Two nucleon transfer reactions

- Two valence nucleons go from core $b$ of nucleus $a$ to core $A$ of nucleus $B$.
- Probing two particle correlations.
- Investigating structure properties such as pairing and superfluidity in a finite fermion system (the atomic nucleus).
- Focus on heavy ions: semiclassical approximation.
- Get absolute values as well as the angular distribution for the cross sections.

Example: $^{138}\text{Ba}(^{14}\text{C},^{12}\text{C})^{140}\text{Ba}$
Because of the residual interaction $V_{res}$, the nuclear hamiltonian is not single particle,

$$H(\vec{r}_1, \ldots, \vec{r}_A) = \sum_{i=1}^{A} H_{MF}(\vec{r}_i) + V_{res}(\vec{r}_1, \ldots, \vec{r}_A)$$

The ground state wave function of the system is not a Slater determinant of nucleon wave functions as soon as we go beyond the mean field approximation.

The wave function of the two transferred particles will not be in a pure configuration.

We can extract the correlated (entangled) wave function of the transferred Cooper pair from BCS theory

$$\Psi(\vec{r}_1, \vec{r}_2) = \sum_{\nu} B_{\nu} \phi_{\nu}(\vec{r}_1) \phi_{\nu}(\vec{r}_2) \quad \nu, \bar{\nu} : \text{time reversal states}$$

Where $B_{\nu} = (-1)^{l_{\nu}} \sqrt{j_{\nu} + 1/2} U_{\nu} V_{\nu}$
As a consequence of pairing, nucleons couple in time reversal states, so all even-even nuclei ground states are $0^+$.

An energy paring gap is established between the ground state and the first excited state, greatly modifying the even-even nuclei low lying spectra.

Many non closed shell nuclei become superfluid.

These last features are static properties of the structure related to the modification of the energy spectrum due to pairing. To get experimental evidence of the correlated wave function of the Cooper pair is a more subtle thing. However this evidence exist:

Enhanced two particle transfer cross section! We will readily try to convince you that this is a consequence of two-particle correlations.
Some scattering theory background

Experimentally, what we see is the **differential cross section**, which is the squared modulus of the **scattering amplitude**,

\[
\frac{d\sigma}{d\Omega}(\theta) = |f(\theta)|^2
\]

We make the partial wave expansion,

\[
f(\theta) = \frac{1}{2k} \sum_l (2l + 1) \exp [2i(\sigma_l + \delta_l)] a(l) P_l(\cos \theta)
\]

\(\sigma_l\) → **Coulomb** scattering phase shift.

\(\delta_l\) → **Elastic scattering** phase shift for some suitable **nuclear optical potential** (generally absorptive): \(U(R) = V(R) + iW(R)\)

\(a(l)\) → **Two nucleon transfer** probability amplitude.

Our task will be to find an expression for this last quantity, and to relate it to the structure features of the nuclei, such as the pairing correlations of the transfered nucleons.
Transfer amplitude: quantum

\[ \Psi_a(\vec{r}_1, \vec{r}_2), \Psi_B(\vec{r}_1, \vec{r}_2) : \text{internal wavefunctions of the transferred nucleons in each nucleus} \]

\[ \chi(R) : \text{distorted wave describing the relative motion in the optical potential } U(R) \]

\[ U(R) = V(R) + iW(R) \]

absorption

\[ \left( \frac{P_R^2}{2\mu} + U(R) \right) \chi(R) = E_{CM} \chi(R) \]

\[ V_A, V_a : \text{mean field potentials of the two nuclei} \]

\( V_A \) is the interaction potential that transfers the nucleons from one nucleus to the other

it is a single particle potential!!
Successive transfer, in which one nucleon is transferred at a time through a set of intermediate states $|\gamma\rangle$, proves to be very important.

\[
\begin{align*}
|\alpha\rangle &= \phi_a \phi_A \\
|\gamma\rangle &= \phi_f \phi_F \\
|\beta\rangle &= \phi_b \phi_B
\end{align*}
\]

with
\[
\begin{align*}
H_a \phi_a &= E_a \phi_a; & H_A \phi_A &= E_A \phi_A \\
H_f \phi_f &= E_f \phi_f; & H_F \phi_F &= E_F \phi_F \\
H_b \phi_b &= E_b \phi_b; & H_B \phi_B &= E_B \phi_B
\end{align*}
\]

Non orthogonal basis!

