

Reazioni nucleari attorno alla barriera coulombiana: una panoramica sperimentale.

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Reazioni nucleari attorno alla barriera coulombiana: una panoramica sperimentale.

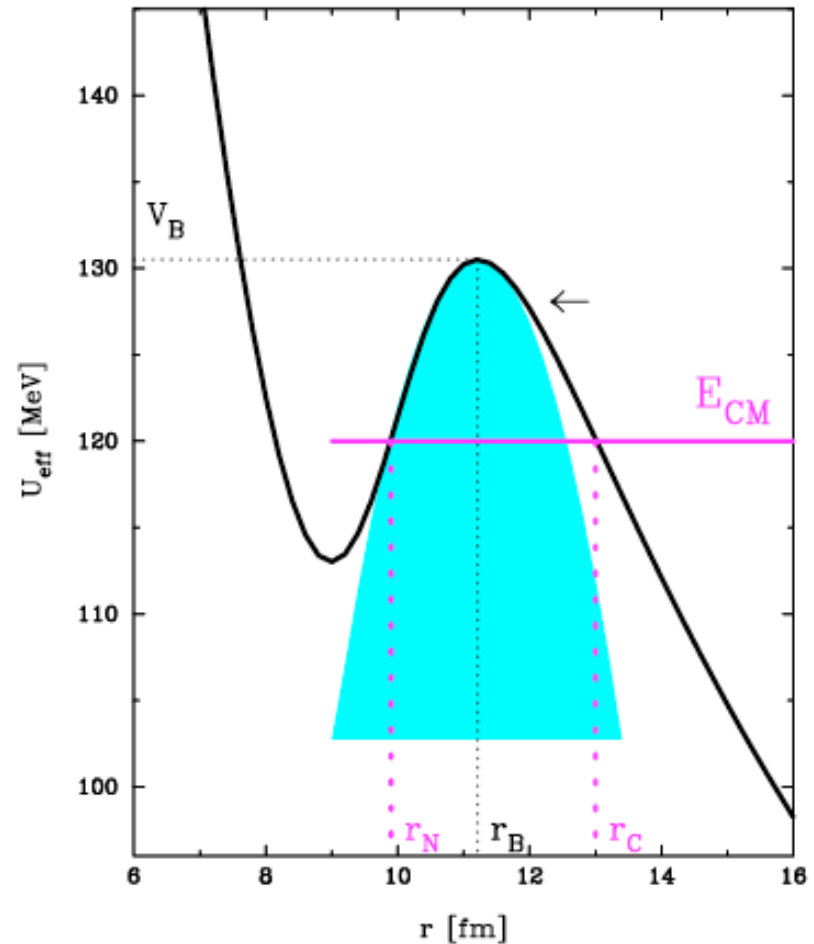
- Fusione attorno alla barriera
 - introduzione
 - risultati sperimentali e interpretazione
 - metodi sperimentali
 - questioni aperte
- Trasferimento di nucleoni
 - perché lo studio di reazioni di trasferimento multiplo di nucleoni
 - risultati sperimentali
 - tecniche di misura
 - questioni aperte e prospettive

FUSION

- Enhancement ed effetti isotopici
- Barrier Distribution
- CC calculation/ Grazing
- Il caso Ca+Zr
- Deep subbarrier fusion
- Misure di sezioni d'urto
- Questioni ancora aperte

Fusione tra ioni pesanti ad energie attorno alla barriera coulombiana

- <http://nr.v.jinr.ru/nrv/>
- Con un database (non completo) sui sistemi di cui è stata misurata la sezione d'urto di fusione, diffusione elastica etc.
- Alcuni elementi di teoria e la possibilità di calcolare la sezione d'urto di fusione nell'ambito del formalismo in CC

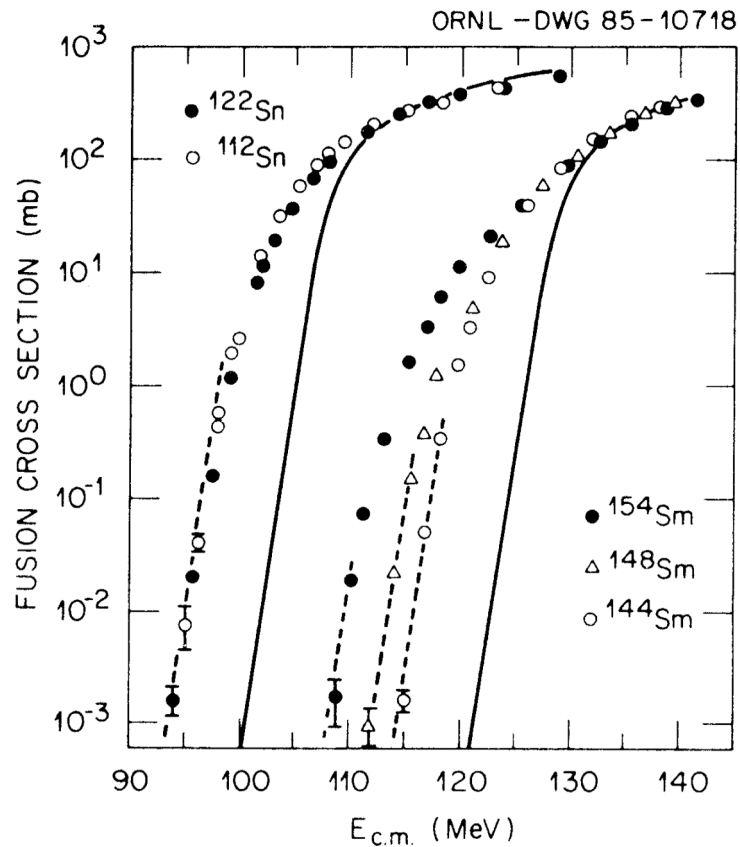


Che cosa caratterizza i sistemi di massa intermedia?

- La fusione tra ioni pesanti ad energie prossime e minori della barriera è influenzata da effetti di accoppiamento del moto relativo con i gradi di libertà intrinseci del sistema.
- I sistemi di massa intermedia sono i più adatti per mettere in evidenza tali effetti per la presenza di canali inelastici di natura collettiva a basse energie di eccitazione (l'accoppiamento di canali inelastici è $\propto Z_t Z_p$) e perché il Nucleo Composto che si forma nel processo di fusione si diseccita quasi esclusivamente per evaporazione di particelle leggere. La sezione d'urto di fusione-evaporazione si può quindi assimilare alla sezione d'urto di fusione.

La sezione d'urto di fusione dipende dalla struttura dei nuclei collidenti e...

W.Reisdorf et al., NPA 438 (1985) 212



$^{40}\text{Ar} + ^{112,122}\text{Sn}$

→ very similar x-sections

$^{40}\text{Ar} + ^{144,148,154}\text{Sm}$

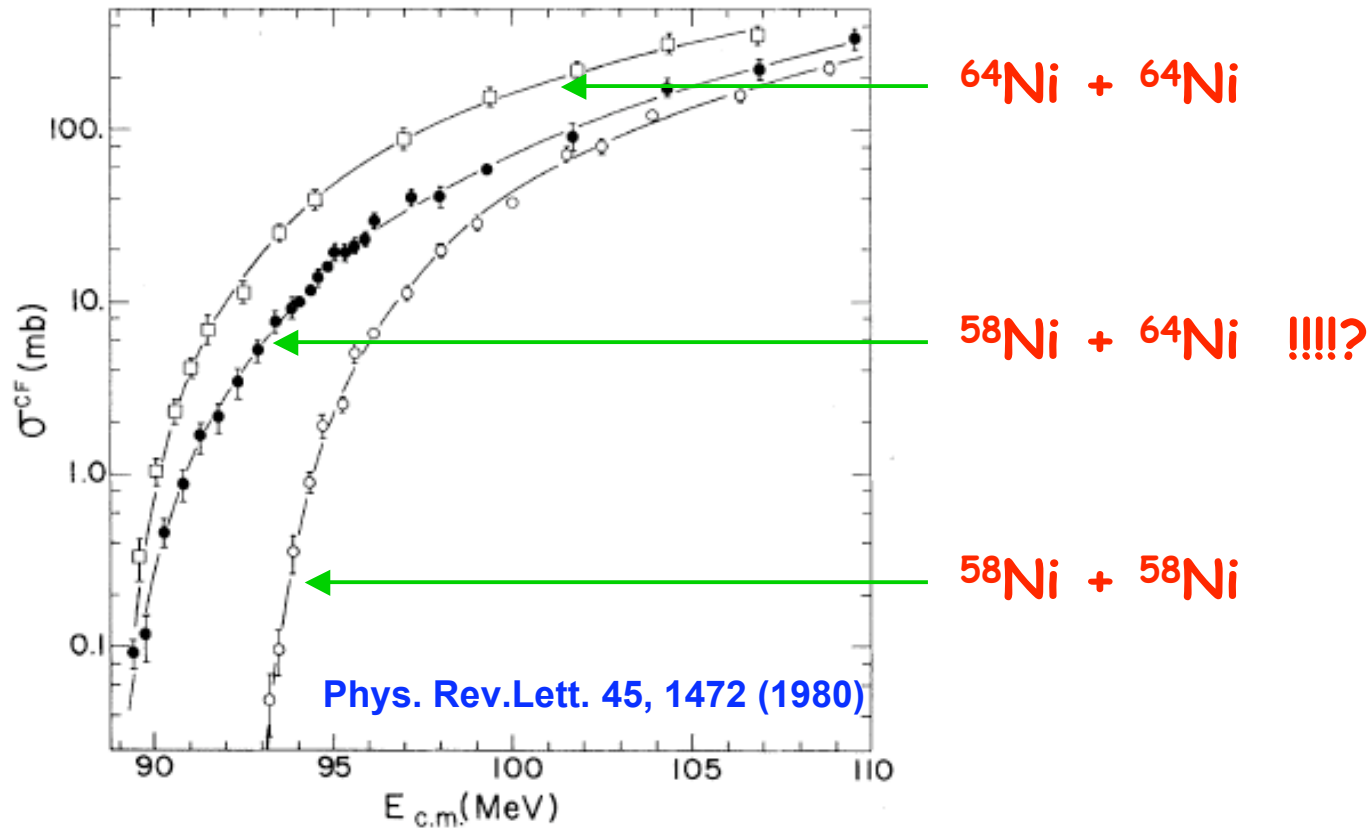
→ very different x-sections

...anche dall'accoppiamento di canali di trasferimento(?)

Dynamic Influence of Valence Neutrons upon the Complete Fusion of Massive Nuclei

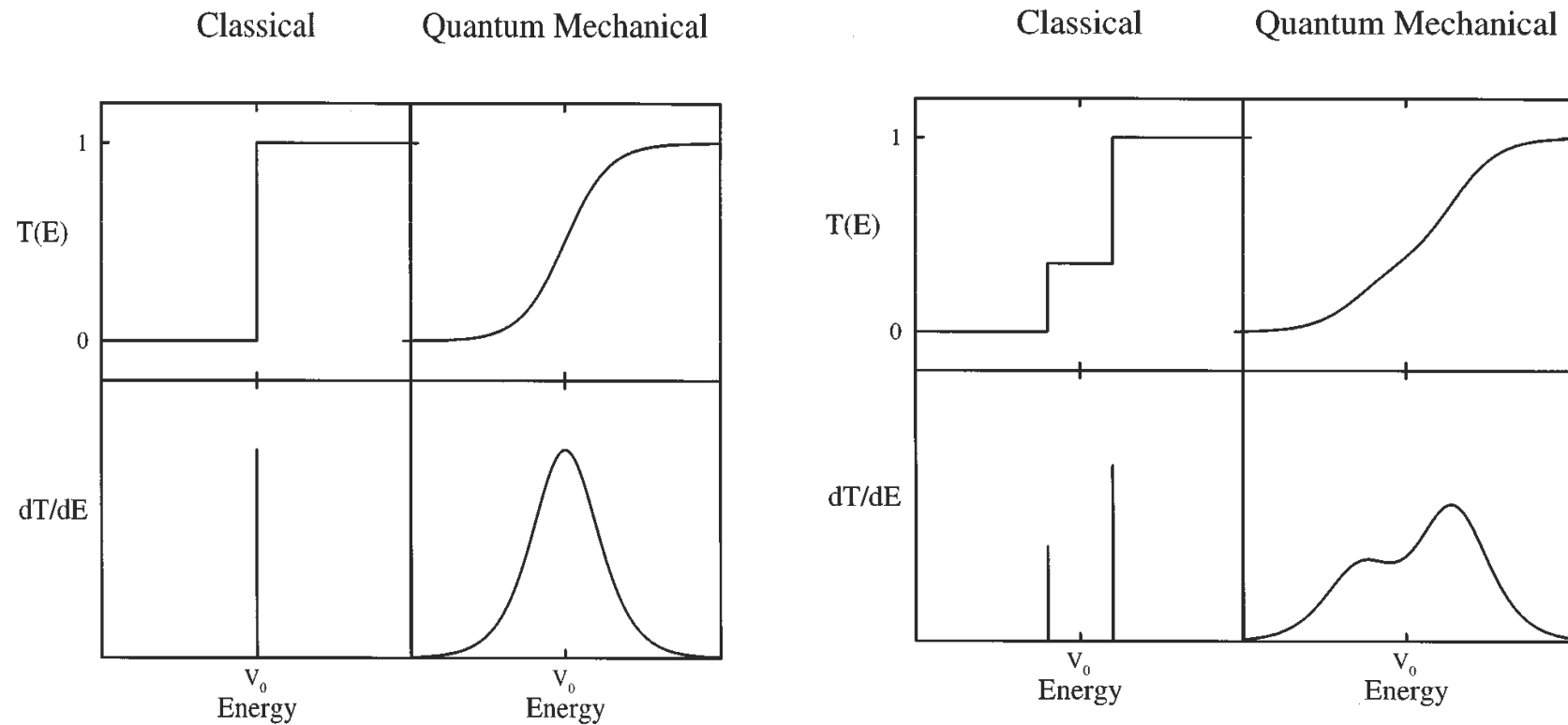
M. Beckerman, M. Salomaa, A. Sperduto,^(a) H. Enge, J. Ball, A. DiRienzo, S. Gazes,
Yan Chen,^(b) J. D. Molitoris, and Mao Nai-feng^(b)

Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139



Distribuzione di barriera

- In presenza degli accoppiamenti a gradi di liberta` intrinseci la barriera V_b non è piu` unica ma si deve parlare di distribuzione di barriere (Esbensen 1981/ Rowley et al. PLB 254 (1991) 25)



A. B. Balantekin and N. Takigawa: Quantum tunneling in nuclear fusion. Rev.Mod.Phys. 70 (1998)77

Accoppiamento a canali inelastici

$$H = H_k + V_o(r) + H_o(\xi) + V_{\text{int}}(\vec{r}, \xi)$$

$$T_l(E) \cong T_o \left(E - \frac{l(l+1)}{2\mu R^2(E)} \right)$$

$$\sigma_{\text{fus}} = \sum_l \sigma_l = \pi \hat{\lambda}^2 \sum_{l=0}^{\infty} (2l+1) T_l$$

