Nonlinear (topological) excitations in 2D spin systems with high spin ($s \geqslant 1$)

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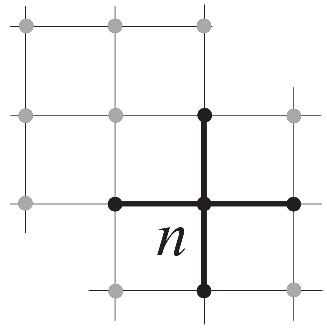
Outline

- Quantum model of 2D spin systems with high spin ($s \geqslant 1$)
  - Bilinear Hamiltonian
  - Mean field approximation. Ordered states
  - Motion equations for large-scale fluctuations
- Classical model as Hamiltonian hierarchy on coadjoint orbits of Lie group
  - Effective Hamiltonians
  - Geometry and topology of orbits
  - An example of topological excitations

Spin system in question

Consider a planar atomic lattice.



Each atom has a spin $s \geqslant 1$ (high spin).

Each atom is assigned to three **spin operators**:

$$\{\hat{S}_n^1,\, \hat{S}_n^2,\, \hat{S}_n^3\} = \hat{\boldsymbol{S}}_n, \qquad [\hat{S}_n^a,\, \hat{S}_m^b] = i\varepsilon_{abc}\hat{S}_n^c\delta_{nm},$$

where $a,\, b,\, c \in \{1,\, 2,\, 3\}$, and δ_{nm} is the Kronecker symbol.
$\{\hat{S}_n^a\}$ are Hermitian operators.

Quantum model

The **quantum model** is described by a **generalized Heisenberg Hamiltonian**:

- as $s=1$ with **biquadratic exchange**

$$\hat{\mathcal{H}} = -\sum_{n,\delta}\Big(J(\hat{\boldsymbol{S}}_n,\hat{\boldsymbol{S}}_{n+\delta}) + K(\hat{\boldsymbol{S}}_n,\hat{\boldsymbol{S}}_{n+\delta})^2\Big),$$

  δ runs over the nearest-neighbour sites;

- as $s=3/2$ with **bicubic exchange**

$$\hat{\mathcal{H}} = -\sum_{n,\delta}\Big(J(\hat{\boldsymbol{S}}_n,\hat{\boldsymbol{S}}_{n+\delta}) + K(\hat{\boldsymbol{S}}_n,\hat{\boldsymbol{S}}_{n+\delta})^2 + L(\hat{\boldsymbol{S}}_n,\hat{\boldsymbol{S}}_{n+\delta})^3\Big);$$

- spin s with $2s$-th power of exchange interaction.

Space of representation ($s=1$)

The spin operators $\{\hat{S}_n^a\}$ are naturally considered over $(2s+1)$-dimensional space of irreducible representation of group $\mathrm{SU}(2)$. *In the case* $s=1$*, the space is 3-dimensional, canonical basis:* $\{|+1\rangle,\ |-1\rangle,\ |0\rangle\}$*.*

The operators $\{\hat{S}_n^a\}$ generate an associative matrix algebra over the space of representation.
In the case $s=1$*, in addition to* $\{\hat{S}_n^a\} \in \mathrm{Mat}_{3\times 3}$ *we introduce* ***quadrupole operators****:* $\{\hat{Q}_n^{12},\ \hat{Q}_n^{13},\ \hat{Q}_n^{23},\ \hat{Q}_n^{[2,2]},\ \hat{Q}_n^{[2,0]}\}$
as tensor operators of weight 2,

$$\hat{Q}_n^{ab} = \hat{S}_n^a \hat{S}_n^b + \hat{S}_n^b \hat{S}_n^a,\ a \neq b, \qquad \hat{Q}_n^{[2,2]} = (\hat{S}_n^1)^2 - (\hat{S}_n^2)^2,$$

$$\hat{Q}_n^{[2,0]} = \sqrt{3}\big((\hat{S}_n^3)^2 - \tfrac{2}{3}\big).$$

Space of representation ($s=3/2$)

In the case $s=3/2$, the space of representation is 4-dim, canonical basis: $\{|+\frac{3}{2}\rangle, |+\frac{1}{2}\rangle, |-\frac{1}{2}\rangle, |-\frac{3}{2}\rangle\}$.