Schematically, in the DWBA, the transfer amplitude will contain the terms:

- \[
\int d\tilde{R} \chi^*(R) \langle \beta | V_A | \alpha \rangle \chi(R) \quad \text{simultaneous transfer}
\]

- \[
\sum_{\gamma} \int d\tilde{R} \chi^*(R) \langle \beta | V_A | \gamma \rangle \langle \gamma | V_A | \alpha \rangle \chi(R) \quad \text{successive transfer}
\]

- \[
\sum_{\gamma} \int d\tilde{R} \chi^*(R) \langle \beta | \gamma \rangle \langle \gamma | V_A | \alpha \rangle \chi(R) \quad \text{non orthogonality term}
\]

\[
f_{nm}(R) = \langle n | V_A | m \rangle_R \quad \text{form factor. It depends only on R}
\]
Transfer amplitude: semiclassical

If the wavelength of the center of mass motion is small, we can localize it along a trajectory, characterized by an impact parameter $b = l/k$.

$^{138}\text{Ba}+^{14}\text{C}$ at 64 MeV $\rightarrow \lambda = 0.03 \text{ fm}$

Quantum

$$\chi(R)$$

Semiclassical

Classical trajectory generated by the optical potential $U(R)$

Interaction potential in each point of the trajectory

$$V_A|_R \rightarrow V_A|_R(t)$$

Integration of the form factor over collision time

$$\int d\vec{R} \chi^*(R)f_{aB}(R)\chi(R) \rightarrow \int dt f_{aB}(R(t)) \exp \left( \frac{i}{\hbar} Q_{\alpha\beta} t + i \sigma_{\alpha\beta} \right)$$

The factor $\exp \left( \frac{i}{\hbar} Q_{\alpha\beta} t + i \sigma_{\alpha\beta} \right)$ arises from differences in energy (Q-value) and center of mass position (recoil effect) between the incoming and the outgoing trajectory.
Form factors and overlaps

\[ |\alpha\rangle = \phi_a(\zeta_b, \vec{r}_1, \vec{r}_2) \phi_A(\zeta_A) \]
\[ |\beta\rangle = \phi_b(\zeta_b) \phi_B(\zeta_A, \vec{r}_1, \vec{r}_2) \]

initial and final channel wave functions. They describe the two nucleons initially in core b and then in core A.

The form factor will be

\[ f_{\alpha\beta}(R(t)) = \int d\zeta_b \, d\zeta_A \, d\vec{r}_1 \, d\vec{r}_2 \phi_b^*(\zeta_b) \phi_B^*(\zeta_A, \vec{r}_1, \vec{r}_2) V_A(\vec{r}_1, \vec{r}_2) \phi_a(\zeta_b, \vec{r}_1, \vec{r}_2) \phi_A(\zeta_A) \]

We will also need the overlap

\[ g_{\alpha\beta}(R(t)) = \int d\zeta_b \, d\zeta_A \, d\vec{r}_1 \, d\vec{r}_2 \phi_b^*(\zeta_b) \phi_B^*(\zeta_A, \vec{r}_1, \vec{r}_2) \phi_a(\zeta_b, \vec{r}_1, \vec{r}_2) \phi_A(\zeta_A) \]
Single particle $f$ and $g$'s

Two nucleons in core $b$

$$\phi_a(\zeta_b, \vec{r}_1, \vec{r}_2) = \chi(\zeta_b) \sum_{\nu} B^b_{\nu} \phi_{\nu}(\vec{r}_1) \phi_{\nu}(\vec{r}_2)$$