$R(E)$ varia debolmente con l'energia

$R(E) \approx R_o$ raggio della barriera per $l = 0$

nell'ipotesi che molti valori di l siano coinvolti nella reazione di fusione

la somma sulle onde parziali si puo` approssimare con l'integrale

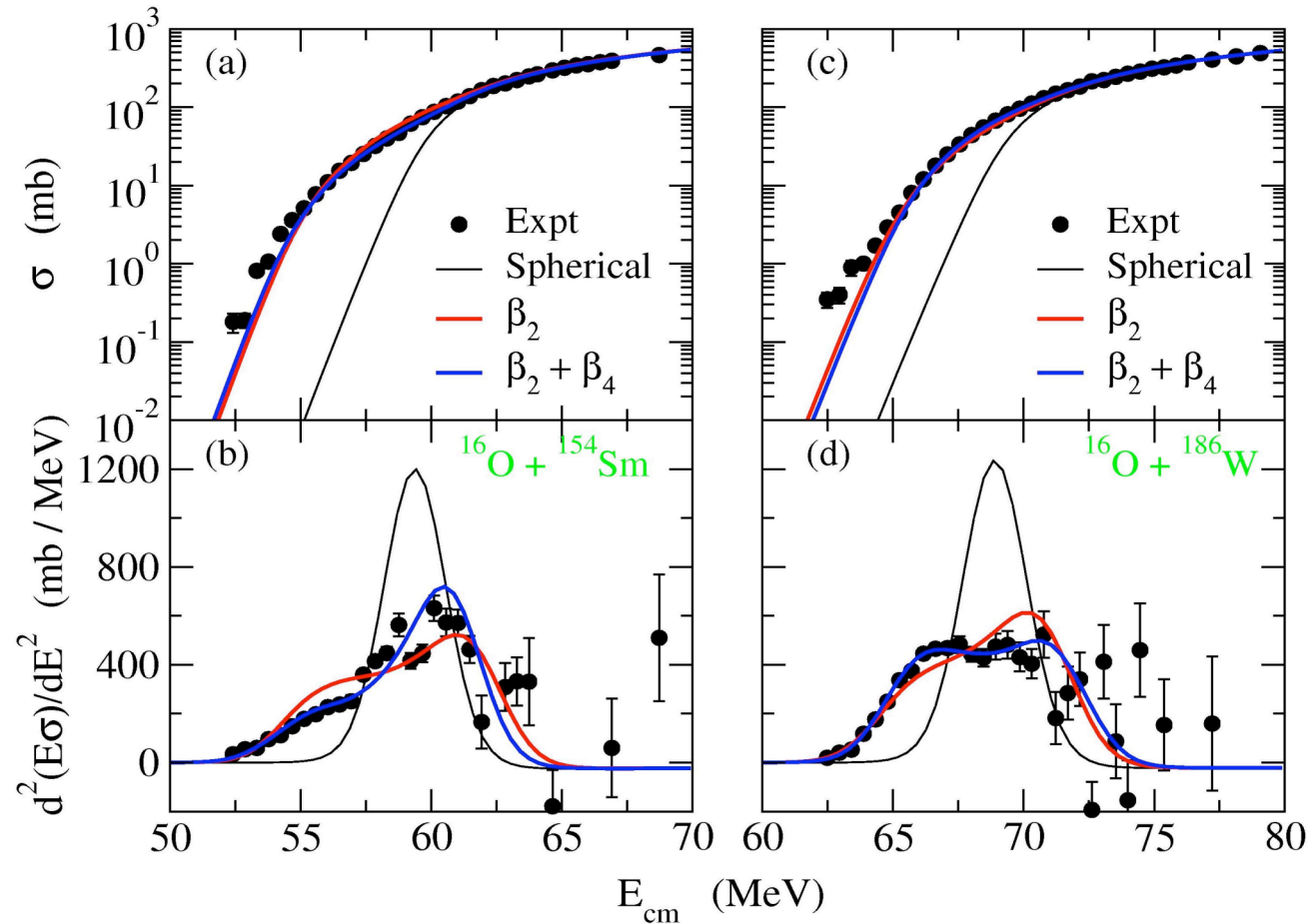
$$\sigma(E) = \sum_{l=0}^{\infty} \sigma_l(E) \rightarrow E\sigma(E) = \pi R^2(E) \int_{-\infty}^E dE' T_o(E') \quad d(E\sigma(E))/dE \propto T_o(E)$$

$$\frac{dT_o(E)}{dE} \cong \frac{1}{\pi R^2(E)} \frac{d^2}{dE^2} [E\sigma(E)] + O\left(\frac{dR}{dE}\right)$$

poiche` $R(E)$ varia debolmente con l'energia possiamo trascurare il secondo termine

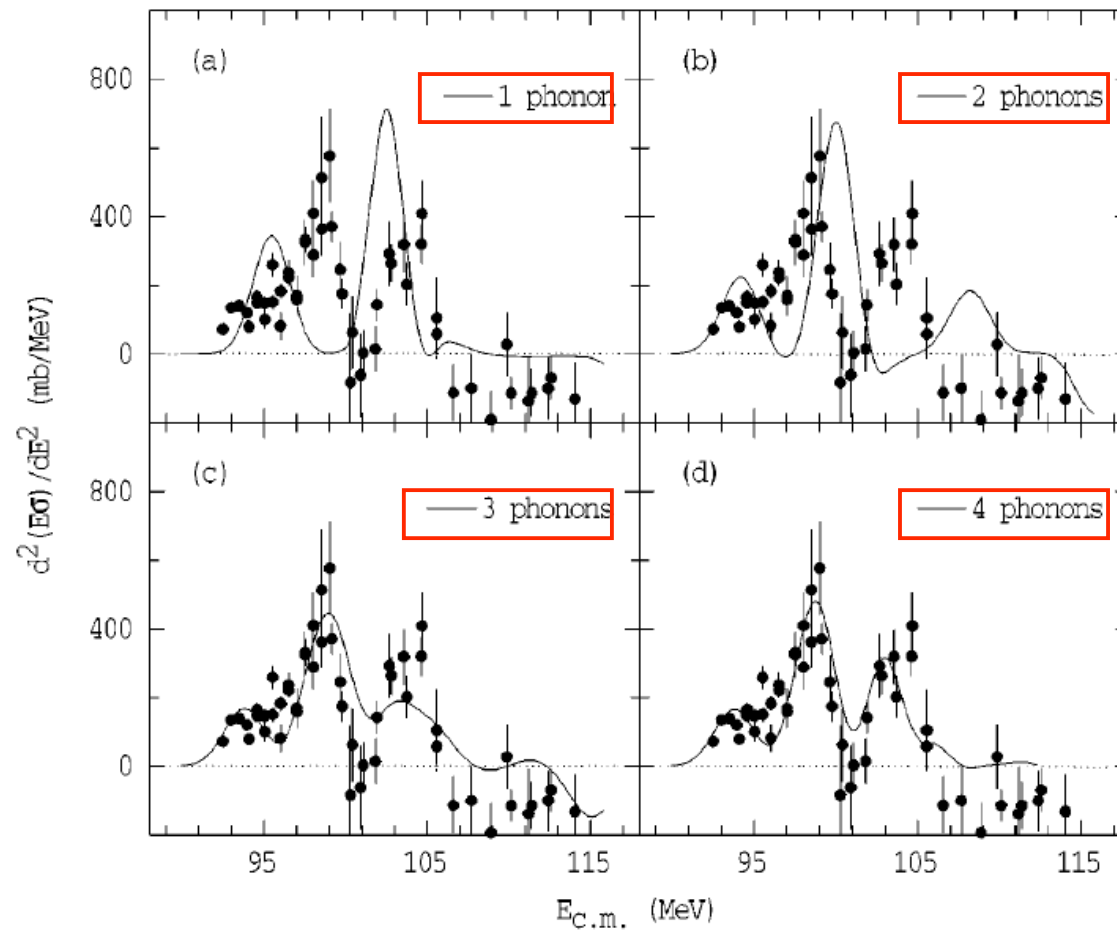
Sperimentalmente è possibile risalire alla distribuzione di barriere ?

Alcuni esempi classici: la rappresentazione della funzione di eccitazione in termini di distribuzione di barriere è sensibile alla deformazione nucleare statica ...



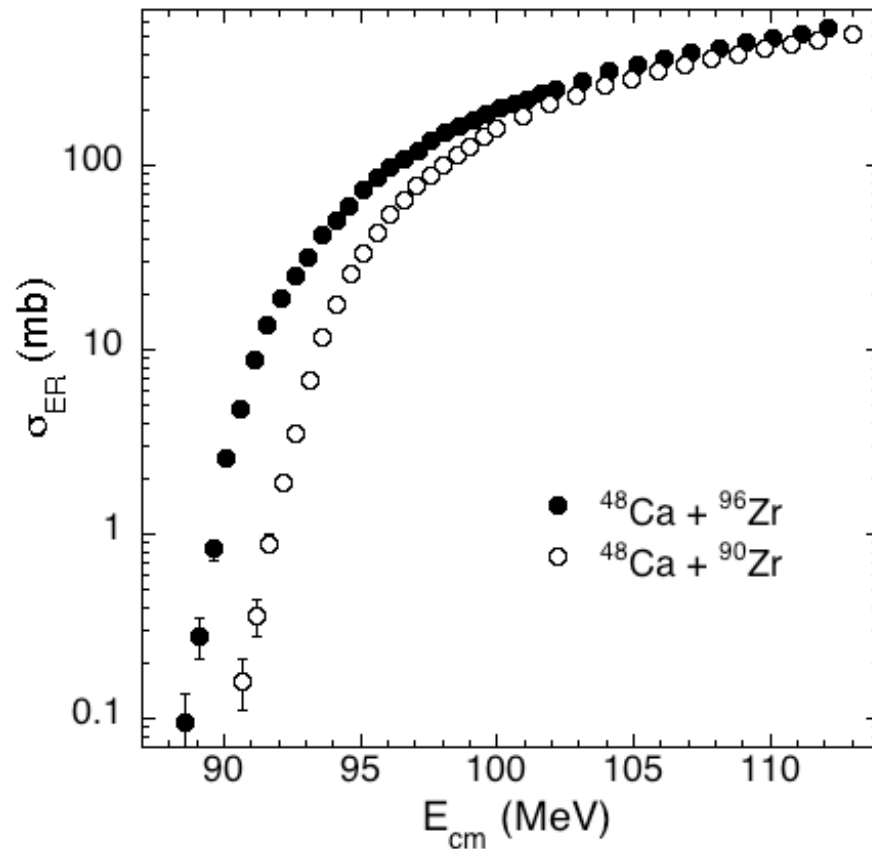
J. Leigh et al., Phys. Rev. C 52, 3151 (1995)

... e all'accoppiamento di modi vibrazionali di superficie in termini di multi-phononi come risulta dallo studio sperimentale del sistema $^{58}\text{Ni}+^{60}\text{Ni}$

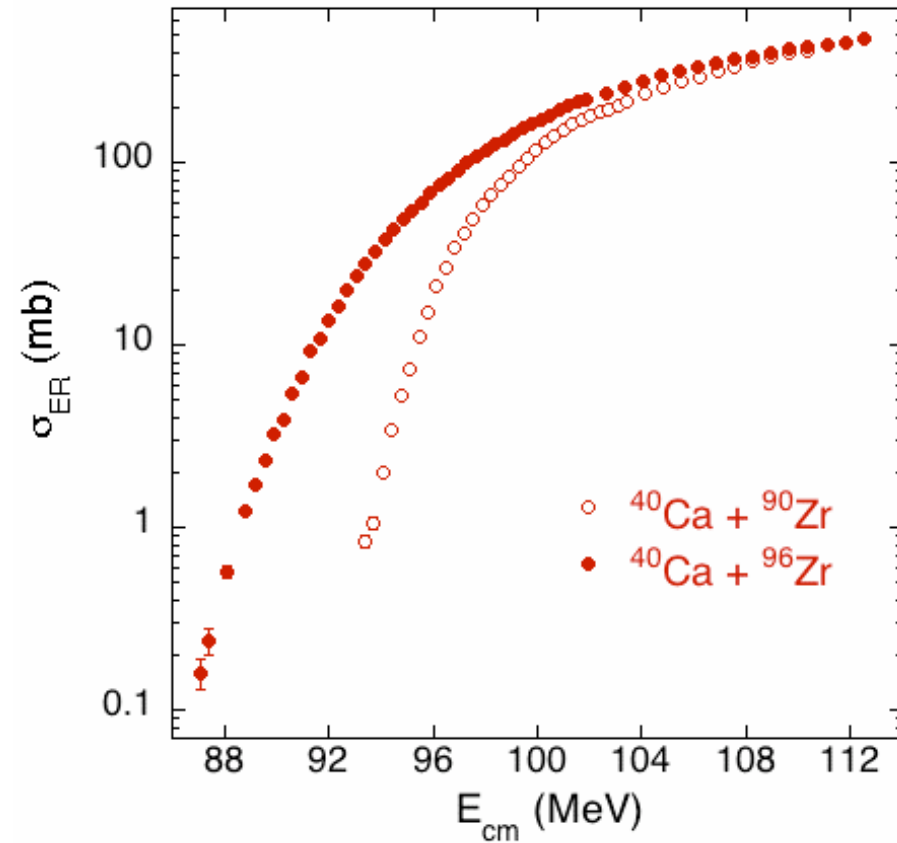


Il caso Ca+Zr

Sezione d'urto di Fusion-Evaporation per $^{40,48}\text{Ca} + ^{90,94,96}\text{Zr}$



A.M.Stefanini et al., PR C, in press



H.Timmers et al., NP A 633, 421 (1998)

I sistemi $^{40,48}\text{Ca}+^{90,96}\text{Zr}$ sono interessanti per un certo numero di motivi:

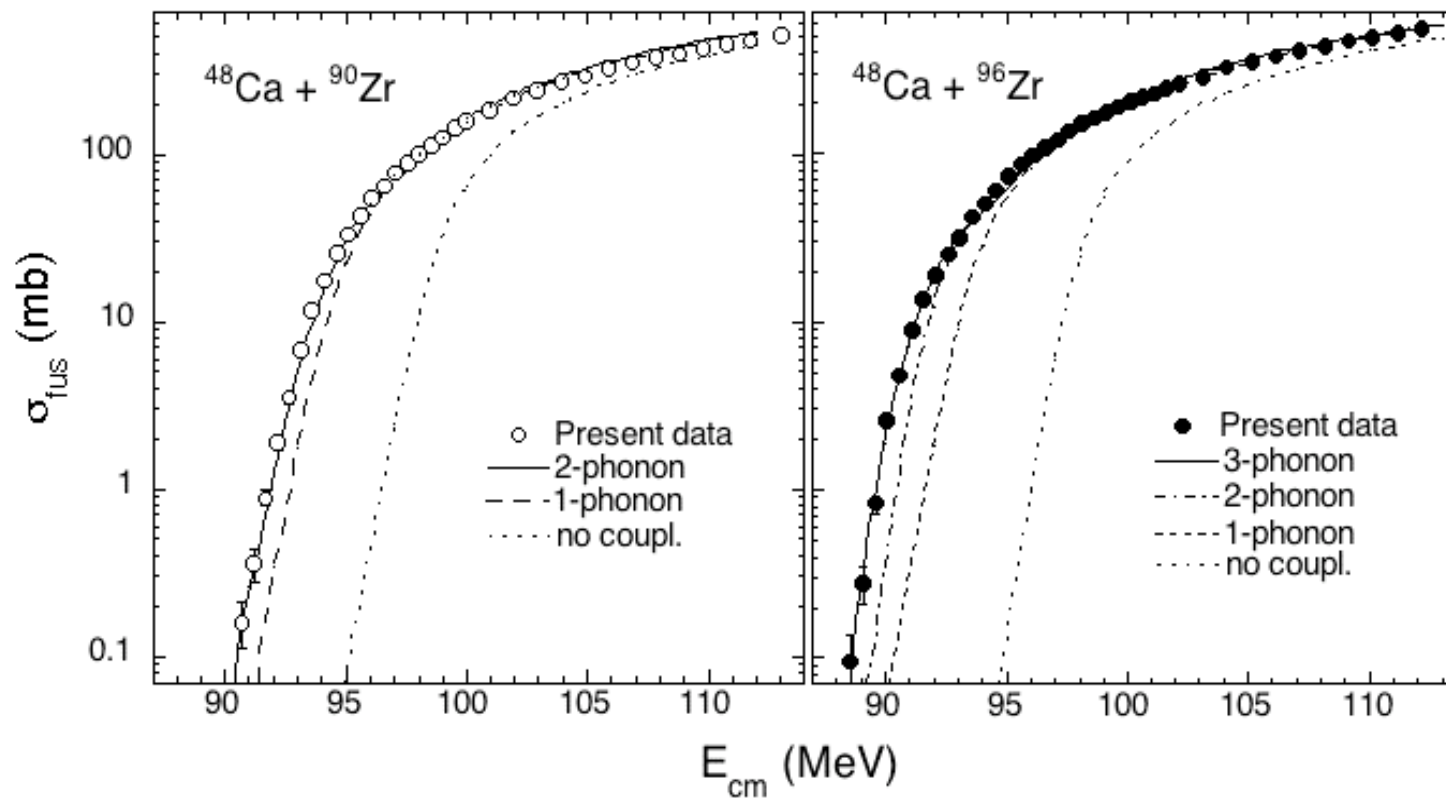
- ^{40}Ca e ^{48}Ca sono doppio magici, i loro stati 2^+ e 3^- hanno un'elevata energia di eccitazione (tutti attorno a 4MeV), lo stato di ottupolo del ^{40}Ca è il solo che rappresenti una forte effetto di accoppiamento

-La vibrazione di quadrupolo di $^{90,96}\text{Zr}$ sono entrambe deboli ed hanno nei due isotopi energie confrontabili

-lo stato di ottupolo di ^{96}Zr è invece significativamente più forte e ad una energia di eccitazione inferiore rispetto al caso ^{90}Zr

- $Q > 0$ per canali di trasferimento di neutroni (pick-up di neutroni) solo per il sistema $^{40}\text{Ca}+^{96}\text{Zr}$

Sezioni d'urto di fusione per $^{48}\text{Ca} + ^{90,96}\text{Zr}$: dati e calcoli