*We complete the associative algebra of $\{\hat{S}^a_n\} \in \mathrm{Mat}_{4\times 4}$ by tensor operators of weight 2 — **quadrupole operators**:*

$$\hat{Q}^{ab}_n = \frac{\sqrt{5}}{2\sqrt{3}}\Big(\hat{S}^a_n\hat{S}^b_n + \hat{S}^b_n\hat{S}^a_n\Big),\ \ a,b\in\{1,2,3\},\ \ a\neq b$$

$$\hat{Q}^{[2,2]}_n = \frac{\sqrt{5}}{2\sqrt{3}}\Big((\hat{S}^1_n)^2 - (\hat{S}^2_n)^2\Big), \quad \hat{Q}^{[2,0]}_n = \frac{\sqrt{5}}{2}\big((\hat{S}^3_n)^2 - \tfrac{5}{4}\big),$$

*and of weight 3 — **sextupole operators**:*

$$\hat{T}^{ab}_n = \frac{1}{\sqrt{6}}\Big((\hat{S}^a_n)^2\hat{S}^b_n + \hat{S}^b_n(\hat{S}^a_n)^2 + \hat{S}^a_n\hat{S}^b_n\hat{S}^a_n - (\hat{S}^b_n)^3\Big),\ \hat{T}^{a3}_n = (\hat{Q}^{a2}_n\hat{S}^3_n + \hat{S}^3_n\hat{Q}^{a2}_n),$$

$$\hat{T}^{3a}_n = \frac{1}{\sqrt{10}}\Big(\hat{Q}^{a3}_n\hat{S}^3_n + \hat{S}^3_n\hat{Q}^{a3}_n + \sqrt{3}(\hat{Q}^{[2,0]}_n\hat{S}^a_n + \hat{S}^a_n\hat{Q}^{[2,0]}_n)\Big),\ \ a,b\in\{1,2\},\ \ a\neq b,$$

$$\hat{T}^{[3,0]}_n = \tfrac{1}{12}\big(41\hat{S}^3_n - 20(\hat{S}^3_n)^3\big).$$

Bilinear Hamiltonian

In terms of tensor operators (together $\{\hat{P}_n^a\}$) over the space of representation we obtain **bilinear Hamiltonians**:
(N is the overall number of sites in the lattice)

$$\hat{\mathcal{H}}^{\text{spin 1}} = -(J - \tfrac{1}{2}K)\sum_{n,\delta}\sum_{b}\hat{S}_n^b\hat{S}_{n+\delta}^b - \tfrac{1}{2}K\sum_{n,\delta}\sum_{\alpha}\hat{Q}_n^\alpha\hat{Q}_{n+\delta}^\alpha - \tfrac{4}{3}KN;$$

$$\hat{\mathcal{H}}^{\text{spin 3/2}} = -(J - \tfrac{1}{2}K + \tfrac{587}{80}L)\sum_{n,\delta}\sum_{b}S_n^bS_{n+\delta}^b - \tfrac{75}{32}(4K - L)N - \tfrac{6}{5}(K - 2L)\sum_{n,\delta}\sum_{\alpha}Q_n^\alpha Q_{n+\delta}^\alpha - \tfrac{9}{10}L\sum_{n,\delta}\sum_{\beta}T_n^\beta T_{n+\delta}^\beta.$$

Remark. *The operators* $\{\hat{P}_n^a\}_{a=1}^8$ *form an orthogonal basis in* $\mathfrak{su}(3)$.
The operators $\{\hat{P}_n^a\}_{a=1}^{15}$ *form an orthogonal basis in* $\mathfrak{su}(4)$.

Mean field Hamiltonian

We introduce a vector field: $\{\mu_a(\boldsymbol{x}_n)\}_{a=1}^{8} = \{\langle\hat{S}_n^1\rangle, \langle\hat{S}_n^2\rangle, \langle\hat{S}_n^3\rangle, \langle\hat{Q}_n^{12}\rangle, \langle\hat{Q}_n^{13}\rangle, \langle\hat{Q}_n^{23}\rangle, \langle\hat{Q}_n^{[2,2]}\rangle, \langle\hat{Q}_n^{[2,0]}\rangle\}$ called a **mean field**.

Here $\langle\cdot\rangle$ denotes a **quasiaverage** (which means a quantum-mechanical and thermodynamical average after a spontaneous breaking of symmetry).

