left[ \cos\left(\frac{E_k}{2}t\right) + i \sin\left(\frac{E_k}{2}t\right) \right] \\
 &= \sum_{j,k} U_{\alpha j} U_{\beta j}^* U_{\alpha k} U_{\beta k} \left\{ \cos\left(\frac{E_j}{2}t\right) \cos\left(\frac{E_k}{2}t\right) + \sin\left(\frac{E_j}{2}t\right) \sin\left(\frac{E_k}{2}t\right) \right. \\
 &\quad \left. + i \left[ \cos\left(\frac{E_j}{2}t\right) \sin\left(\frac{E_k}{2}t\right) - \sin\left(\frac{E_j}{2}t\right) \cos\left(\frac{E_k}{2}t\right) \right] \right\} \\
 &= \sum_{j,k} U_{\alpha j} U_{\beta j}^* U_{\alpha k} U_{\beta k} \left[ \cos\left(\frac{E_j - E_k}{2}t\right) + i \sin\left(\frac{E_k - E_j}{2}t\right) \right]
 \end{aligned}$$

$\sin(-x) = -\sin(x)$

$\sum_{j,k} U_{\alpha j} U_{\beta j}^* U_{\alpha k} U_{\beta k} \sin\left(\frac{E_k - E_j}{2}t\right) = 0$

$\cos x = 1 - 2 \sin^2 \frac{x}{2}$

$$\begin{aligned}
 P(\nu_\alpha \rightarrow \nu_\beta; t) &= \sum_{j,k} U_{\alpha j} U_{\beta j}^* U_{\alpha k} U_{\beta k} \cos\left(\frac{E_k - E_j}{2}t\right) \\
 &= \sum_{j,k} U_{\alpha j} U_{\beta j}^* U_{\alpha k} U_{\beta k} \left[ 1 - 2 \sin^2\left(\frac{E_k - E_j}{2}t\right) \right] \tag{28}
 \end{aligned}$$

nota questa è  $\sin^2(-x) = \sin^2(x)$