System	V_o (MeV)	r_o (fm)	a (fm)	V_b (MeV)	R_b (fm)	$\hbar\omega$ (MeV)
$^{48}\text{Ca} + ^{90}\text{Zr}$	73.7	1.18	0.68	96.9	11.08	3.58
	113.9	1.12	0.68	98.3	10.95	3.76
$^{48}\text{Ca} + ^{96}\text{Zr}$	73.3	1.18	0.68	95.9	11.21	3.51
	127.8	1.05	0.85	97.5	10.82	3.25

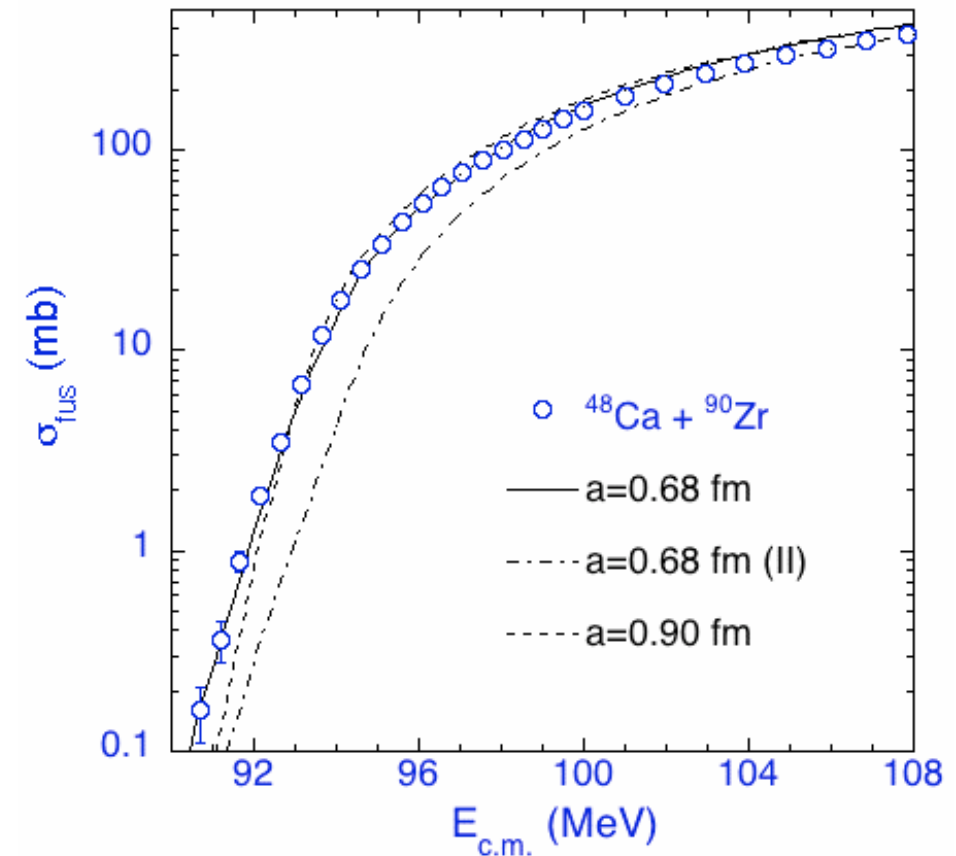
Akyuz-Winther potential

CC potential

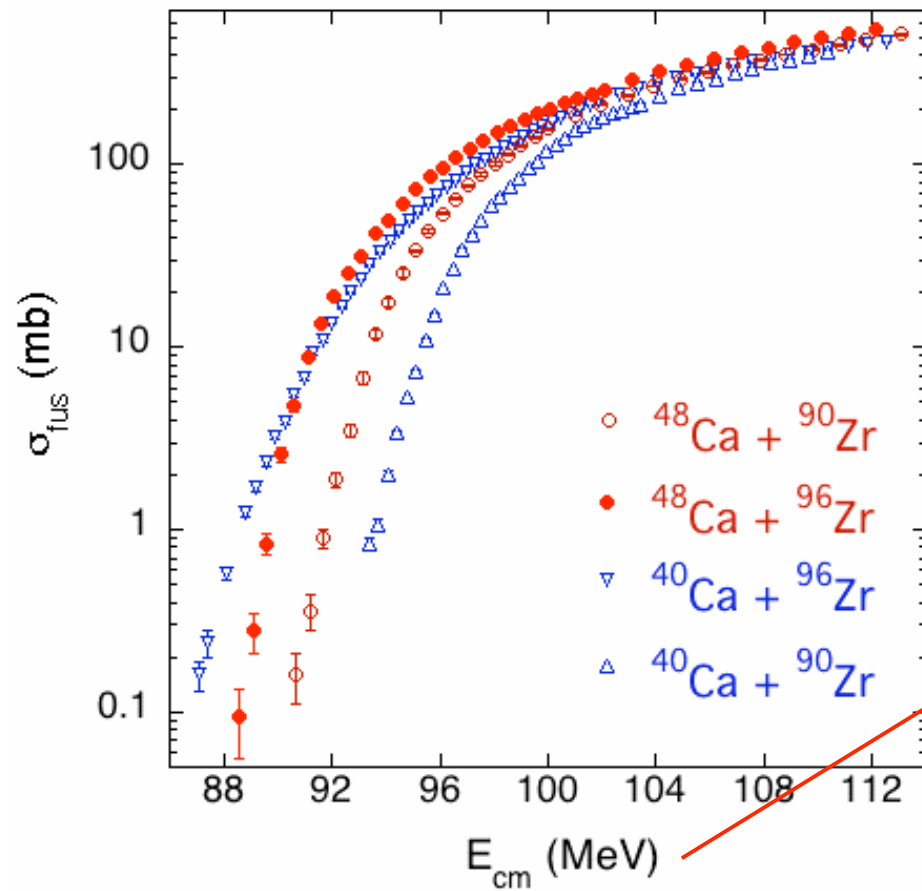
Sensitivity of the calculated x-sects. to the diffuseness parameter of the Woods-Saxon potential

$a = 0.68$ fm gives the correct slope below the barrier, but we overpredict somewhat the data at high energies

$a = 0.90$ fm (taken as a representative value of large diffuseness) gives an exc. fct. too steep below the barrier



Se confrontiamo i quattro sistemi $^{40,48}\text{Ca} + ^{90,96}\text{Zr}$...

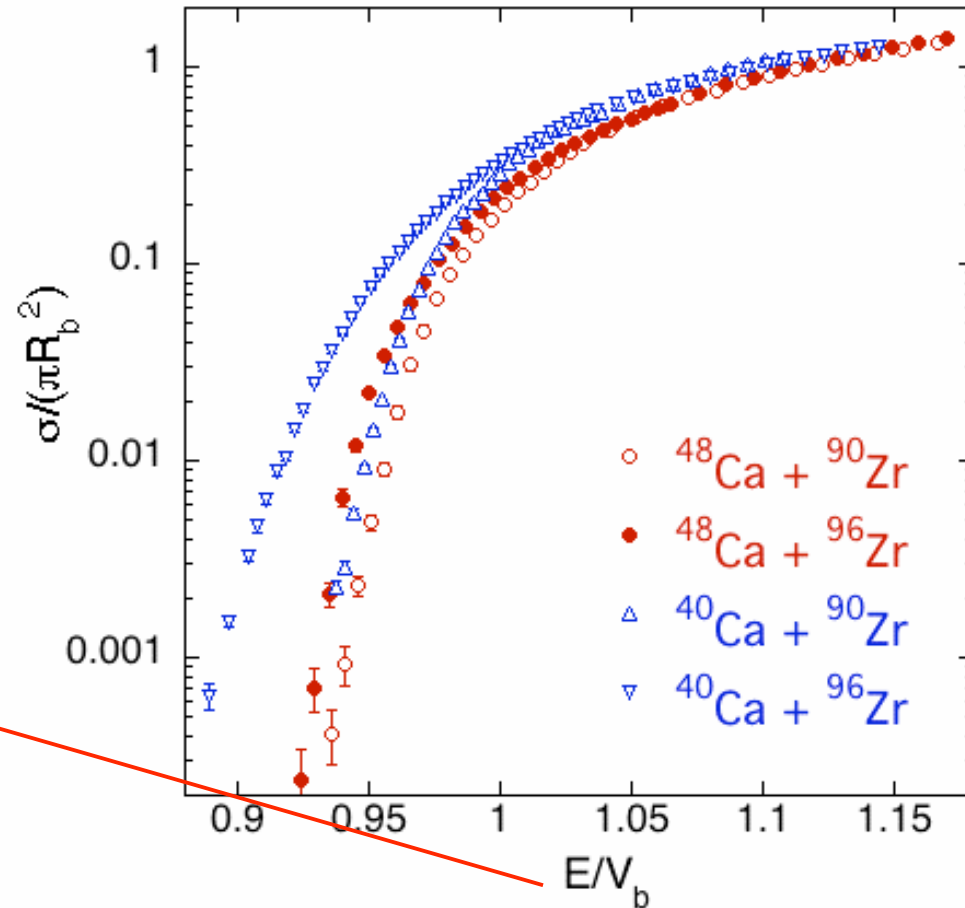


(the energy scale is in MeV)

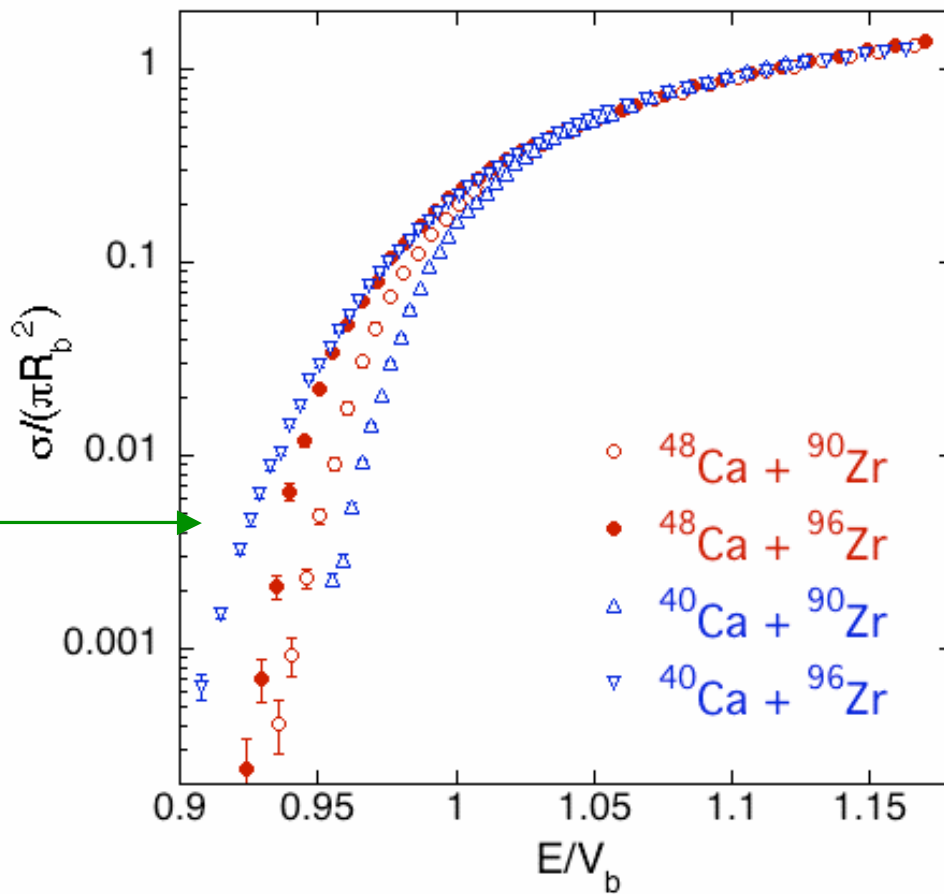
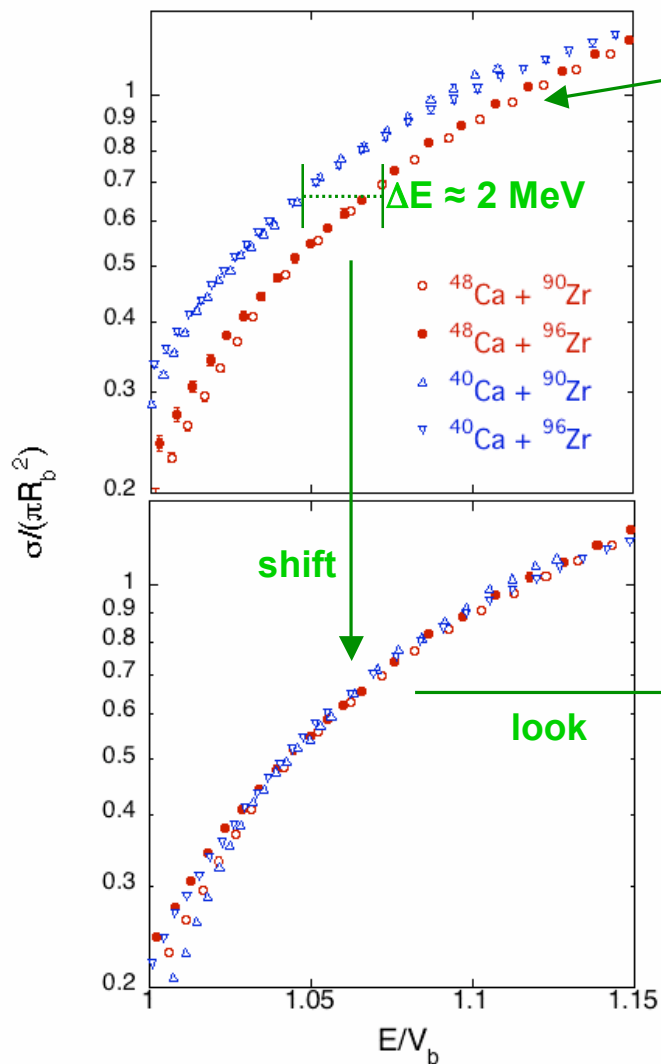
... occorre tener presente che i quattro sistemi $^{40,48}\text{Ca} + ^{90,96}\text{Zr}$ hanno barriere coulombiane diverse e quindi in una scala "ridotta" si ottiene:....

	V_b A.W.
○	96.9
●	95.9
△	99.6
▽	98.3

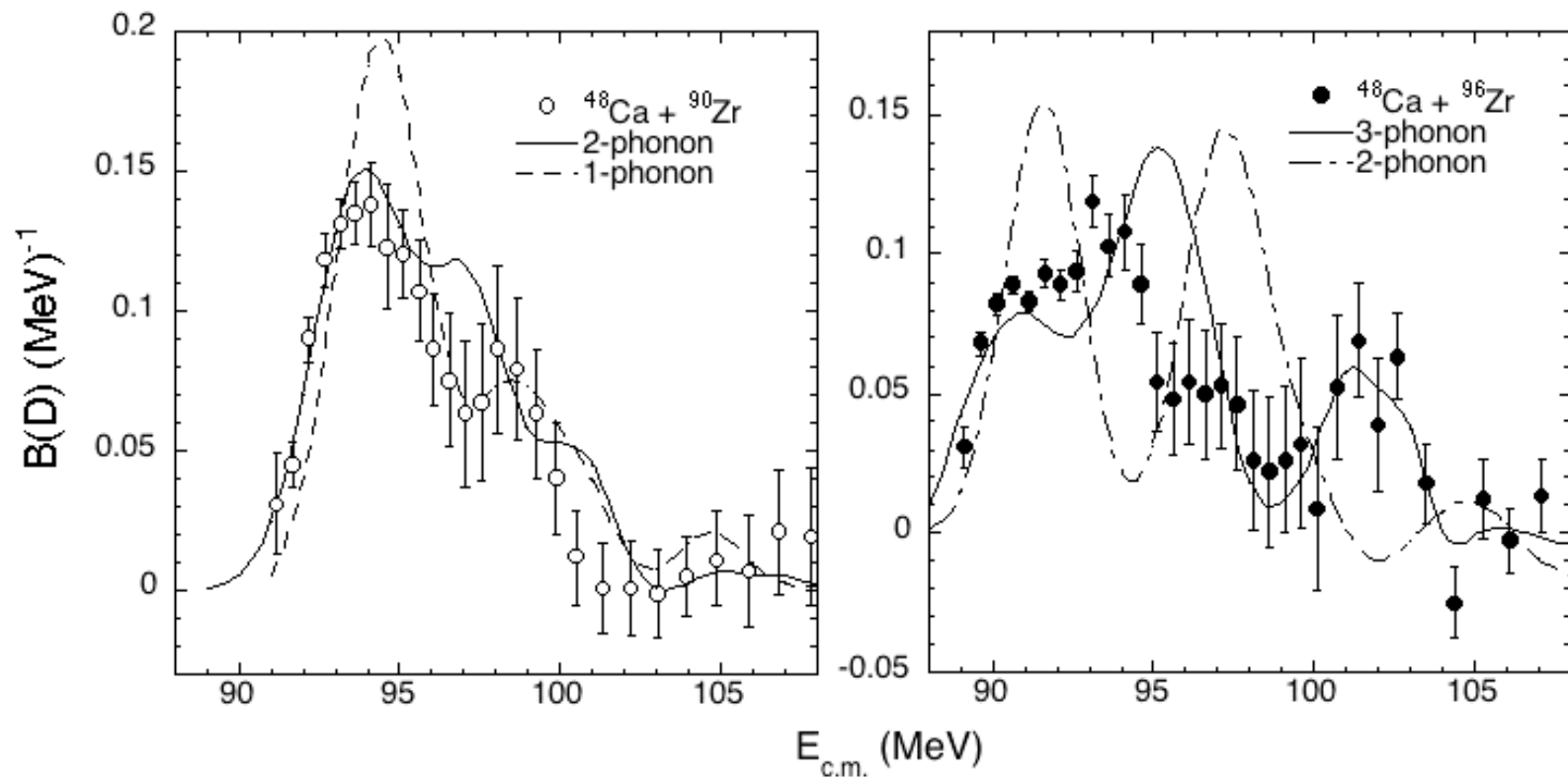
but ... what is V_b ?