*In the case $s=1$, a **mean field Hamiltonian**:*

$$\hat{\mathcal{H}}_{\textit{MF}}^{\textit{spin 1}} = -(J-\tfrac{1}{2}K)z\sum_n\sum_{a=1}^{3}\hat{P}_n^a\mu_a(x_n) - \frac{1}{2}Kz\sum_n\sum_{a=4}^{8}\hat{P}_n^a\mu_a(x_n) - \frac{4}{3}KzN,$$

z is a number of the nearest-neighbour sites.

Order parameters

The mean field Hamiltonian is $\mathrm{SU}(2)$-invariant.
By action of $\mathrm{SU}(2)$ it can be reduced to the diagonal form:

$$\hat{\mathcal{H}}_{\text{MF}}^{\text{spin 1}} = -z \sum_n \Big((J - \tfrac{1}{2}K)\hat{S}_n^3 \mu_3(x_n) + \tfrac{1}{2}K \hat{Q}_n^{[2,0]} \mu_8(x_n) \Big) - \tfrac{4}{3} K z N.$$

In the case of thermodynamical equilibrium and unlimited lattice, μ_3 and μ_8 are constants. We call them **order parameters**.

μ_3 describes a normalized **magnetization** (a ratio of z-projection of magnetic moment to a saturation magnetization)

μ_8 is a normalized **projection of quadrupole moment**.

Self-consistent equations (SCEq)

A mean field exists if self-consistent relations are held: ($\hat{h}_{\text{MF}}$ is a one-site Hamiltonian: $\hat{h}_{\text{MF}} = \hat{\mathcal{H}}_{\text{MF}}/N$)

$$\mu_3 = \langle \hat{S}^3 \rangle_{\text{MF}} = \frac{\operatorname{Tr} \hat{S}^3 e^{-\frac{\hat{h}_{\text{MF}}}{kT}}}{\operatorname{Tr} e^{-\frac{\hat{h}_{\text{MF}}}{kT}}}, \qquad \mu_8 = \langle \hat{Q}^{[2,0]} \rangle_{\text{MF}} = \frac{\operatorname{Tr} \hat{Q}^{[2,0]} e^{-\frac{\hat{h}_{\text{MF}}}{kT}}}{\operatorname{Tr} e^{-\frac{\hat{h}_{\text{MF}}}{kT}}}.$$

*In the case $s{=}1$, **self-consistent equations** are*

$$\mu_3 = \frac{2\sinh \frac{(J-K/2)\mu_3}{kT}}{\exp\left\{-\frac{\sqrt{3}\,K\mu_8}{2kT}\right\} + 2\cosh \frac{(J-K/2)\mu_3}{kT}},$$

$$\mu_8 = \frac{2}{\sqrt{3}} \frac{\cosh \frac{(J-K/2)\mu_3}{kT} - \exp\left\{-\frac{\sqrt{3}\,K\mu_8}{2kT}\right\}}{\exp\left\{-\frac{\sqrt{3}\,K\mu_8}{2kT}\right\} + 2\sinh \frac{(J-K/2)\mu_3}{kT}}.$$

This is a kind of Weiss equation

Phase diagram

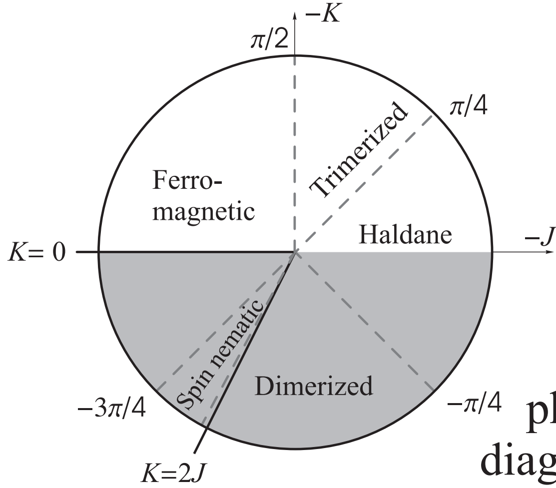
There exist **4 solutions of SCEq** (as $T=0$, $J>0$):