Two nucleons in core $A$

$$\phi_B(\zeta_A, \vec{r}_1, \vec{r}_2) = \chi(\zeta_A) \sum_{\nu'} B^A_{\nu'} \phi_{\nu'}(\vec{r}_1) \phi_{\nu'}(\vec{r}_2)$$

If we assume that the potential is separable

$$V_A(\vec{r}_1, \vec{r}_2) = V_A(\vec{r}_1) + V_A(\vec{r}_2)$$

We then can write

$$f_{\alpha\beta}(R(t)) = \sum_{\nu, \nu'} B^b_{\nu} B^A_{\nu'} f_{\nu\nu'}(R(t))$$

$$g_{\alpha\beta}(R(t)) = \sum_{\nu, \nu'} B^b_{\nu} B^A_{\nu'} g_{\nu\nu'}(R(t))$$

$$f_{\nu\nu'}(R(t)) = \left[ \int d\vec{r} \phi^*_{\nu'}(\vec{r}) V_A(\vec{r}) \phi_{\nu}(\vec{r}) \right]_{R(t)}$$

$$g_{\nu\nu'}(R(t)) = \left[ \int d\vec{r} \phi^*_{\nu'}(\vec{r}) \phi_{\nu}(\vec{r}) \right]_{R(t)}$$

Single particle form factors and overlaps
Transfer amplitude $a(l)$

$$[a(l)]_{(1)} = -i \frac{(-1)^{\lambda}}{2\lambda + 1} \sum_{\nu, \nu'} B^A_{\nu} B^b_{\nu'} \left( \frac{2j'_1 + 1}{2j_1 + 1} \right)^{1/2}$$

$$\times 2 \int_{-\infty}^{\infty} \frac{dt}{\hbar} f_{\nu\nu'}(R(t)) g_{\nu\nu'}(R(t)) \exp [(i/\hbar)Q_{\alpha\beta} t + i\sigma_{\alpha\beta}]$$

$$[a(l)]_{\text{orth}} = i \frac{(-1)^{\lambda}}{2\lambda + 1} \sum_{\nu, \nu'} B^A_{\nu} B^b_{\nu'} \left( \frac{2j'_1 + 1}{2j_1 + 1} \right)^{1/2}$$

$$\times 2 \int_{-\infty}^{\infty} \frac{dt}{\hbar} f_{\nu\nu'}(R(t)) g_{\nu\nu'}(R(t)) \exp [(i/\hbar)Q_{\alpha\beta} t + i\sigma_{\alpha\beta}]$$

$$[a(l)]_{\text{succ}} = -i \frac{(-1)^{\lambda}}{2\lambda + 1} \sum_{\nu, \nu'} B^A_{\nu} B^b_{\nu'} \left( \frac{2j'_1 + 1}{2j_1 + 1} \right)^{1/2}$$

$$\times 2 \int_{-\infty}^{\infty} \frac{dt}{\hbar} f_{\nu\nu'}(R(t)) \exp [(i/\hbar)Q_{\beta\gamma} t + i\sigma_{\beta\gamma}]$$

$$\times \int_{-\infty}^{t} \frac{dt'}{\hbar} f_{\nu\nu'}(R(t')) \exp [(i/\hbar)Q_{\gamma\alpha} t' + i\sigma_{\gamma\alpha}]$$

$$a(l) = [a(l)]_{(1)} + [a(l)]_{\text{succ}} + [a(l)]_{\text{orth}}$$

- $B^A_{\nu} B^b_{\nu'}$ → spectroscopic amplitudes
- $f_{\nu\nu'}(R(t))$ → single particle form factors
- $g_{\nu\nu'}(R(t))$ → single particle overlaps

For each $l$, one trajectory $R(t)$

→ evaluate the overlaps and form factors in many of its points
→ perform the numerical integration over time
→ get $a(l)$
Complex turning points