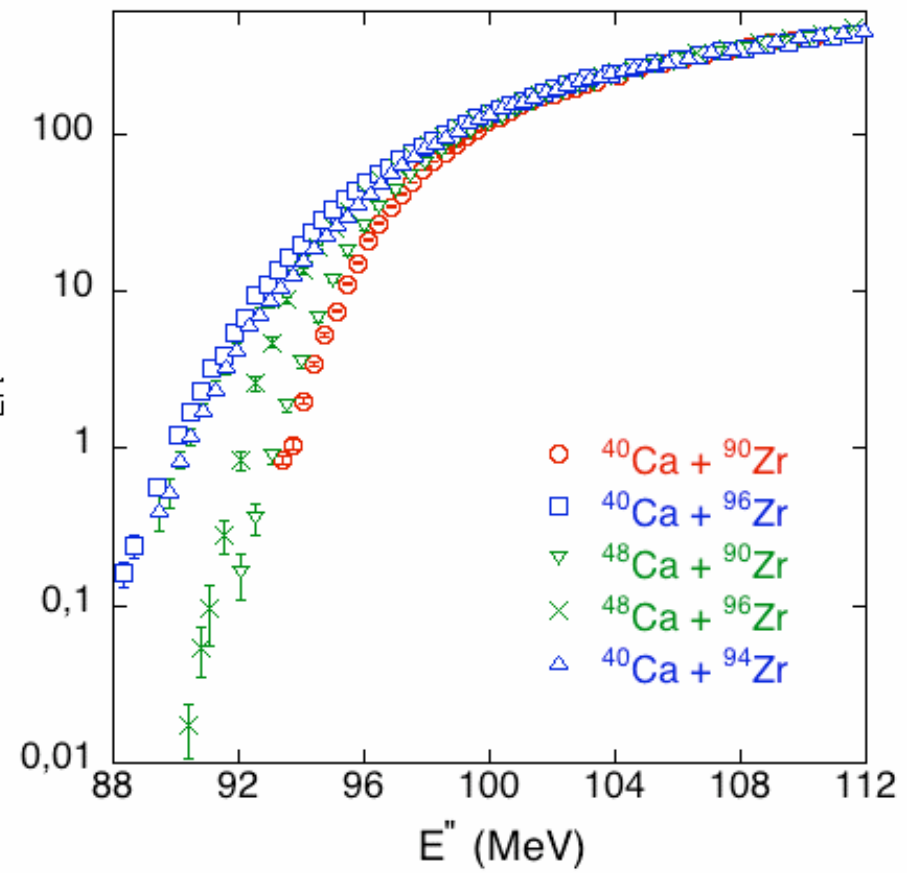
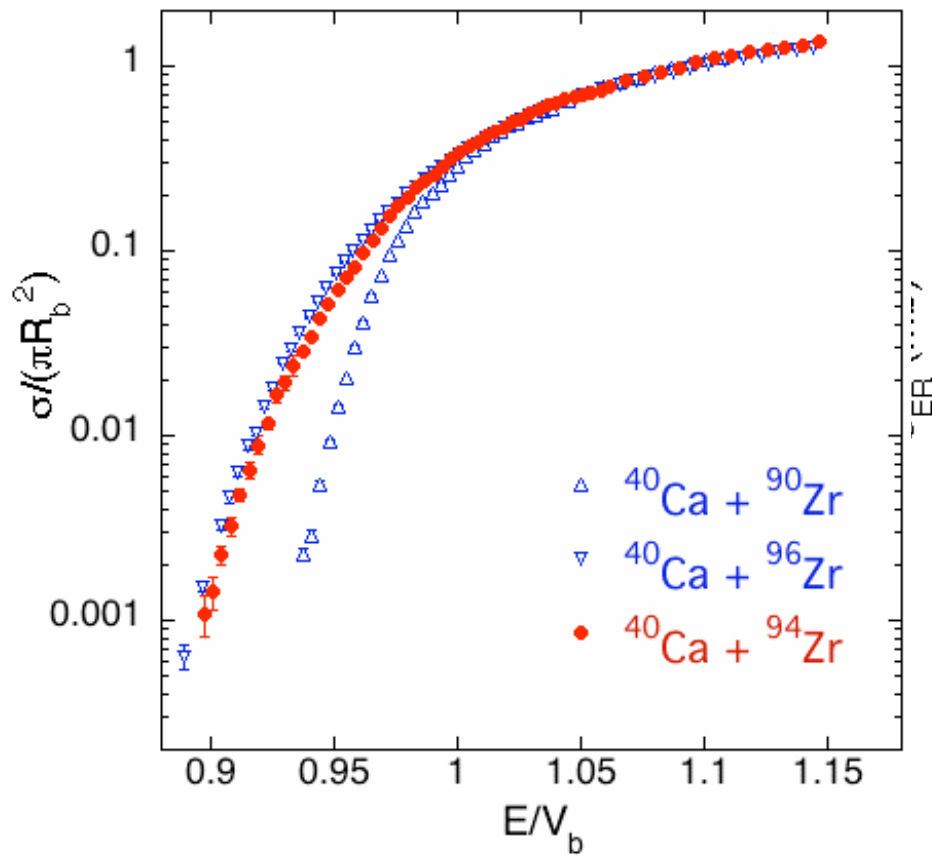
Using the A.W. potential to estimate the barrier height does not lead to a correct normalization of the excitation functions above the barrier



Fusion barrier distributions of $^{48}\text{Ca} + ^{90,96}\text{Zr}$



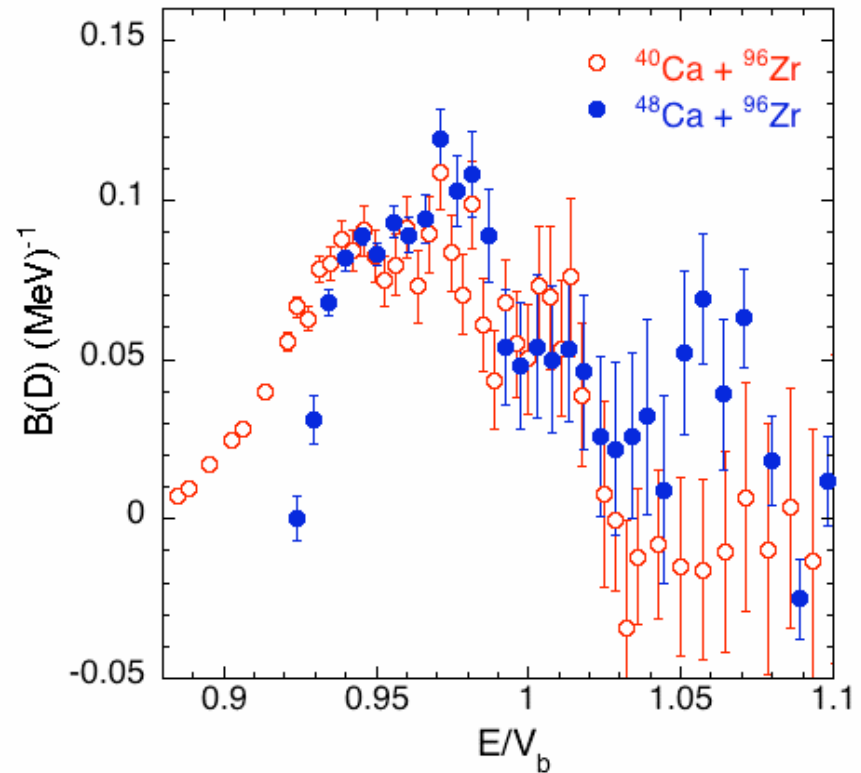
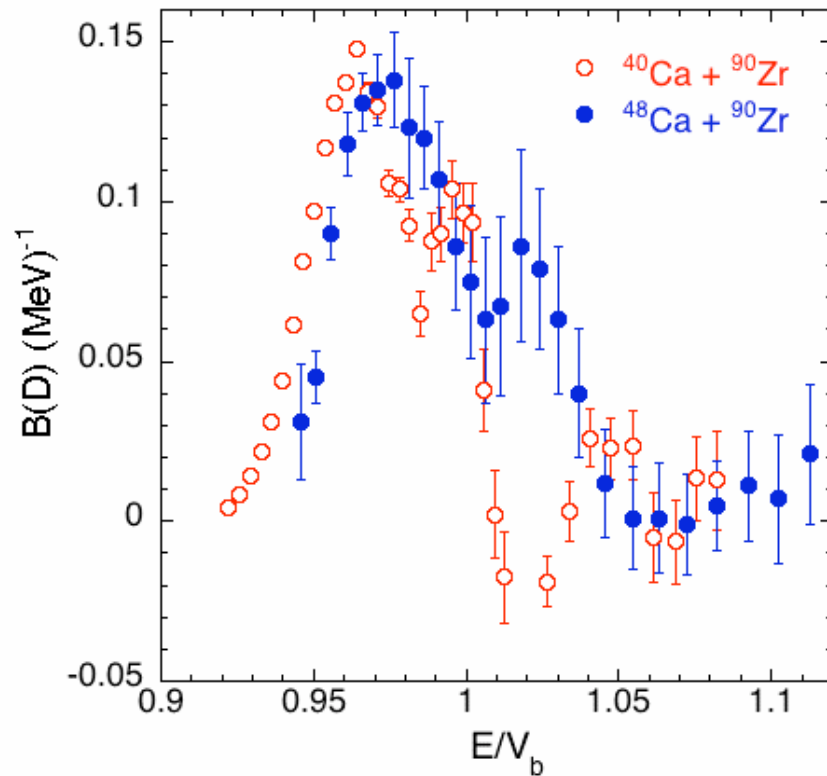
Confrontiamo con $^{40}\text{Ca} + ^{94}\text{Zr}$...



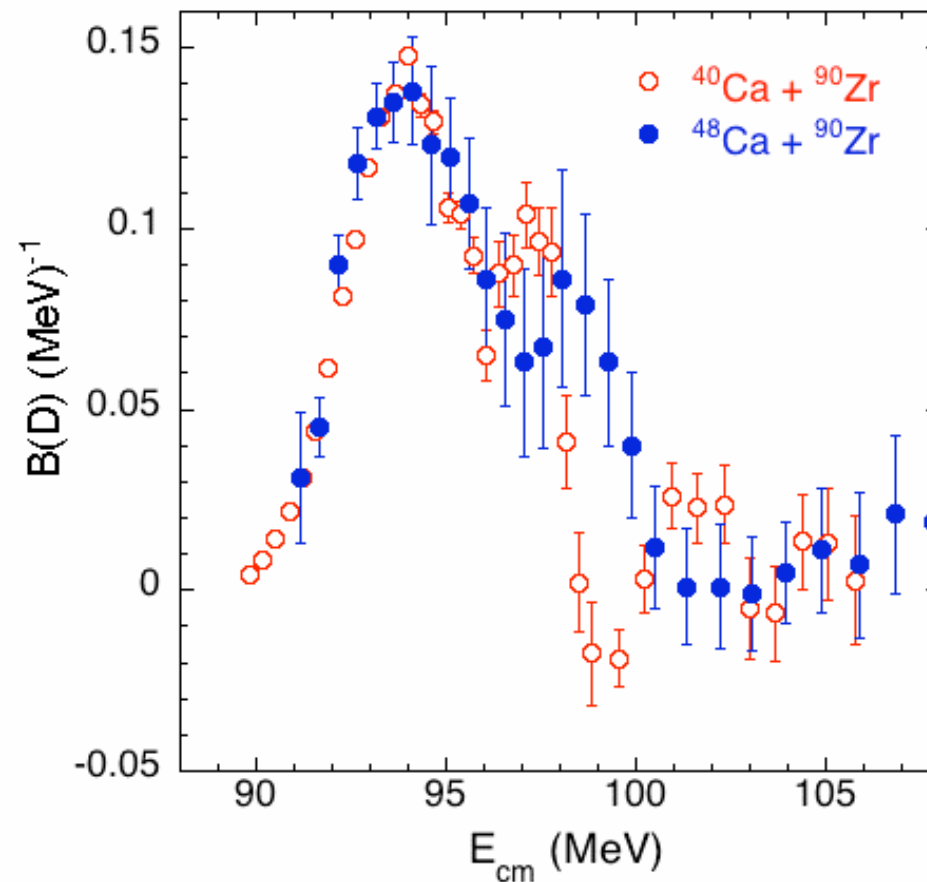
Ground state Q-values (in MeV) for neutron pick-up channels in $^{40,48}\text{Ca} + ^{90,94,96}\text{Zr}$

	40+90	40+94	40+96	48+90	48+96
1n	-3.61	+0.14	+0.51	-6.83	-2.71
2n	-1.44	+4.89	+5.53	-9.79	-2.82
3n	-5.86	+4.19	+5.24	-17.70	-6.60
4n	-4.17	+8.12	+9.64	-	-
5n	-9.65	+3.57	+8.42		
6n	-9.05	+4.65	+11.6		

