$$= \frac{1}{4\pi}\frac{2n+1}{2}\frac{\partial}{\partial z}\sum_m \phi_m(z)\left[(m+1)\frac{2}{2n+1}\delta_{n,m+1} + m\frac{2}{2n+1}\delta_{n,m-1}\right]$$

$$= \frac{1}{4\pi}\frac{2n+1}{\cancel{2}}\left[\frac{\partial \phi_{n-1}}{\partial z}\frac{\cancel{2}}{\cancel{2n+1}} n + (n+1)\frac{\cancel{2}}{\cancel{2n+1}}\frac{\partial \phi_{n+1}}{\partial z}\right]$$

$$= \frac{1}{4\pi}\left[(n+1)\frac{d\phi_{n+1}}{dz} + n\frac{d\phi_{n-1}}{dz}\right]$$

Terzo termine

$$\int \sum \Phi \frac{2n+1}{2} P_n(\mu)\,d\mu = \sum\sum_{m=0}^{\infty} \phi_m(z)\frac{2m+1}{4\pi}\int_{-1}^{1} P_m(\mu)P_n(\mu)\,d\mu\,\frac{2n+1}{2}$$

$$= \frac{1}{4\pi}(2n+1)\sum \phi_n(z)$$

Quarto termine (lo prendiamo da pag. RN12)

$$\frac{1}{2\pi}\Sigma(z)\int_0^{2\pi} d\varphi'\int_{-1}^{1} d\mu'\int_0^{2\pi} d\varphi\, F(\vec{\Omega}'\to\vec{\Omega})\,\Phi(z,\mu')$$

$$= \frac{\Sigma(z)}{2\pi}\int_0^{2\pi} d\varphi'\int_{-1}^{1} d\mu'\int_0^{2\pi} d\varphi \sum_{l_1 m_1} F_{l_1} Y^*_{l_1 m_1}(\vartheta',\varphi')\, Y_{l_1 m_1}(\vartheta,\varphi)\sum_m \frac{2m+1}{4\pi}\phi_m(z)P_m(\mu')$$

$$\int_0^{2\pi} d\varphi\, Y_{lm}(\vartheta,\varphi) = \delta_{m,0}\, 2\pi\sqrt{\frac{2l+1}{4\pi}}\, P_l(\cos\vartheta)$$

$$= \frac{\Sigma(z)}{\cancel{2\pi}}\int_{-1}^{1} d\mu' \sum_{l_1} F_{l_1}\cancel{2\pi}\sqrt{\frac{2l_1+1}{4\pi}}P_{l_1}(\mu')\; 2\pi\sqrt{\frac{2l_1+1}{4\pi}}P_{l_1}(\mu)\sum_m \frac{2m+1}{4\pi}\phi_m(z)P_m(\mu')$$

$$= \Sigma(z)\sum_{l_1} F_{l_1} 2\pi\frac{2l_1+1}{4\pi}\sum_m \frac{2m+1}{4\pi}\underbrace{\int_{-1}^{1} d\mu' P_{l_1}(\mu')P_m(\mu')}_{\frac{2}{2l_1+1}\delta_{l_1,m}}\phi_m(z)P_{l_1}(\mu)$$

$$= \Sigma(z)\sum_{l_1}\frac{2l_1+1}{4\pi}F_{l_1}\phi_{l_1}(z)P_{l_1}(\mu)$$