Tidal interactions in compact binaries systems

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Summary

- Introduction
  - Compact Objects
  - Coalescing binaries
- A model for Tidal interactions
  - Post-Newtonian Affine Model
  - Tidal Interaction
- Results

Compact objects

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$$0.1 \lesssim \frac{2GM}{Rc^2} \lesssim 1$$

Fully relativistic treatment is required

Compact objects

Sources of potentially observable gravitational waves (GWs)

Gravitational luminosity

$$\frac{dE}{dt} \sim \frac{G}{c^5} M^2 R^4 \nu^6$$

Adding the characteristic internal velocity $v \sim \nu R$

$$\frac{dE}{dt} \sim \frac{c^5}{G} \left(\frac{2GM}{Rc^2}\right)^2 \left(\frac{v}{c}\right)^6$$

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What we need:

- ☑ extremely compact objects $(2GM/Rc^2 \sim 1)$
- ☑ relativistic internal velocities

Coalescing binaries

- Most promising source of gravitational waves for terrestrial interferometers

- NS can be tidally disrupted only from a BH or another NS: we can derive useful informations about equation of state

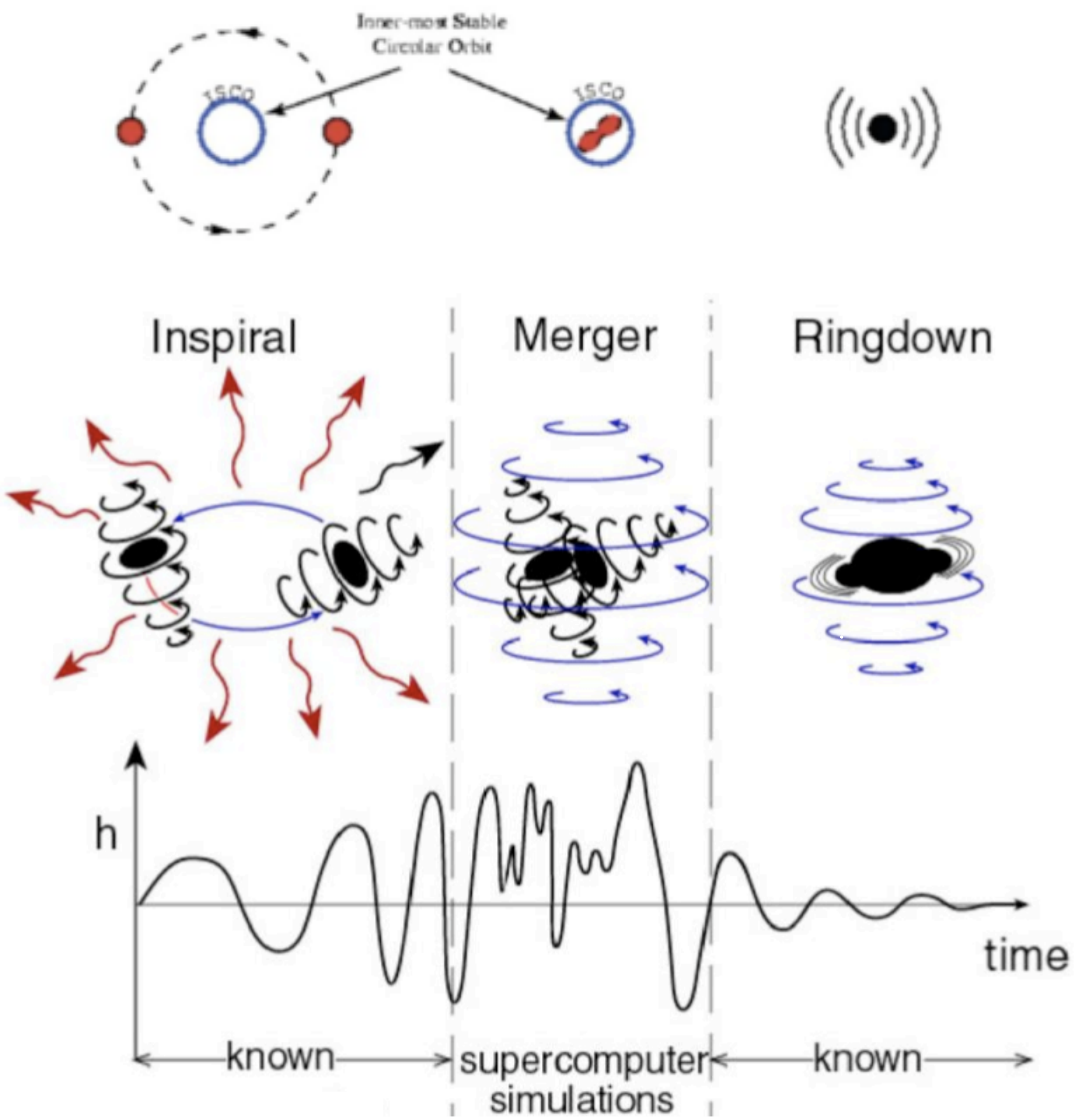
*V.Ferrari, L.Gualtieri, F.Pannarale, PRD **81**, 0604026 (2010)*