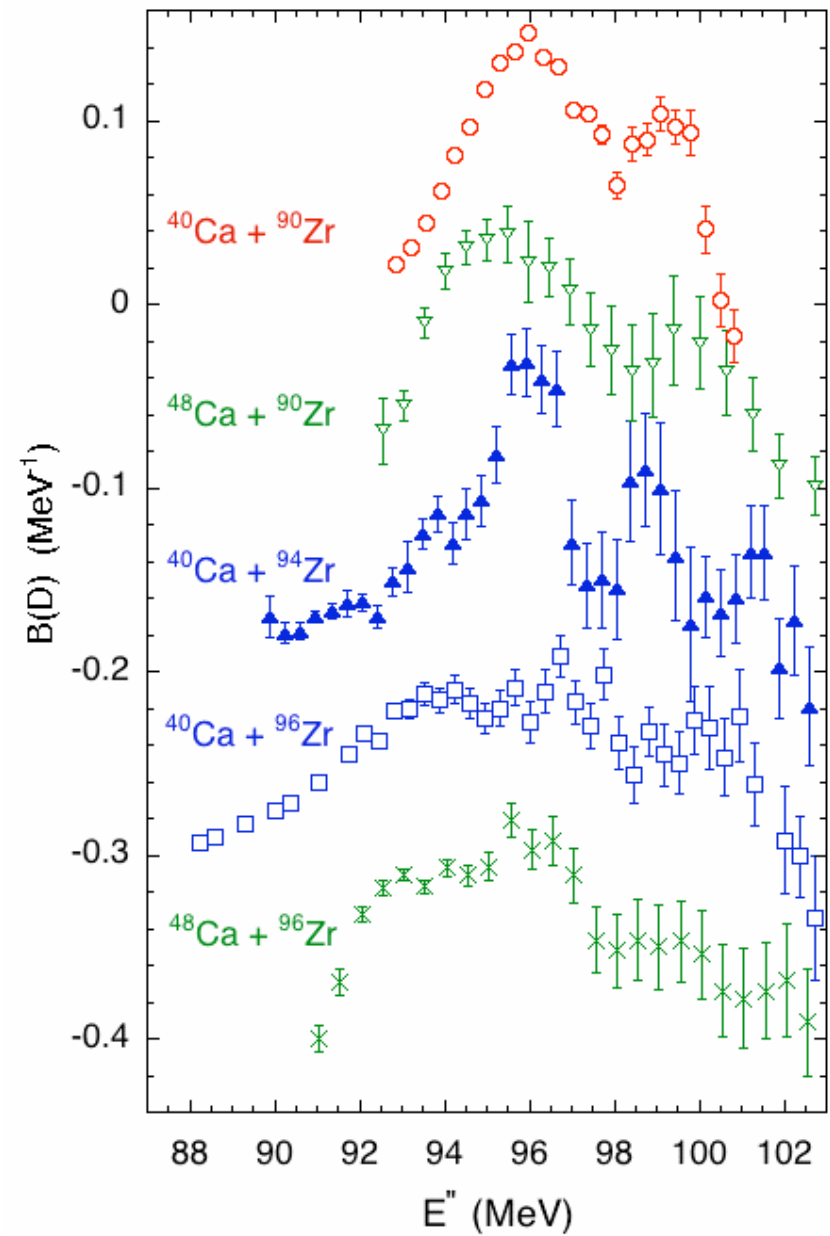
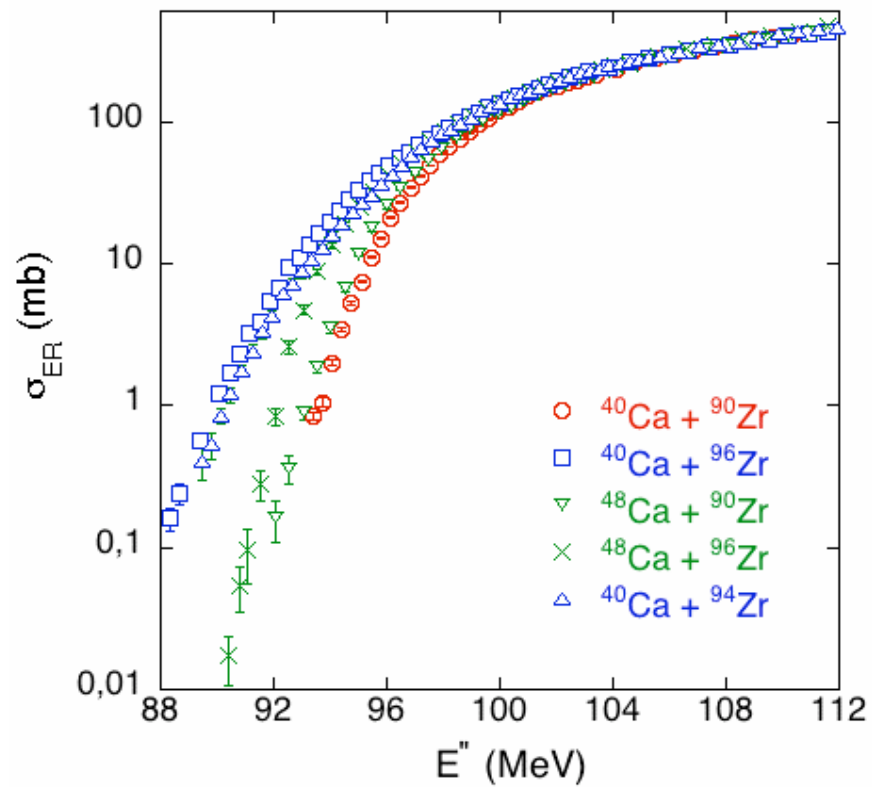
La distribuzione di barriere dei quattro sistemi $^{40,48}\text{Ca} + ^{90,96}\text{Zr}$



Per i due sistemi le distribuzioni di barriere sono molto simili e la separazione tra i due picchi ben visibili è maggiore nel caso di $^{48}\text{Ca} + ^{90}\text{Zr}$

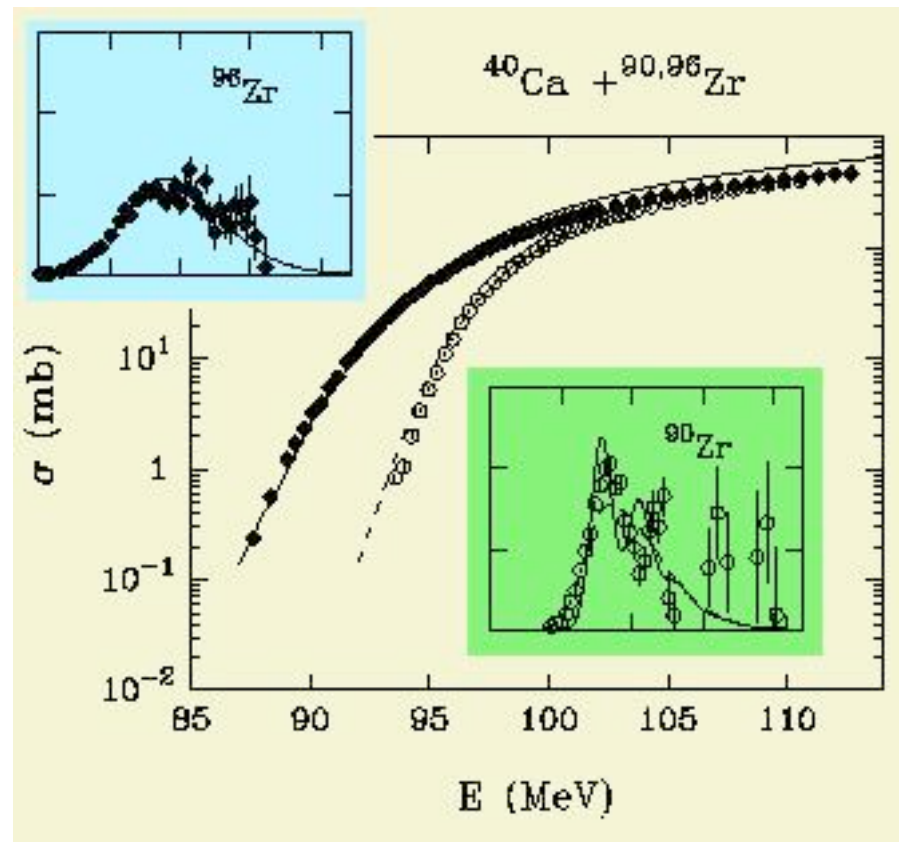


Per completezza anche con $^{40}\text{Ca}+^{94}\text{Zr}$

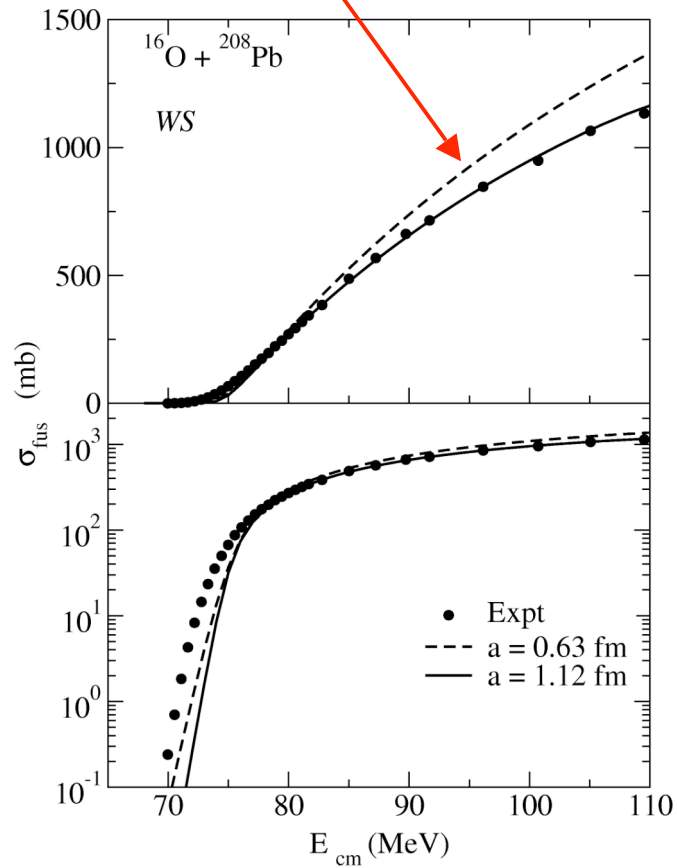


Funzione di eccitazione e distribuzione di barriera: un approccio semiclassico.

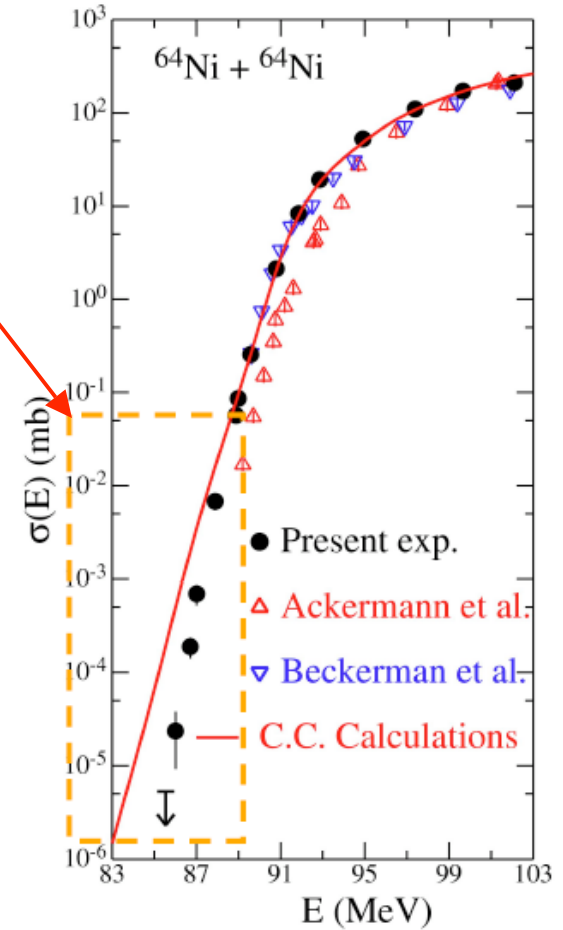
G. Pollarolo and Aage Winther PRC 62, 054611 (2000)



Sopra barriera ... e a basse energie

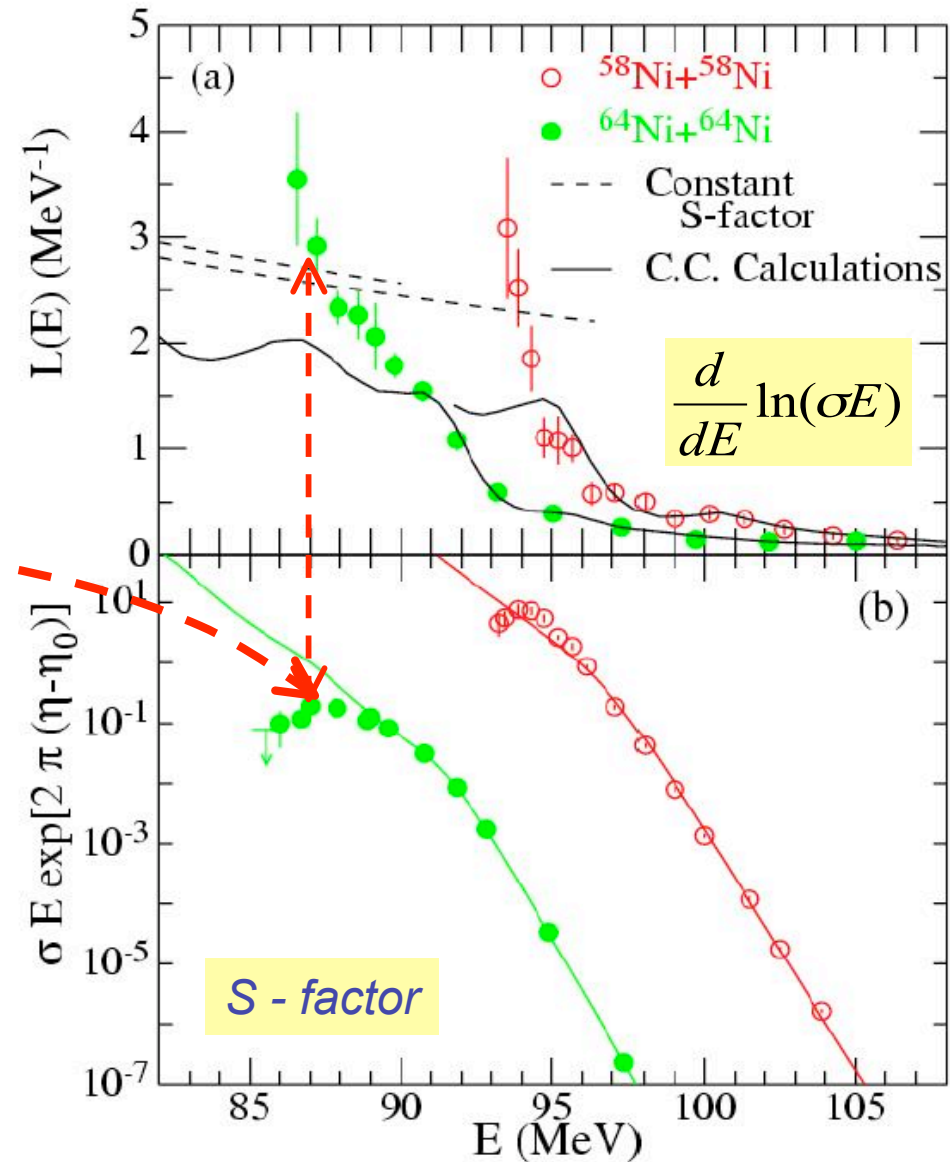
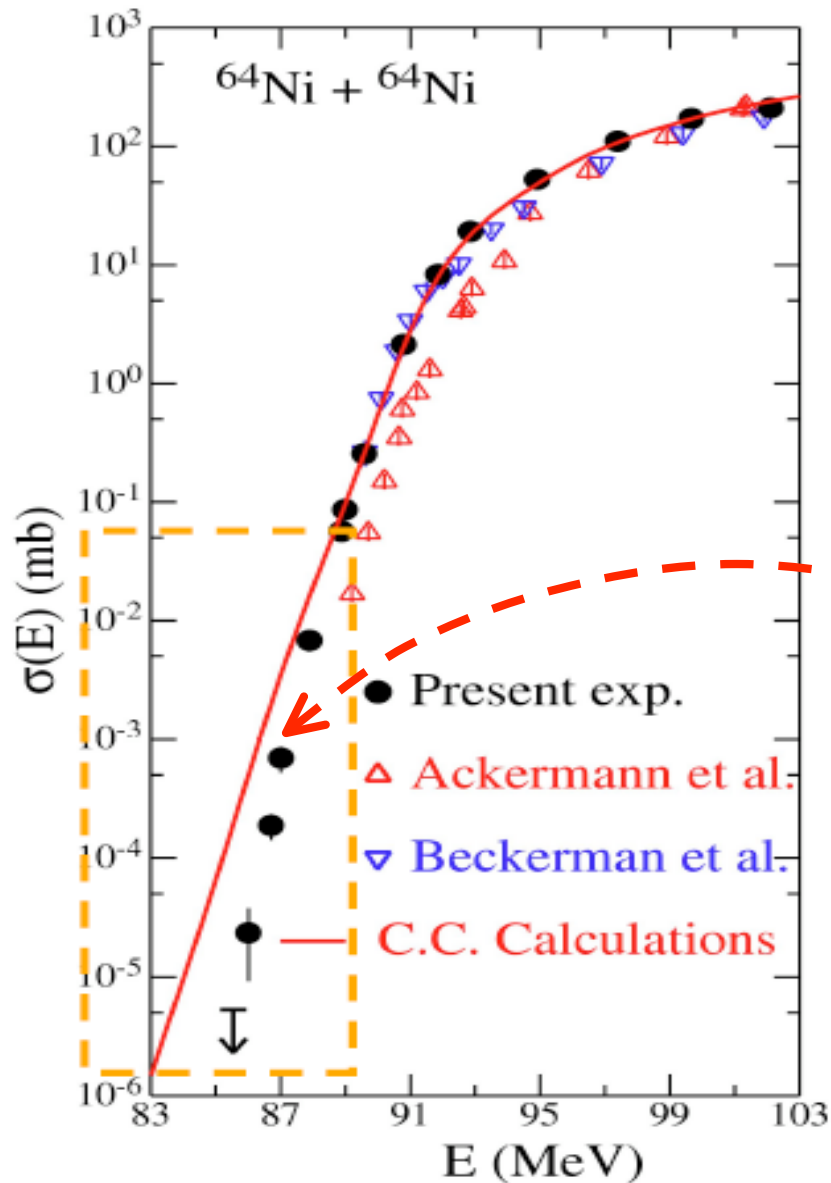