$\lvert\mu_3\rvert = 1,\ \mu_8 = \frac{2J-K}{\sqrt{3}\,K}$	ferromagnetic state
$\lvert\mu_3\rvert = \frac{1}{2},\ \mu_8 = \frac{J-K/2}{\sqrt{3}\,K}$	partly ordered ferromagnetic state ($K>0$), unstable
$\mu_3 = 0,\ \lvert\mu_8\rvert = \frac{2}{\sqrt{3}}$	nematic state ($K>0$)
$\mu_3 = 0,\ \lvert\mu_8\rvert = \frac{1}{\sqrt{3}}$	partly ordered nematic state ($K>0$), unstable

Matveev V. M., Quantum quadrupole magnetizm and changes of phase under biquadratic exchange, *J. Exper. Theor. Phys.*, **65** (1973), 1627–1636.

According to SCEq, **nematic states** appear in the dark region if $T<T_{\text{crit}}$, where $T_{\text{crit}}\approx\frac{K}{3k}$.

Ferromagnetic states appear in the region $0<K<2J$.



1D phase diagram

Motion equations

Motion equations **in the quantum model** are

$$i\hbar \frac{d\hat{P}_n^a}{dt} = [\hat{P}_n^a, \hat{\mathcal{H}}]. \tag{1}$$

Suppose we take the quasiaverage as explained above (**mean field average**) of (1), and take a **large-scale limit** with zero correlations between fluctuations, then the equations (1) transform into

$$\frac{\partial \mu_a}{\partial t} = \frac{Jz}{\hbar} C_{abc} \mu_b (\mu_{c,xx} + \mu_{c,yy}), \tag{2}$$

where C_{abc} are structure constants for the Lie algebra of operators $\{\hat{P}_n^a\}$: $[\hat{P}_n^a, \hat{P}_m^b] = iC_{abc}\hat{P}_n^c \delta_{nm}$.

Remark*.** *The equations* (2) *are a* ***generalization of Landau-Lifshits equation *to the case of vector field* $\{\mu_a\}$*.*

Classical model

The generalized Landau-Lifshits equation (2) can be interpreted as a **Hamiltonian hierarchy on a coadjoint orbit of a Lie group**:
$\mathrm{SU}(3)$ in the case $s = 1$,
$\mathrm{SU}(4)$ in the case $s = 3/2$,
$\mathrm{SU}(2s+1)$ in the case of spin s.

That is why we use the method of Hamiltonian systems on coadjoint orbits of Lie groups (**orbital method**) to investigate a **generalized Landau-Lifshits equation, which is a classical model** for the system in question.

The matrices $\{\hat{P}^a\}$ serve as a basis in the Lie algebra $\mathfrak{su}(2s+1)$. The components of mean field $\{\mu_a\}$ serve as coordinates in the dual space to $\mathfrak{su}(2s+1)$.

Motion equations on orbit

By the orbital method we obtain **Hamiltonian equations on coadjoint orbits** of the Lie group $\mathrm{SU}(2s+1)$.

Depending on an orbit we have **different equations implied by different ways of the mean field averaging**. For example, if we neglect correlations between fluctuations of the quantum fields $\{\hat{P}^a_n\}$ we come to Hamiltonian equations on the maximal degenerate orbit.

In the case $s = 1$:

$$\frac{\partial \mu_a}{\partial t} = \tfrac{1}{3h_0} C_{abc}\mu_b(\mu_{c,xx} + \mu_{c,yy}), \quad h_0 = \big(\mu^0_8(T)\big)^2, \qquad (3)$$

the parameter h_0 *depends on initial conditions, which generally depend on a temperature* T*.*

Effective Hamiltonians

Each orbit has a Hamiltonian. We call the Hamiltonians on all orbits of $\mathrm{SU}(2s+1)$ **effective Hamiltonians** of the model. Equations determining orbits serve as **constrains**.