- Possible progenitors of short gamma-ray burst

*Luciano Rezzolla et a., Astrophys. J. Lett. **732**, L6 (2011)*

Coalescing binaries

Several approaches have been introduced to study differents phases of the coalescence



- Inspiral: approximate methods
- Merger: numerical relativity
- Ringdown: perturbation theory

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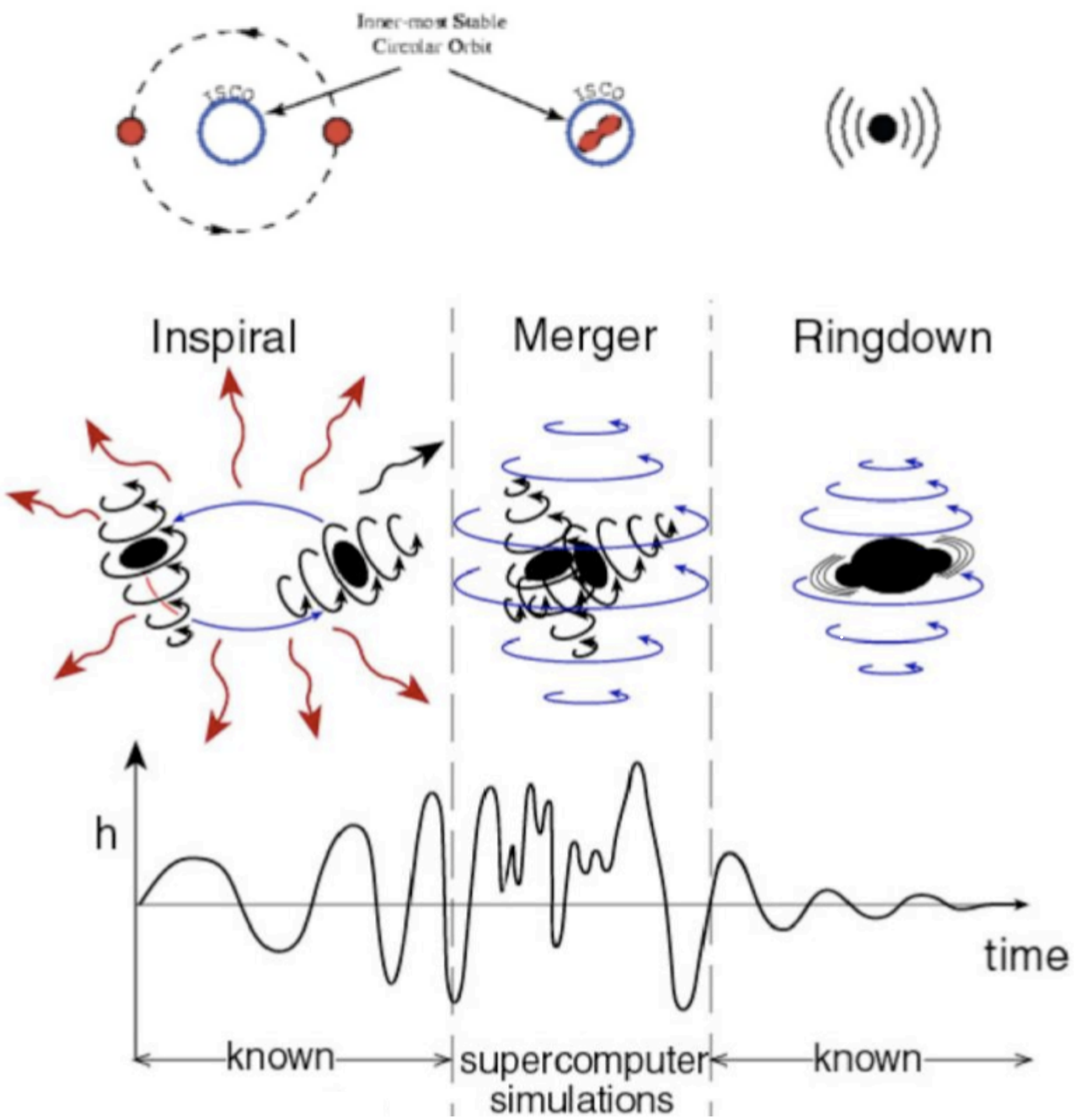