J.O. Newton et al., PLB 586 (2004) 219 and refs. therein



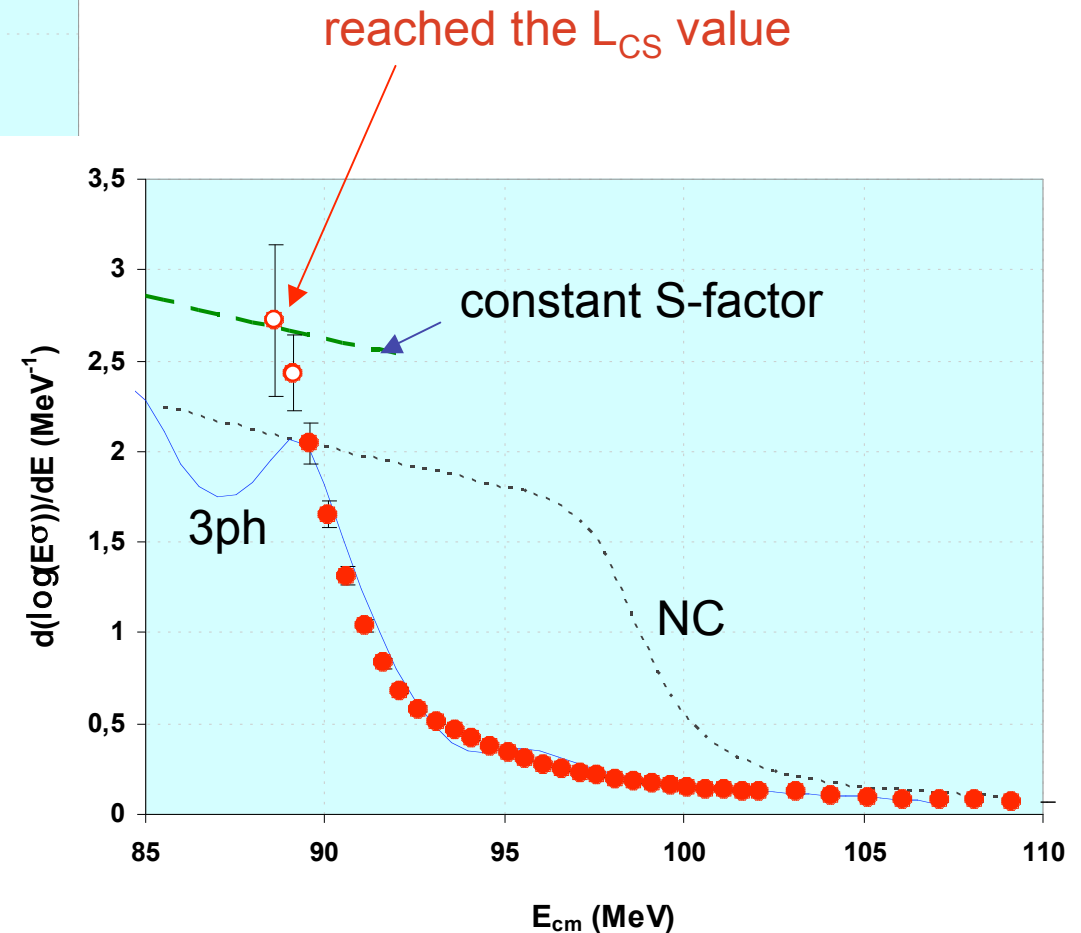
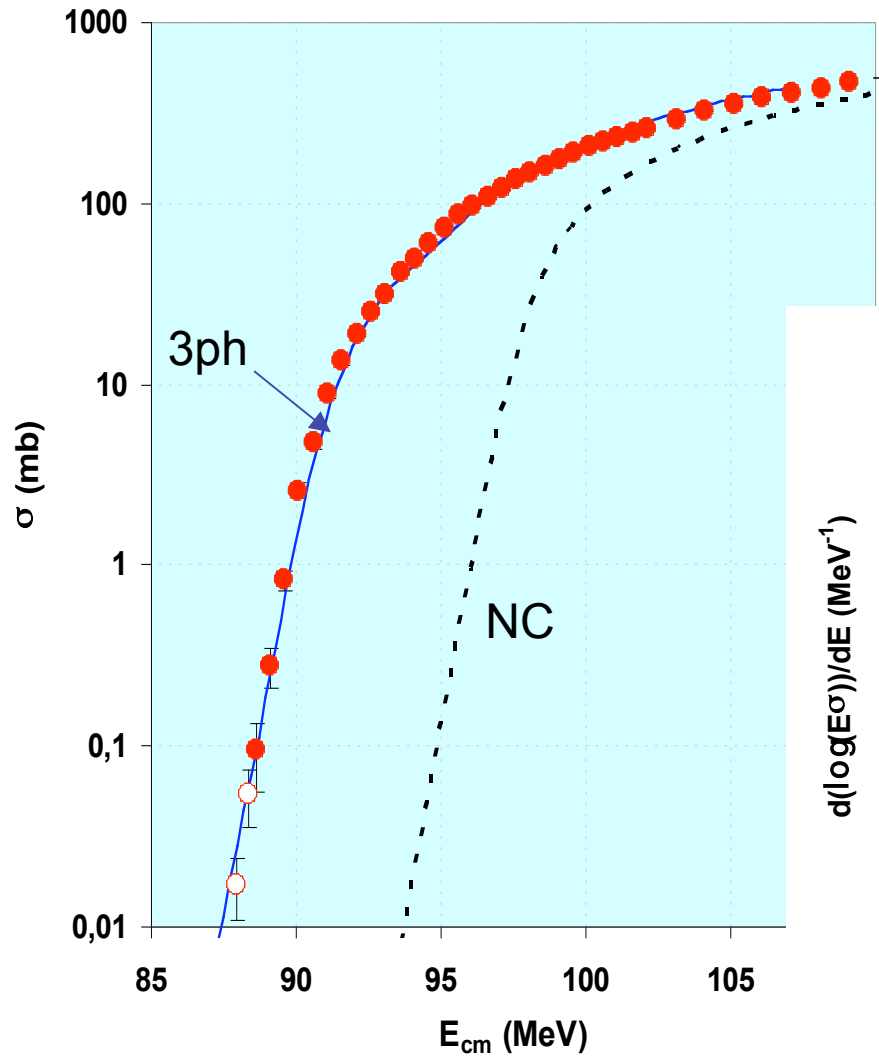
C.L. Jiang et al., PRL 93, 012701 (2004)

Fusion cross sections at deep sub-barrier energies show an unexpected behaviour

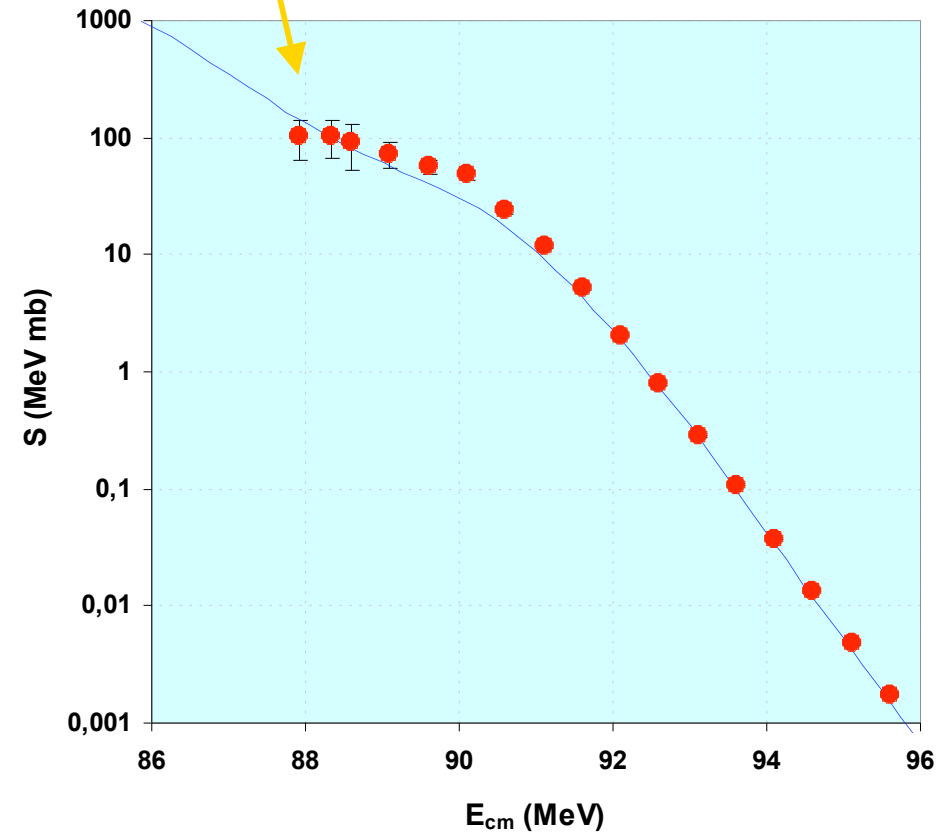
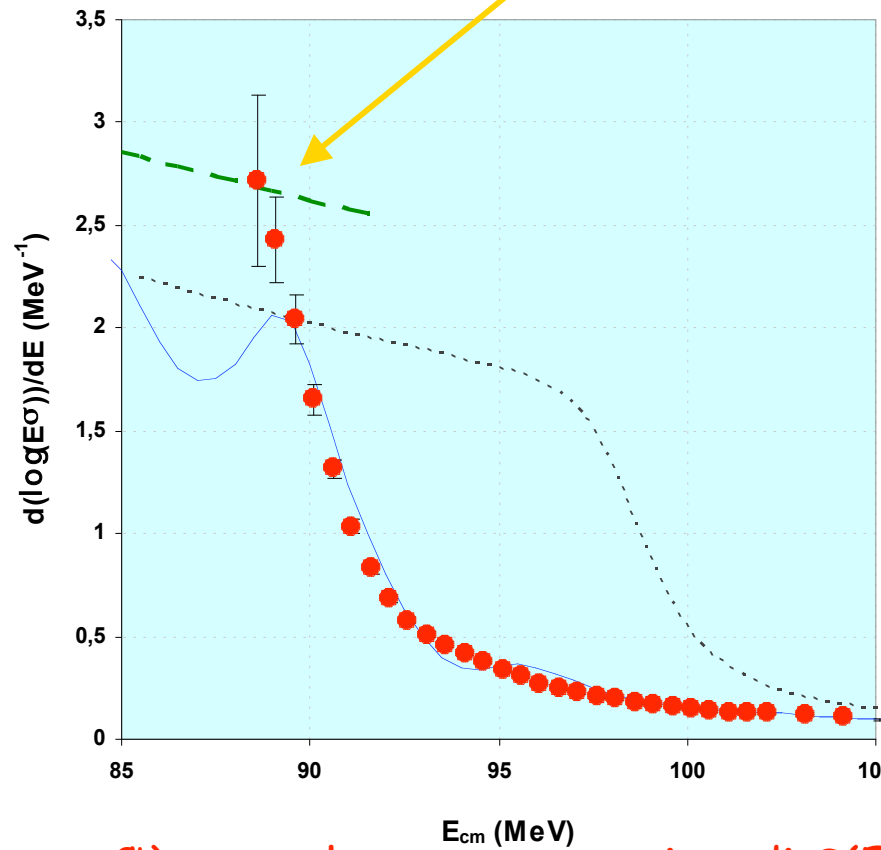


C.L.Jiang et al., PRL 93, 012701 (2004) PRC 73, 014613 (2006)

Ad energie $E \ll V_b$ qual'è l'andamento della funzione di eccitazione?
La sezione d'urto di fusione per $^{48}\text{Ca} + ^{96}\text{Zr}$ è stata misurata ad energie dove ci si aspetta un andamento esponenziale

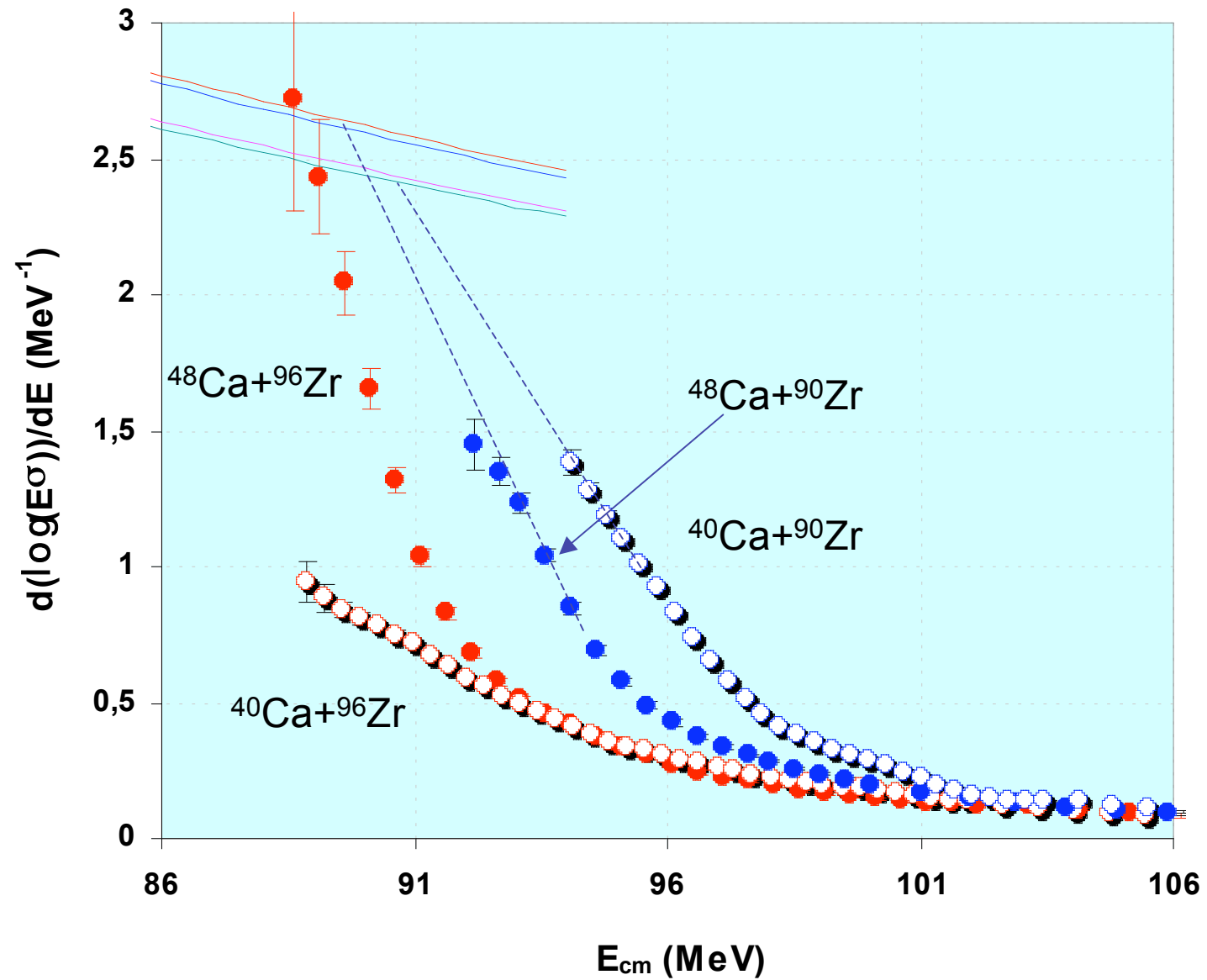


I dati sembrano indicare l'inizio del fenomeno di "hindrance" della fusione ad un'energia attorno a 88 MeV