*In the case $s = 1$, we deal with the group $\mathrm{SU}(3)$, which has **two types of orbits**: degenerate and generic. Thus, we propose **two effective Hamiltonians**.*

$$\mathcal{H}_{\textit{eff,1}} = \frac{1}{6h_0} \int \sum_{a=1}^{8} \Big((\mu_{a,x})^2 + (\mu_{a,y})^2 \Big) \, dx dy,$$

$$d_{abc}\mu_b\mu_c + \sqrt{h_0/3}\,\mu_a - \frac{2h_0}{3} = 0, \qquad \textbf{\textit{for a degenerate orbit}}$$

where $d_{abc} = \frac{1}{4}\,\mathrm{Tr}(\hat{P}_a\hat{P}_b\hat{P}_c + \hat{P}_b\hat{P}_a\hat{P}_c)$ *is a symmetric tensor.*

***Remark*. *The Hamiltonian* $\mathcal{H}_{\textit{eff,1}}$ *gives rise to the generalized Landau-Lifshits equation* (3), *which coincides with* (2).*

Effective Hamiltonians

Another Hamiltonian is

$$\mathcal{H}_{\textit{eff,2}} = \frac{1}{8(h_0^3 - 3f_0^2)} \sum_{a=1}^{8} \Big(h_0^2 (\nabla \mu_a)^2 + 3h_0 (\nabla \xi_a)^2 - 6 f_0 \langle \nabla \mu_a, \nabla \xi_a \rangle \Big),$$

$$d_{abc}\mu_b\xi_c - h_0\mu_a - \tfrac{2}{3} f_0 = 0,$$ ***for a generic orbit***

ξ_a *is a quadratic form in* $\{\mu_a\}$*:* $\xi_a = d_{abc}\mu_b\mu_c$*.*
The parameters h_0*,* f_0 *depend on initial conditions, and generally depend on a temperature* T*.*

Remark. *One can quantize* $\mathcal{H}_{\textit{eff,1}}$*-, and* $\mathcal{H}_{\textit{eff,2}}$*-models. Such effective models are called* ***σ-models*** *in quantum field theory. Evidently, they describe slow fluctuations. One can take into account quick fluctuations by means of a renormalization group connected to the coefficients* $\frac{1}{8(h_0^3(T) - 3f_0^2(T))}$ *and* $\frac{1}{6h_0(T)}$*.*

Geometry of orbits

Effective Hamitonians have a geometrical nature.

For an **orbit** is a homogeneous space that admits a **Kählerian structure**, we use a complex parameterization (by means of *generalized stereographic projection*) and reduce effective Hamiltonians to the form:

$$\mathcal{H}_{\text{eff}} = \int \sum_{\alpha,\beta} h_{\alpha\bar{\beta}} \Big(\frac{\partial w_\alpha}{\partial z}\frac{\partial w_\beta}{\partial \bar{z}} + \frac{\partial w_\alpha}{\partial \bar{z}}\frac{\partial w_\beta}{\partial z} \Big)\, dz d\bar{z},$$

$h_{\alpha\bar{\beta}}$ are components of a Kählerian metrics on an orbit;
$\{w_\alpha\}$ are complex parameters on an orbit.

***Remark*.** *The geometric form of an effective Hamiltonian does not depend on an orbit. A density of Hamiltonian is defined by a Kahlerian metrics.*

Generalized stereographic projection

$$\mu_1 = \frac{\mu_3^0 - \sqrt{3}\,\mu_8^0}{2\sqrt{2}} \cdot \frac{w_2 + w_3 + \bar{w}_2 + \bar{w}_3}{1 + |w_2|^2 + |w_3|^2} - \frac{\mu_3^0}{\sqrt{2}} \frac{(1 - w_1)(\bar{w}_3 - \bar{w}_1\bar{w}_2) + (1 - \bar{w}_1)(w_3 - w_1 w_2)}{1 + |w_1|^2 + |w_3 - w_1 w_2|^2}$$

$$\mu_2 = \frac{\mu_3^0 - \sqrt{3}\,\mu_8^0}{2i\sqrt{2}} \cdot \frac{w_3 - w_2 - \bar{w}_3 + \bar{w}_2}{1 + |w_2|^2 + |w_3|^2} + \frac{i\mu_3^0}{\sqrt{2}} \frac{(1 + w_1)(\bar{w}_3 - \bar{w}_1\bar{w}_2) - (1 + \bar{w}_1)(w_3 - w_1 w_2)}{1 + |w_1|^2 + |w_3 - w_1 w_2|^2}$$

$$\mu_3 = -\frac{\mu_3^0 - \sqrt{3}\,\mu_8^0}{2} \cdot \frac{|w_2|^2 - |w_3|^2}{1 + |w_2|^2 + |w_3|^2} + \frac{\mu_3^0(1 - |w_1|^2)}{1 + |w_1|^2 + |w_3 - w_1 w_2|^2}$$