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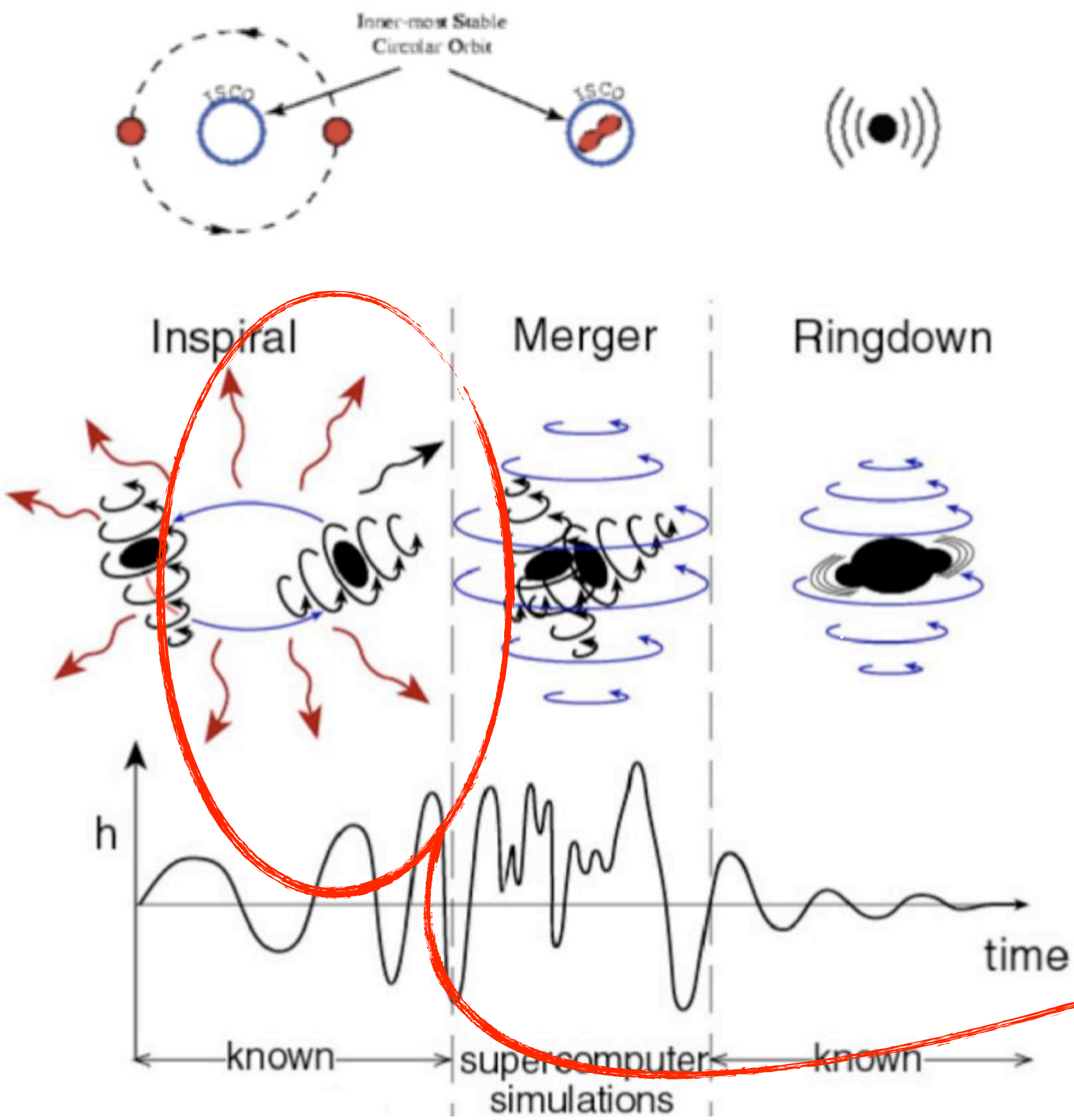