Example: $^{138}\text{Ba}(^{14}\text{C},^{12}\text{C})^{140}\text{Ba}$

The turning points are the solutions of $E_{CM} - V_{eff} = 0$

with $V_{eff}(r;l) = U(r) + V_{Coul}(r) + \frac{\hbar^2 l(l + 1)}{2mr^2}$

Optical potential is complex (absorption) ➔ complex turning points
Contribution of the trajectories

Let us have a closer look on how the nature of the classical trajectories affect the cross sections for $^{138}\text{Ba}(^{14}\text{C},^{12}\text{C})^{140}\text{Ba}$
We have a grazing reaction

Angular distribution determined by classical dynamics

\[ ^{138}\text{Ba}(^{14}\text{C},^{12}\text{C})^{140}\text{Ba} \]

Absorption for deflection angles larger than the grazing angle (close trajectories)

Rutherford scattering without transfer for deflection angles smaller than the grazing angle (far trajectories)
Absolute values. Q value and recoil

To get the correct absolute values of transfer reaction cross sections is a greater challenge. First of all, we have to account for the mismatch among the trajectories in the incoming ($\alpha$) and outgoing ($\beta$) channel.

Mismatch in energy due to differences in reduced mass, internal energy and optical potential $\rightarrow$ effective Q-value:

$$Q_{\alpha\beta} = \frac{1}{2} v^2 (\mu_\alpha - \mu_\beta) + U_\alpha - U_\beta + E_\alpha - E_\beta$$

Mismatch in space due to the difference in the center of mass position $\rightarrow$ recoil:

$$\sigma_{\alpha\beta} = \kappa (R_\alpha - R_\beta)$$

The integrals along the trajectories will be affected by a factor $\exp \left( \frac{i}{\hbar} Q_{\alpha\beta} t + i \sigma_{\alpha\beta} \right)$.
Absolute values. Structure

Details of the structure, as well as successive transfer, are very important. We made the calculation for $^{208}\text{Pb}(^{16}\text{O},^{18}\text{O})^{206}\text{Pb}$ at 86 MeV:

Transfered neutrons exhibit BCS correlations described by the spectroscopic amplitudes.

Single particle form factors contribute coherently to give an enhancement with respect to pure configurations.

From structure theory, we need:

- single particle wavefunctions $\phi^a_\nu, \phi^B_\nu$
- spectroscopic amplitudes $B^B_\nu, B^a_\nu$
- interaction potential
- optical potential (for trajectory)
Comparison with earlier results

\[ ^{208}\text{Pb}(^{16}\text{O}, ^{18}\text{O})^{206}\text{Pb} \text{ at 86 MeV} \]

Bayman and Chen, 1982
DWBA calculation

Our calculation

Maglione, Pollarolo, Vitturi, Broglia and Winther, 1985
Semiclassical calculation

\[ \frac{d\sigma}{d\Omega} (\mu b/sr) \]

\[ \text{cross section (\mu b)} \]

\[ \frac{d\sigma}{d\Omega} (\text{mb/sr}) \]
### Comparison with earlier results

\( ^{208}\text{Pb}(^{16}\text{O},^{18}\text{O})^{206}\text{Pb} \text{ at 86 MeV} \)

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Maglione, Pollarolo, Vitturi, Broglia and Winther, 1985

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\( \varepsilon \): enhancement factor

Cross sections for pure two-particle configurations in the target compared with those obtained with the correlated wave functions, all measured at 112°.
Conclusions and perspectives

Done:
- We have developed a tool to get **reliable absolute values**, along with angular distributions, for the **two particle transfer nuclear reactions**.
- We allowed for the accommodation a **variety of structure inputs** in the calculation.
- We have **checked our results** with earlier calculations and experiments for different structure inputs, investigating the influence of the structure in the outcomes.

To do:
- **Make calculations**! There is a great interest in reactions involving **exotic beams** ($^{11}\text{Li}$, $^6\text{He}$, $^{14}\text{Be}$) recently available.
- Adapt the program to make a **full-quantum calculation**.
- Extend the **self-consistency** (calculation of interaction potentials, optical potentials, wave functions and cross sections within a single framework).