$$\mu_4 = \frac{\mu_3^0 - \sqrt{3}\,\mu_8^0}{2i} \cdot \frac{\bar{w}_2 w_3 - w_2\bar{w}_3}{1 + |w_2|^2 + |w_3|^2} + \frac{i\mu_3^0(w_1 - \bar{w}_1)}{1 + |w_1|^2 + |w_3 - w_1 w_2|^2}$$

$$\mu_5 = \frac{\mu_3^0 - \sqrt{3}\,\mu_8^0}{2i\sqrt{2}} \cdot \frac{w_2 + w_3 - \bar{w}_2 - \bar{w}_3}{1 + |w_2|^2 + |w_3|^2} + \frac{i\mu_3^0}{\sqrt{2}} \frac{(1 - \bar{w}_1)(w_3 - w_1 w_2) - (1 - w_2)(\bar{w}_3 - \bar{w}_1\bar{w}_2)}{1 + |w_1|^2 + |w_3 - w_1 w_2|^2}$$

$$\mu_6 = \frac{\mu_3^0 - \sqrt{3}\,\mu_8^0}{2\sqrt{2}} \cdot \frac{w_3 - w_2 + \bar{w}_3 - \bar{w}_2}{1 + |w_2|^2 + |w_3|^2} - \frac{\mu_3^0}{\sqrt{2}} \frac{(1 + w_1)(\bar{w}_3 - \bar{w}_1\bar{w}_2) + (1 + \bar{w}_1)(w_3 - w_1 w_2)}{1 + |w_1|^2 + |w_3 - w_1 w_2|^2}$$

$$\mu_7 = \frac{\mu_3^0 - \sqrt{3}\,\mu_8^0}{2} \cdot \frac{\bar{w}_2 w_3 + w_2\bar{w}_3}{1 + |w_2|^2 + |w_3|^2} - \frac{\mu_3^0(w_1 + \bar{w}_1)}{1 + |w_1|^2 + |w_3 - w_1 w_2|^2}$$

$$\mu_8 = -\frac{\mu_3^0 - \sqrt{3}\,\mu_8^0}{2\sqrt{3}} \cdot \frac{2 - |w_2|^2 - |w_3|^2}{1 + |w_2|^2 + |w_3|^2} + \frac{\mu_3^0}{\sqrt{3}} \cdot \frac{1 + |w_1|^2 - 2|w_3 - w_1 w_2|^2}{1 + |w_1|^2 + |w_3 - w_1 w_2|^2}.$$

Topology of orbits

A coadjoint orbit $\mathrm{SU}(3)$ is parameterized by $\{w_1,\, w_2,\, w_3\}$. ***Kählerian potentials*** *are*

$$\Phi = \mu_3^0\,\Phi_1 + \tfrac{\sqrt{3}\mu_8^0 - \mu_3^0}{2}\,\Phi_2,$$

$$\Phi_1 = \ln(1 + |w_1|^2 + |w_3 - w_1 w_2|^2), \quad \Phi_2 = \ln(1 + |w_2|^2 + |w_3|^2).$$

On a generic orbit $\dim H^2 = 2$ *(*H^2 *is a cohomology class).*
On a degenerate orbit: $\mu_3^0{=}0$, $w_1{=}0$ *or* $\mu_3^0{=}\sqrt{3}\mu_8^0$, $w_2{=}0$, *evidently,* $\dim H^2 = 1$.

Density of the effective Hamiltonians

$$h_{\alpha,\bar\beta} = \frac{\partial^2\Phi_1}{\partial w_\alpha \partial \bar w_\beta} + \frac{\partial^2\Phi_2}{\partial w_\alpha \partial \bar w_\beta} + \frac{2\omega_{\alpha\bar\beta}}{e^{\Phi_1}e^{\Phi_2}},$$

$$\omega_{2\bar 2} = |w_1|^2,\ \omega_{2\bar 3} = \bar\omega_{3\bar 2} = -w_1,\ \omega_{2\bar 2} = 1.$$

Topological charge

Introduce a **topological charge of a mean field configuration** on a Kählerian manifold by

$$\mathcal{Q} = \frac{1}{4\pi} \int \sum_{\alpha,\beta} i h_{\alpha\bar{\beta}} \Big(\frac{\partial w_\alpha}{\partial z} \frac{\partial w_\beta}{\partial \bar{z}} - \frac{\partial w_\alpha}{\partial \bar{z}} \frac{\partial w_\beta}{\partial z} \Big)\, dz \wedge d\bar{z}.$$

The expressions for $\mathcal{Q}$ and $\mathcal{H}_{\text{eff}}$ differ only in the sign.

