RN22

Dato che

$\gamma^2 = K^2 + \alpha^2 + \beta^2$ abbiamo $\gamma_{mn}^2 = K^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$ RN15

m e n dispari

Quindi $Z_{mn}(z) = C_{mn} e^{-\gamma_{mn} z}$

$$\phi(x,y,z) = \sum_{\substack{mn\\ \text{dispari}}} A_m \cos\left(\frac{m\pi}{a}x\right) B_n \cos\left(\frac{n\pi}{b}y\right) C_{mn} e^{-\gamma_{mn} z}$$

Definiamo $S_{mn} = A_m B_n C_{mn}$ e determiniamo S_{mn} dalla condizione sulla sorgente.

$$Q_0 = S\delta(z)\overbrace{}^{\delta(y)\delta(x)} = \sum_{\substack{mn\\ \text{dispari}}} S_{mn} \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

Moltiplico per $\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$ e integro nei limiti

$$S\delta(z)\int_{-\frac{a}{2}}^{a/2} dx \cos\left(\frac{m\pi}{a}x\right)\int_{-\frac{b}{2}}^{b/2} dy \cos\left(\frac{n\pi}{b}y\right)\delta(x)\delta(y)$$

$$= S_{mn}\int_{-a/2}^{a/2} dx \cos^2\left(\frac{m\pi}{a}x\right)\int_{-b/2}^{b/2}\cos^2\left(\frac{n\pi}{b}y\right)dy$$

$\int \cos(\alpha x) = \frac{1}{\alpha}\sin \alpha x + K$ $\qquad \cos^2\alpha = \frac{1}{2}(1+\cos 2\alpha)$

$$\int \cos^2(\alpha x)dx = \int \frac{1}{2}(1+\cos 2\alpha x)dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2\alpha x)dx$$

$$= \frac{x}{2} + \frac{1}{2}\frac{1}{2\alpha}\sin(2\alpha x) + K$$

$$\int \cos(x)\delta(x)\,dx = 1$$