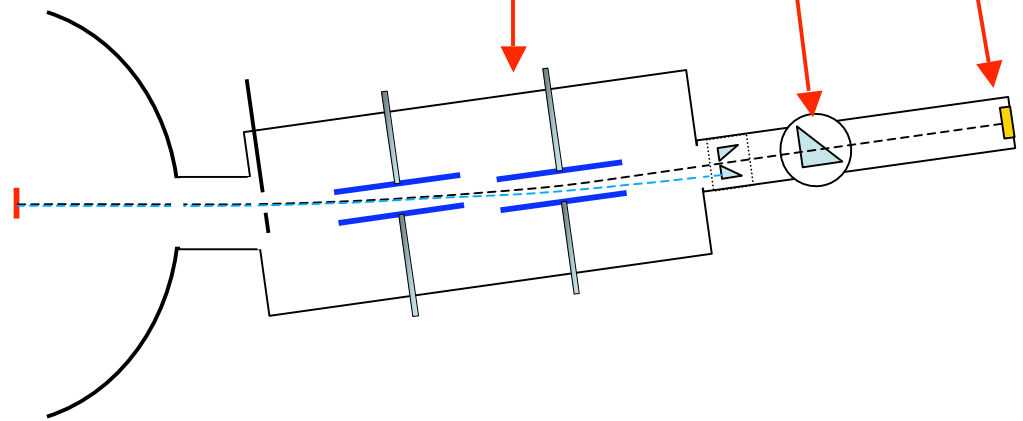
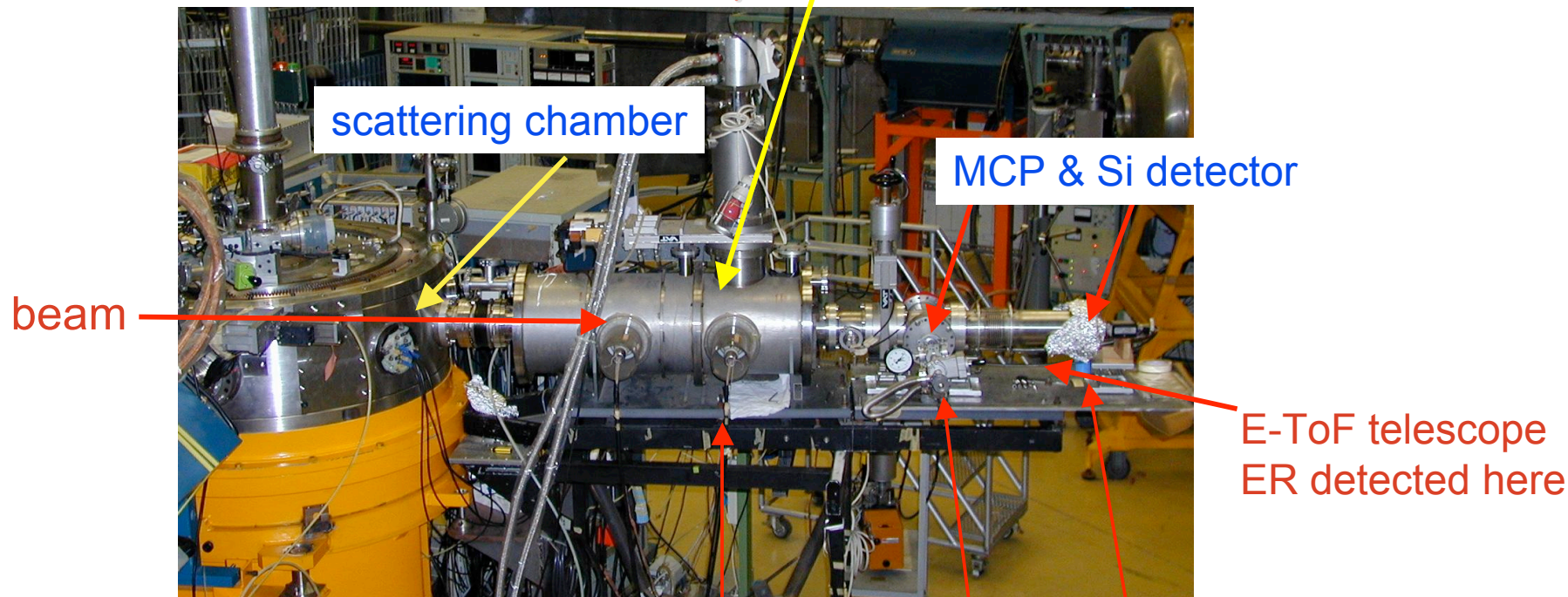


C'è un realmente un massimo di $S(E)$ a quest'energia ?

Una sistematica per i sistemi Ca + Zr

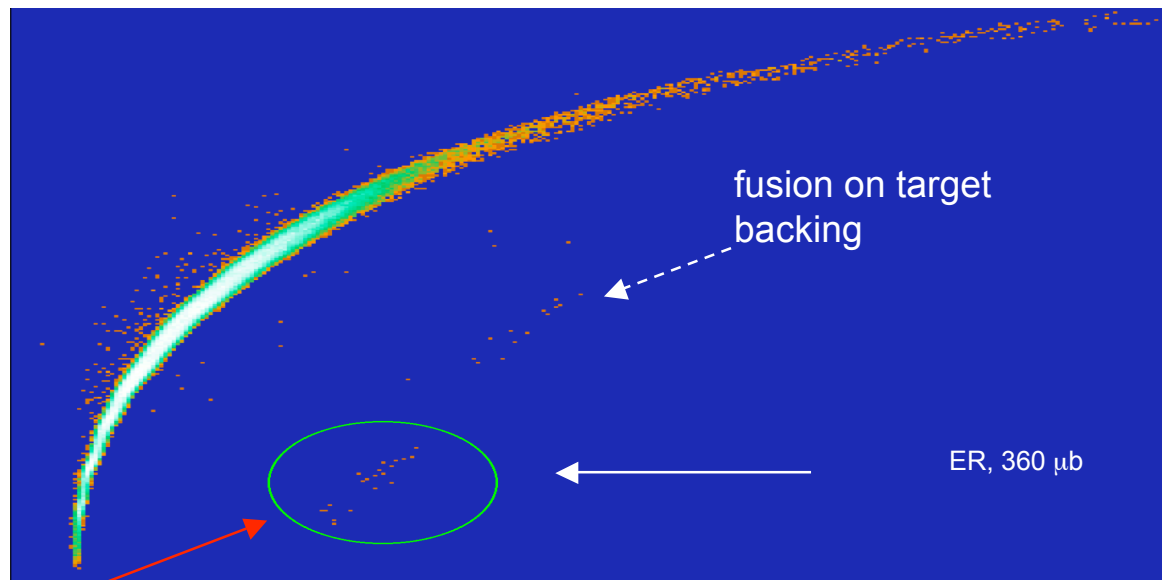
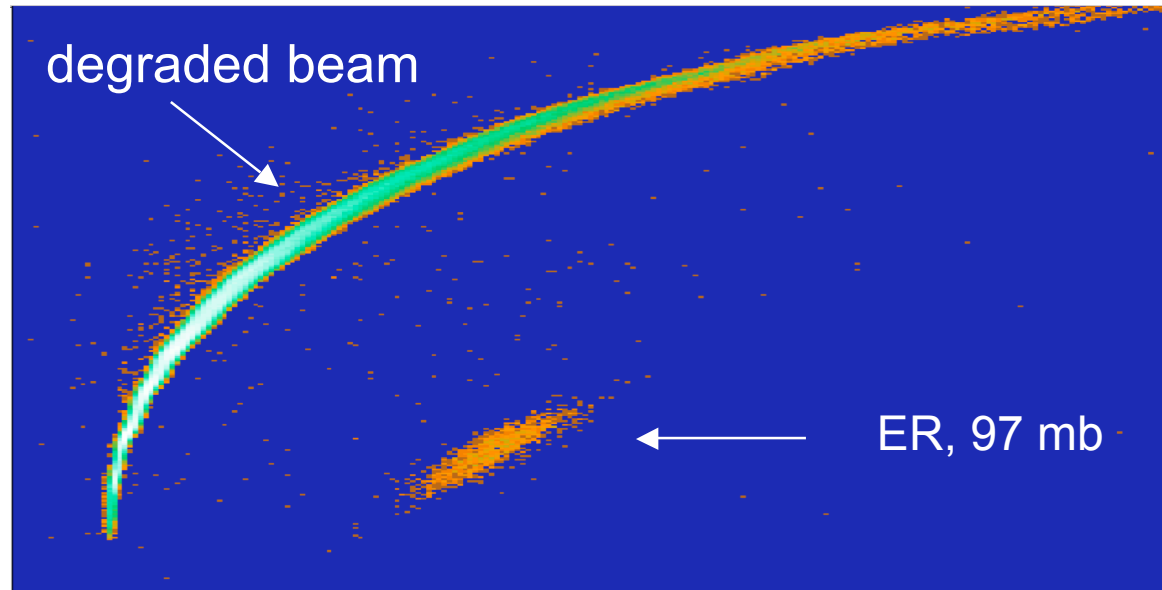


Come misuriamo le sezioni d'urto di fusione-evaporazione?
Il separatore elettrostatico dei LNL permette di rivelare i
residui di evaporazione a 0°





Time of Flight



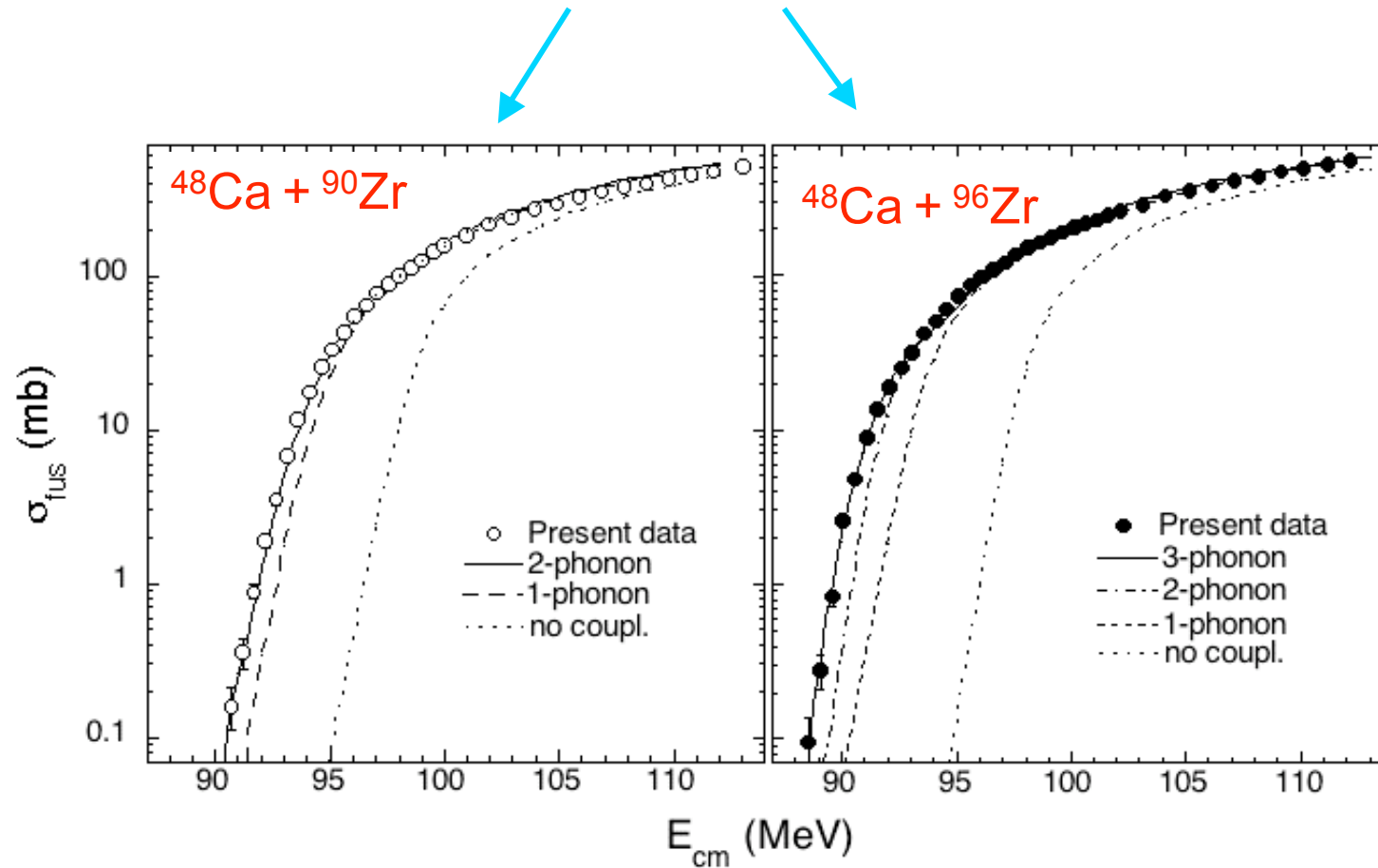
2 hours, beam 5 pA, target 50 $\mu\text{g}/\text{cm}^2$

Energy

Open questions e prospettive

- Ruolo del trasferimento nel processo di fusione, lo strumento a disposizione per trattare fusione e trasferimento di nucleoni nello stesso ambito teorico è il modello semiclassico GRAZING a cui bisogna fornire ulteriori dati completi
- Misure del trasferimento ad energie $E < V_b$
- Processo di fusione con fasci instabili il ruolo di nuclei con alone, fusione per sistemi N-rich con elevati Q-valori positivi per il trasferimento
- Competizione di reazioni di break-up
- Deep subbarrier fusion-fusion hindrance a basse energie
- Fusione in sistemi pesanti per la produzione di superheavy, 3 laboratori in competizione

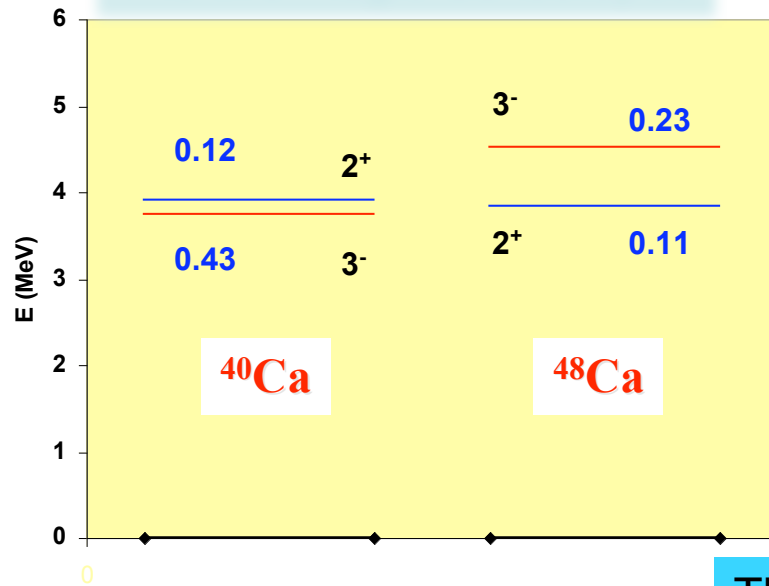
Extended the previous measurement of $^{48}\text{Ca} + ^{96}\text{Zr}$ to lower energy



A.M.Stefanini et al. Phys. Rev. C73, 034606 (2006)

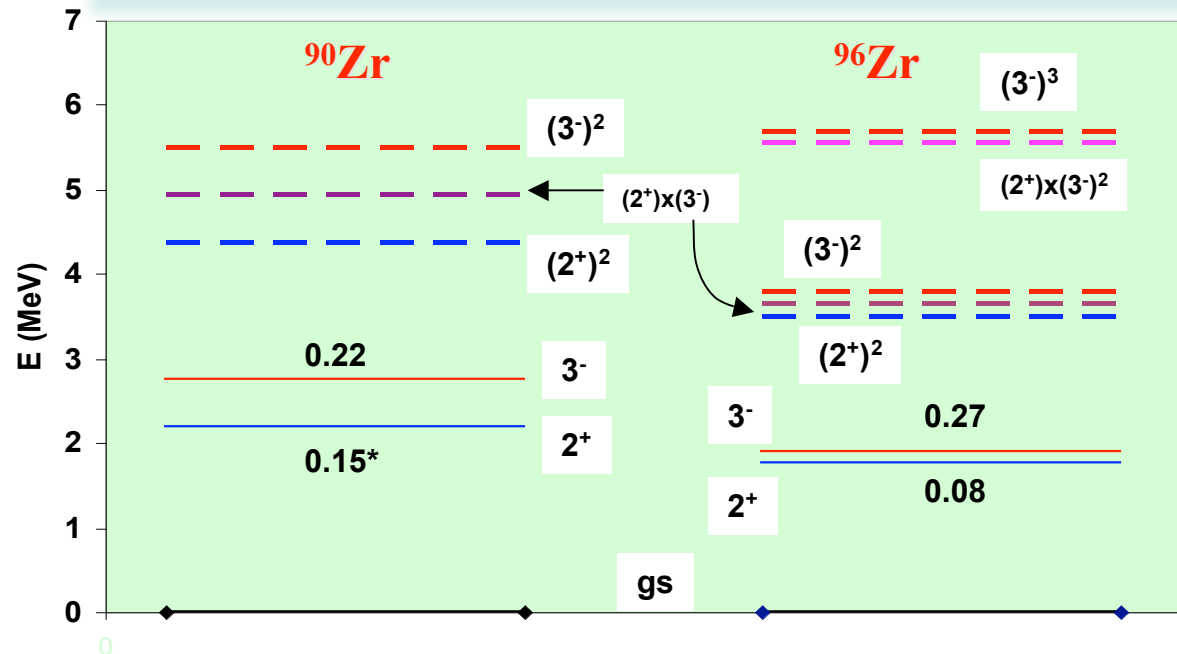
Calculations: CCFULL [K.Hagino et al., Comput. Phys. Comm. 123, 143 (1999)]