***Remark*.** *Compare with* $\mathcal{H}_{\text{eff}} = \int \sum_{\alpha,\beta} h_{\alpha\bar{\beta}} \left(\frac{\partial w_\alpha}{\partial z} \frac{\partial w_\beta}{\partial \bar{z}} + \frac{\partial w_\alpha}{\partial \bar{z}} \frac{\partial w_\beta}{\partial z} \right) dz d\bar{z}.$

Evidently,

$$\mathcal{H}_{\text{eff}} \geqslant 4\pi |\mathcal{Q}|.$$

A **minimum of $\mathcal{H}_{\text{eff}}$** is realized if the equality holds, that takes place if $\{w_\alpha\}$ **are holomorphic or antiholomorphic**.

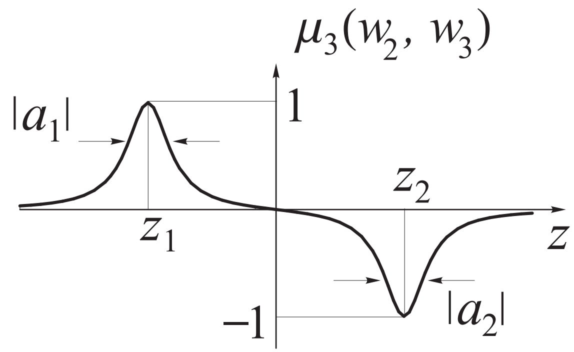
Large-scale topological excitations

Example. Consider a planar magnet with spin 1, and the effective $\mathcal{H}_{\text{eff},1}$-model. This corresponds to a degenerate orbit of $\mathrm{SU}(3)$. Assign $\mu_3^0 = 0$, $\mu_8^0 = -\frac{2}{\sqrt{3}}$ (an equilibrium nematic state).

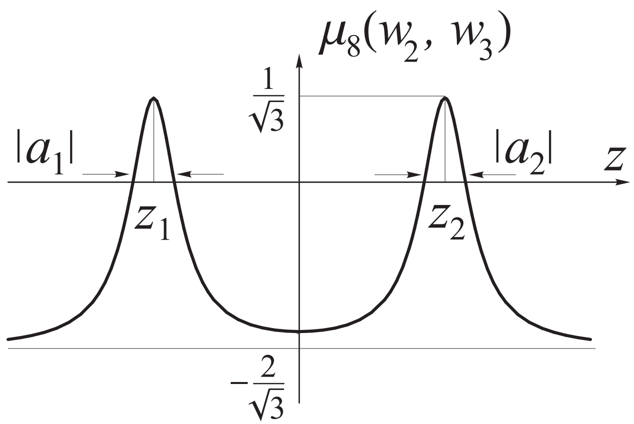
Take a mean field configuration with the holomorphic functions ($\mathcal{Q} = 2$):

$$w_2(z) = \frac{a_1}{z - z_1}, \quad w_3(z) = \frac{a_2}{z - z_2},$$

$$a_1,\ z_1,\ a_2,\ z_2 \in \mathbb{C}\,.$$



This is a kind of Belavin-Polyakov soliton.



$\mathcal{H}_{\text{eff},1}$ does not depend on a_1, z_1, a_2, z_2. Thus, the excitation can infinitely enlarge without energy input, that causes destruction of the nematic order.

Results

- The example shows that **in a 2D magnet an ordered state is easily destroyed at any temperature** $T>0$ (that agrees with Mermin-Wagner theorem). Moreover, **we propose a mechanism of destruction of ordered states in 2D magnets.** That is, we suggest that an order exists, but any excitation easily destroys it.
- The well-known fact: an order exists in 3D magnets. How does it disappear in 2D? **A prospect**: to construct a quasi2D theory that considers a planar magnet with a fixed thickness, and takes into account anisotropic effects (demagnetization in normal direction).

The end