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Tidal interaction

Coalescing binaries (BH-NS)

The contributions to gravitational signal are given by orbital motion and size effects

- Perturbative methods treat bodies as point particles
- Numerical relativity parameter space is poorly explored
  - High number of parameter: spins, mass ratio M_{BH}/M_{NS} , equation of state
  - Lack of simmetry make numerical simulation an hard task

We need approximations to reduce computational cost

Post-Newtonian affine model

The NS is an extended body subjected to its internal pressure, self-gravity and tidal field

- It is an ellipsoid and preserves this shape during the orbital evolution (affine hypotesis)

- NS gravity is determined by a potential constructed using relativistic stellar structure equations

  - Possibility to use realistic equations of state (EoS)

*V.Ferrari, L.Gualtieri, F.Pannarale, CQG **26**, 125004 (2009)*

BH isn't affected by tidal field due to its higher compactness

Post-Newtonian affine model

Low-velocity (*v/c* small) and weak-field (*M/R* small) expansion of the Einstein equations

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
&= [\eta_{\mu\nu} + h_{\mu\nu}] \, dx^\mu dx^\nu
\end{aligned}
$$

$$
\begin{aligned}
g_{00} &= -1 + \frac{2}{c^2} V(t, \mathbf{x}) - \frac{2}{c^4} V(t, \mathbf{x})^2 + \mathcal{O}(6) \\
g_{0i} &= -\frac{4}{c^3} V_i(t, \mathbf{x}) + \mathcal{O}(5) \\
g_{ij} &= \delta_{ij} \left(1 + \frac{2}{c^2} V(t, \mathbf{x}) \right) + \mathcal{O}(4) \ .
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$$V = \frac{Gm_1}{r_1} - \frac{Gm_1}{r_1 c^2} \left[\frac{(n_1 v_1)^2}{2} - 2v_1^2 + \frac{Gm_2}{r_{12}} \left(\frac{r_1^2}{4r_{12}^2} + \frac{5}{4} - \frac{r_2^2}{4r_{12}^2}\right)\right] + 1 \leftrightarrow 2 + \mathcal{O}(3)$$

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Post-Newtonian order

$$V = \frac{Gm_1}{r_1} - \frac{Gm_1}{r_1 c^2} \left[\frac{(n_1 v_1)^2}{2} - 2v_1^2 + \frac{Gm_2}{r_{12}} \left(\frac{r_1^2}{4r_{12}^2} + \frac{5}{4} - \frac{r_2^2}{4r_{12}^2} \right) \right] + 1 \leftrightarrow 2 + \mathcal{O}(3)$$

Post-Newtonian affine model

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We derive from the metric

- ☑ Tidal field
- ☑ Equations of motion including spin-spin and spin-orbit couplings

Tidal interaction

We study the deviation of two geodesics due to the BH-NS gravitatational interaction

$$\frac{d^2 n^\alpha}{d\tau^2} = -R^\alpha_{\beta\gamma\delta} u^\beta u^\gamma n^\delta \longrightarrow \frac{d^2 x^i}{d\tau^2} = -C^i_j x^j$$

$$C_{ij} = R_{\alpha\beta\gamma\delta} e^\alpha_{(0)} e^\beta_{(i)} e^\mu_{(0)} e^\nu_{(j)}$$

- ☐ We estimate the tidal tensor up to the $1/c^5$ order
- ☐ Using the tidal tensor we can calculate the NS deformation during the inspiral

