$$\phi_m(z) = \int d\vec{\Omega}\ \Phi(z,\mu)\, P_m(\mu) = 2\pi \int_{-1}^{1} \sum_n \frac{2n+1}{4\pi} \phi_n(z) P_n(\mu) P_m(\mu)\, d\mu$$

$$= 2\pi \sum_n \frac{2n+1}{4\pi} \phi_n(z) \frac{2}{2m+1} \delta_{m,n} = \phi_m(z)$$

Moltiplichiamo la (RN5) per $\frac{2n+1}{2} P_n(\mu)$ e integriamo $\int_{-1}^{1} d\mu$

Primo termine

$$\frac{1}{v} \int_{-1}^{1} \frac{\partial}{\partial t} \sum_n \frac{2m+1}{4\pi} \phi_m(z) P_m(\mu)\, P_n(\mu)\, d\mu\ \frac{2n+1}{2}$$

$$= \frac{1}{v} \frac{\partial}{\partial t} \sum_n \frac{2m+1}{4\pi} \phi_m(z) \underbrace{\int_{-1}^{1} P_m(\mu) P_n(\mu)\, d\mu}_{\frac{2}{2m+1}\delta_{m,n}}\ \frac{2n+1}{2}$$

$$= \frac{2n+1}{4\pi} \frac{1}{v} \frac{\partial}{\partial t} \phi_n(z)$$

Secondo termine

$$\mu \frac{\partial \Phi}{\partial z} = \mu \frac{\partial}{\partial z} \sum_m \frac{2m+1}{4\pi} \phi_m(z) P_m(\mu)$$

Sfrutto la relazione di ricorrenza

$$(2m+1)\mu P_m(\mu) = (m+1) P_{m+1}(\mu) + m P_{m-1}(\mu)$$

$$\mu \frac{\partial \Phi}{\partial z} = \frac{\partial}{\partial z} \sum_m \frac{1}{4\pi} \phi_m(z) \left[(m+1) P_{m+1}(\mu) + m P_{m-1}(\mu) \right]$$

$$\frac{2n+1}{2} \int_{-1}^{1} d\mu\ P_n(\mu)\, \mu\frac{\partial \Phi}{\partial z} = \frac{1}{4\pi} \frac{2n+1}{2} \frac{\partial}{\partial z} \sum_m \phi_m \left[(m+1) \int_{-1}^{1} P_n(\mu) P_{m+1}(\mu)\, d\mu + m \int_{-1}^{1} d\mu\, P_n(\mu) P_{m-1}(\mu) \right] =$$

→ segue