The two magic Ca isotopes

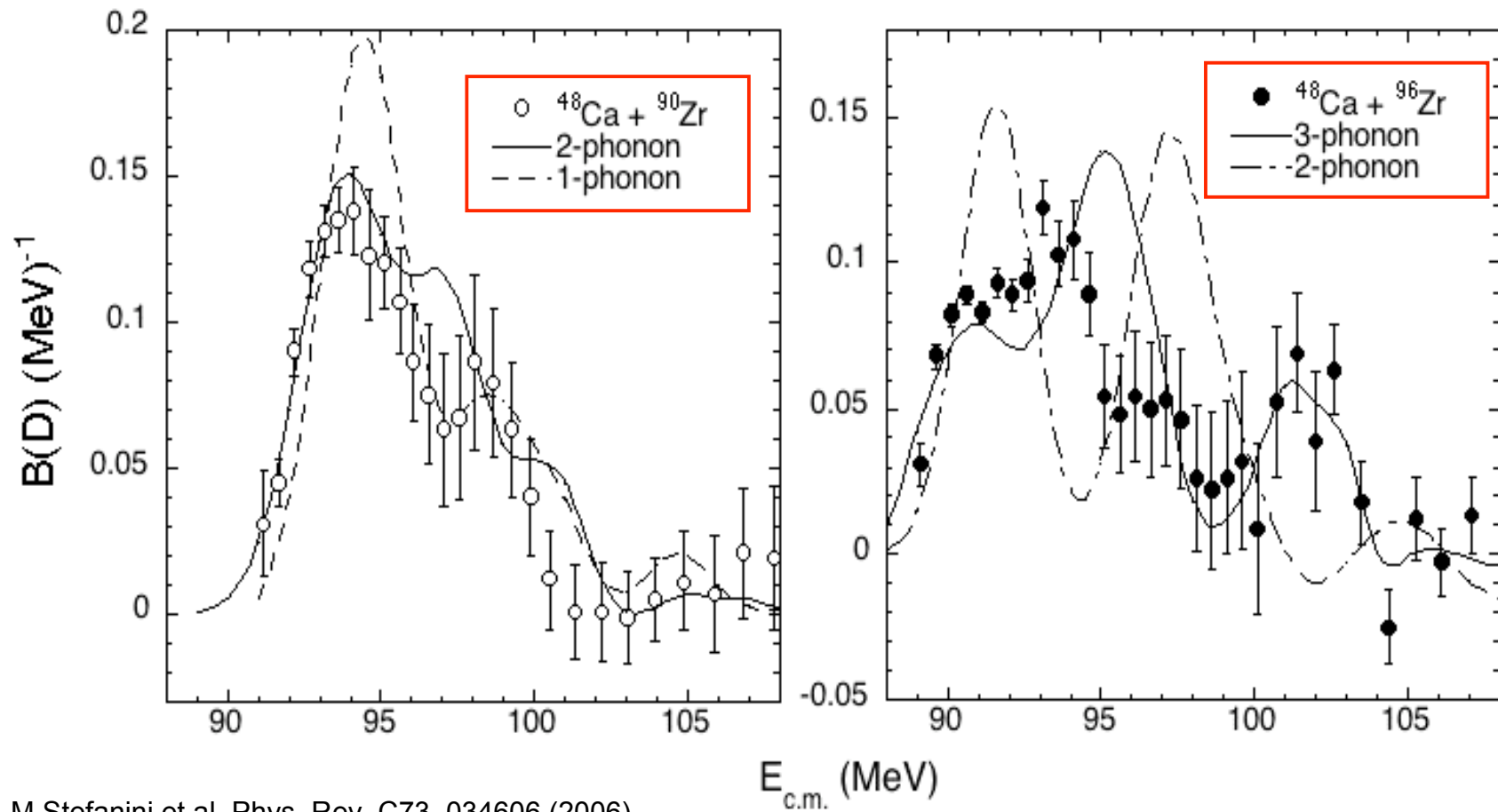


Scheme of CC calculations

The lowest 2^+ and 3^- excitations in the Zr isotopes



Barrier distributions are also “reasonably” reproduced



A.M.Stefanini et al. Phys. Rev. C73, 034606 (2006)

Appendice

SB-fusion some formula

- Approximate the barrier shape by an inverted parabola di curvatura

$$\varepsilon = \frac{\hbar}{2\pi} \left[-\frac{1}{\mu} \frac{\partial^2 V(r)}{\partial r^2} \Big|_{r=R_b} \right]^{1/2}$$

- The Hill-Wheeler Transmission coefficients

$$T_l = \frac{1}{1 + \exp\left\{ \left[V_b + l(l+1)\hbar^2 / 2\mu R_b^2 - E \right] / \varepsilon \right\}}$$

- at low energies

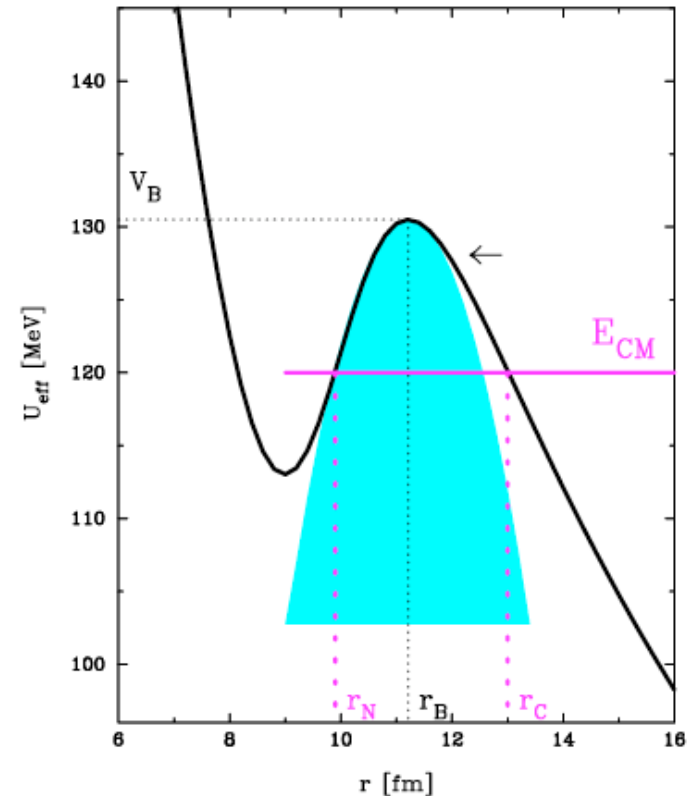
$$T_l \approx \exp\left[-(V_b - E) / \varepsilon\right] \exp\left\{-\left[l(l+1)\hbar^2 / 2\mu R_b^2\right] / \varepsilon\right\}$$

and $T_l / T_0 \approx \exp\left\{-\left[l(l+1)\hbar^2 / 2\mu R_b^2\right] / \varepsilon\right\}$

which is energy-independent.

One also has:

at sub-barrier energies. $\langle l \rangle \approx \frac{4}{3} \left(\mu R_b^2 \varepsilon / \hbar^2 \right)^{1/2}$



- Barrier Penetration

$$V = V_n(r) + \frac{Z_1 Z_2 e^2}{r} + \frac{l(l+1)}{2\mu \cdot r^2} \hbar^2$$

Approximate the barrier by an inverted parabola:

then we have the Hill-Wheeler

Transmission coefficients:

$$T_l(E) = \left[1 + \exp \left\{ \frac{2\pi}{\hbar\omega_l} (V_{bl} - E) \right\} \right]^{-1}$$

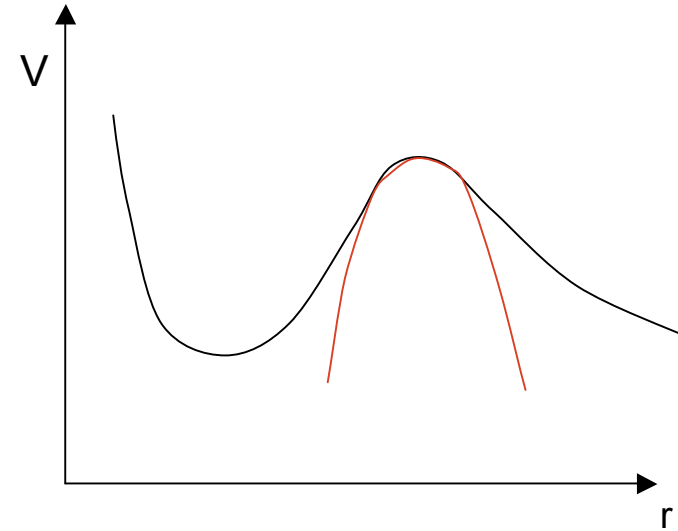
dove

$$\hbar\omega_l = \left[-\frac{\hbar^2}{\mu} \frac{\partial^2 V(r,l)}{\partial r^2} \Big|_{r=R_{bl}} \right]^{1/2}$$

Assuming further

$$\hbar\omega_l \approx \hbar\omega_{l=0} = \hbar\omega$$

$$R_{b,l} = R_{b,l=0} = R_b$$



$$\sigma_l = \frac{\pi}{k^2} (2l+1) \left[1 + \exp \left\{ \frac{2\pi}{\hbar\omega} \left(V_b + \frac{l(l+1)\hbar^2}{2\mu R_b^2} \right) \right\} \right]^{-1} \quad (V_b = V_{b,l=0})$$

$$\sigma_{fus}(E) = \sum_l \sigma_l = \frac{R_b^2 \hbar\omega}{2E} \ln \left\{ 1 + \exp \left[\frac{2\pi}{\hbar\omega} (E - V_b) \right] \right\}$$

$\rightarrow \mu R_b^2$ is taken to be independent of l and E ,
where μ is the reduced mass and R_b the unperturbed
barrier radius. One has

$$\sigma_l(E) = \frac{\pi}{k^2} (2l+1) \left[E' \frac{d\sigma_{fus}}{dE} \Big|_{E=E'} + \sigma_{fus}(E') \right]$$

$$\text{dove } E' = E - \frac{l(l+1)\hbar^2}{2\mu R_b^2}$$

Fusion excitation Function & spin distributions

One expects that the energy dependence of fusion σ - sects. determines the spin distribution uniquely

By assuming that :

- 1- σ - sects. correspond to the flux transmitted by one or more barrier which are energy independent,
 - 2- the angular momentum only increases the barrier by an amount corresponding to the centrifugal term, where μR_b^2 is l - and E - independent,
 - 3- μ and R_b are taken as the asymptotic reduced mass and unperturbed barrier radius,
- one obtains

$$\sigma_l(E) = \frac{2l+1}{k^2 R_b^2} \left[E' \frac{d\sigma}{dE} \Big|_{E=E'} + \sigma(E') \right]$$

dove

$$E' = E - \frac{l(l+1)\hbar^2}{2\mu R_b^2}$$

at very low energies $E \ll V_b$

$$\langle l \rangle \approx \sqrt{\langle l^2 \rangle} = \frac{4}{3} \left(\frac{\mu R_b^2}{\hbar^2} \cdot \frac{\hbar\omega}{2\pi} \right)^{1/2} \text{ becomes constant, and}$$

$$\frac{2\pi}{\hbar\omega} = \frac{d}{dE} \left[\ln(E\sigma_{fus}(E)) \right] = \frac{1}{\varepsilon}$$

H.Esbensen and S.Landowne, Nucl. Phys. A467 (1987)136

Higher-order coupling effects in low-energy heavy-ion fusion reactions (H.Esbensen and S.Landowne, Phys. Rev. C35 (1987))

Surface vibrational degrees of freedom are normally treated as independent harmonic oscillators linearly coupled to the relative motion, through the parameter

$$\sigma = \frac{\beta R}{\sqrt{4\pi}},$$

often truncating the model space at 1-phonon level.

The elements of the nuclear interaction matrix V_n are written in terms of the optical potential $U(r)$, its derivatives and powers of σ .

For instance,

$$\langle 1|V_n|0\rangle = -\sigma \frac{dU(r)}{dr}$$

is the matrix element for 1-phonon excitation.

Coupling to second order, and considering also 2-phonon states, leads to matrix elements where terms in σ^2 appear. Their effect on low-energy fusion of heavy symmetric systems is very significant.

→ application to the case of $^{58}\text{Ni} + ^{58}\text{Ni}$

$$\beta_\lambda = \frac{[4\pi(2\lambda + 1)B(E_\lambda)_{w.u}]^{1/2}}{Z(3 + \lambda)}$$

The “distribution of barriers” interpretation of heavy-ion fusion

If $D(B)$ = distribution of barrier heights B

$\sigma(e,B)$ = fusion x - sect. across the barrier B

we assume

$$\sigma(E) = \int_0^{\infty} \sigma(E,B)D(B)dB \quad (1)$$

This is justified in a CC treatment with 0 exc. energies (adiabatic limit), where

$$\sigma(E) = \sum_{\alpha} \omega_{\alpha} \sigma_{\alpha}(E) \quad (2)$$

By defining the discrete spectrum of barriers

$$D(B) = \sum_{\alpha} \omega_{\alpha} \delta(E - B)$$

equation (2) may be identified with eq. (1).

When exc. energies are not negligible, the position and relative weights of the adiabatic barrier are slightly changed.

If then

$$\sigma(E, B) \equiv \sigma_{\text{wong}}(E, B) = \frac{\hbar\omega R^2}{2E} \ln \left\{ 1 + \exp \left[\frac{2\pi}{\hbar\omega} (E - B) \right] \right\}$$

we differentiate twice and get

$$\frac{1}{\pi R^2} \frac{d^2(E\sigma_w)}{dE^2} = \frac{2\pi}{\hbar\omega} \frac{e^x}{(1 + e^x)^2} = G(x)$$

$$x = \frac{2\pi}{\hbar\omega} (E - B)$$

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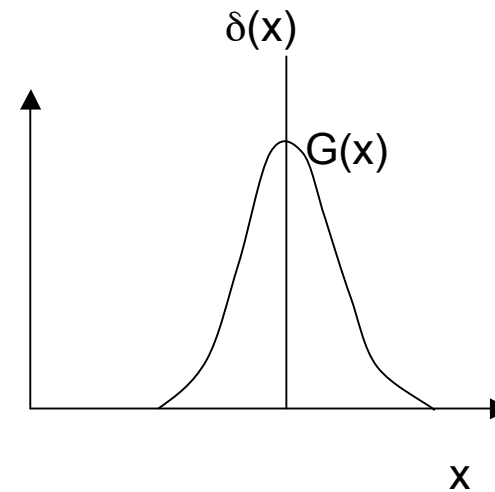
$$x = \frac{2\pi}{\hbar\omega} (E - B)$$

$G(x)$ is a generalization of $\delta(x)$, since

$$\lim_{\hbar\omega \rightarrow 0} G(x) = \delta(x) \equiv \delta(E - B)$$

it is smeared by tunneling over an energy region with

$$\text{FWHM} = 0.56\hbar\omega \approx 2 - 3 \text{ MeV}$$



Now we differentiate twice eq (1):

$$\frac{1}{\pi R^2} \frac{d^2(E\sigma)}{dE^2} = \int_0^\infty \frac{1}{\pi R^2} \frac{d^2(E\sigma_w)}{dE^2} D(B) dB = \int_0^\infty G(x) D(B) dB \equiv \bar{D}(E)$$

i.e. the exp. data are directly related to $\bar{D}(E)$

that is the "true" distribution of barriers $D(B)$ smoothed by the tunneling factor $G(x)$

Rowley, Satchler and Stelson, PL B254 (1991)25

- Obtaining the distribution of barriers from the data

For a set of fusion x-sects. Measured with a fixed energy spacing ΔE one can approximate the derivatives by the point-difference formulae

$$\frac{d(E\sigma)}{dE} = \frac{(E\sigma)_{n+1} - (E\sigma)_n}{\Delta E}$$

$$\frac{d^2(E\sigma)}{dE^2} = \frac{2(E\sigma)_n - (E\sigma)_{n+1} - (E\sigma)_{n-1}}{\Delta E^2}$$

if the statistical errors are

$\delta\sigma_n = f\sigma_n$ fixed fractions of the x - sects.

we have

$$\delta\left(\frac{d(E\sigma)}{dE}\right) \cong \sqrt{2}fE\sigma/\Delta E$$

$$\delta\left(\frac{d^2(E\sigma)}{dE^2}\right) \cong \sqrt{6}fE\sigma/\Delta E^2$$

The errors increase as the x-sects. increase, and the distributions are less well defined if ΔE is small, but it has to be small enough to resolve any interesting structure. No fine structure on a scale $\Delta E \leq 0.56\hbar\omega \sim 2-3\text{MeV}$ should show up due to tunneling effects. If it does, it is likely coming from the statistical error in σ .