Tidal interaction

$$C_{xx} = -\frac{1}{c^2}\partial_{xx}V^{(0)} - \frac{1}{c^4}\Bigg\{4\partial_{xt}V_x^{(0)} - 4v_y\partial_{xx}V_y^{(0)} + 4v_y\partial_{xy}V_x^{(0)} + (\partial_y V^{(0)})^2 + \partial_{xx}V^{(2)}$$

$$-\Big[\partial_{tt} + v_y^2(\partial_{yy} + 2\partial_{xx}) + 2v_y\partial_{yt} - (2Q_{xy} + v_x v_y)\,\partial_{xy}\Big]V^{(0)}$$

$$+2\partial_{xx}V^{(0)}V^{(0)} + 2(\partial_x V^{(0)})^2\Bigg\} - \frac{4}{c^5}\left\{(\partial_{xt} + v_y\partial_{xy})V_x^{(1)} - v_y\partial_{xx}V_y^{(1)}\right\}$$

To do:

- ☑ estimate the derivatives of scalar and vectorial potentials
- ☑ compute the tidal tensor at the source location $[C_{ij}]_{NS}$
- ☑ express all quantities in the center of mass frame
- ☑ switch to the principal frame

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spin

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Orbital evolution

We use an hamiltonian approach to describe the BH-NS orbital evolution

$$\mathcal{H} = \mathcal{H}_{orb} + \mathcal{H}_T + \mathcal{H}_I$$

$$\mathcal{H}_{orb} = \frac{p^2}{2\mu} - \frac{Gm\mu}{r} + \mathcal{O}(2) \qquad \mathcal{H}_T = -\mathcal{L}_T = \frac{1}{2} C_{ij} I_{ij}$$

- We place a spherical star in equilibrium at $r \gg R_{NS}$ from the black hole
- We evolve numerically the equations of motion until the system reaches the distance r_{shed} at which the mass flow from the NS starts

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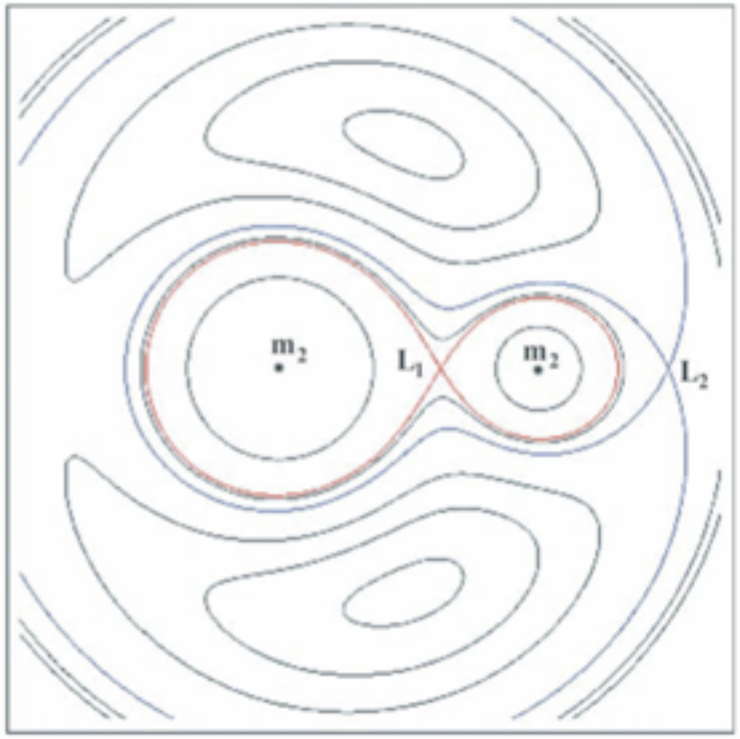
Tidal disrutption

We define r_{shed} identifying the Roche lobe surface embedding the neutron star and the Innermost Circular Orbit (ICO)

We have to compare

- ❑ NS and Roche lobe radii
- ❑ BH-NS orbital separation and ICO



$$R_{NS} > R_{roche}$$

$$r_{orb} > r_{ICO}$$

→ Tidal disruption

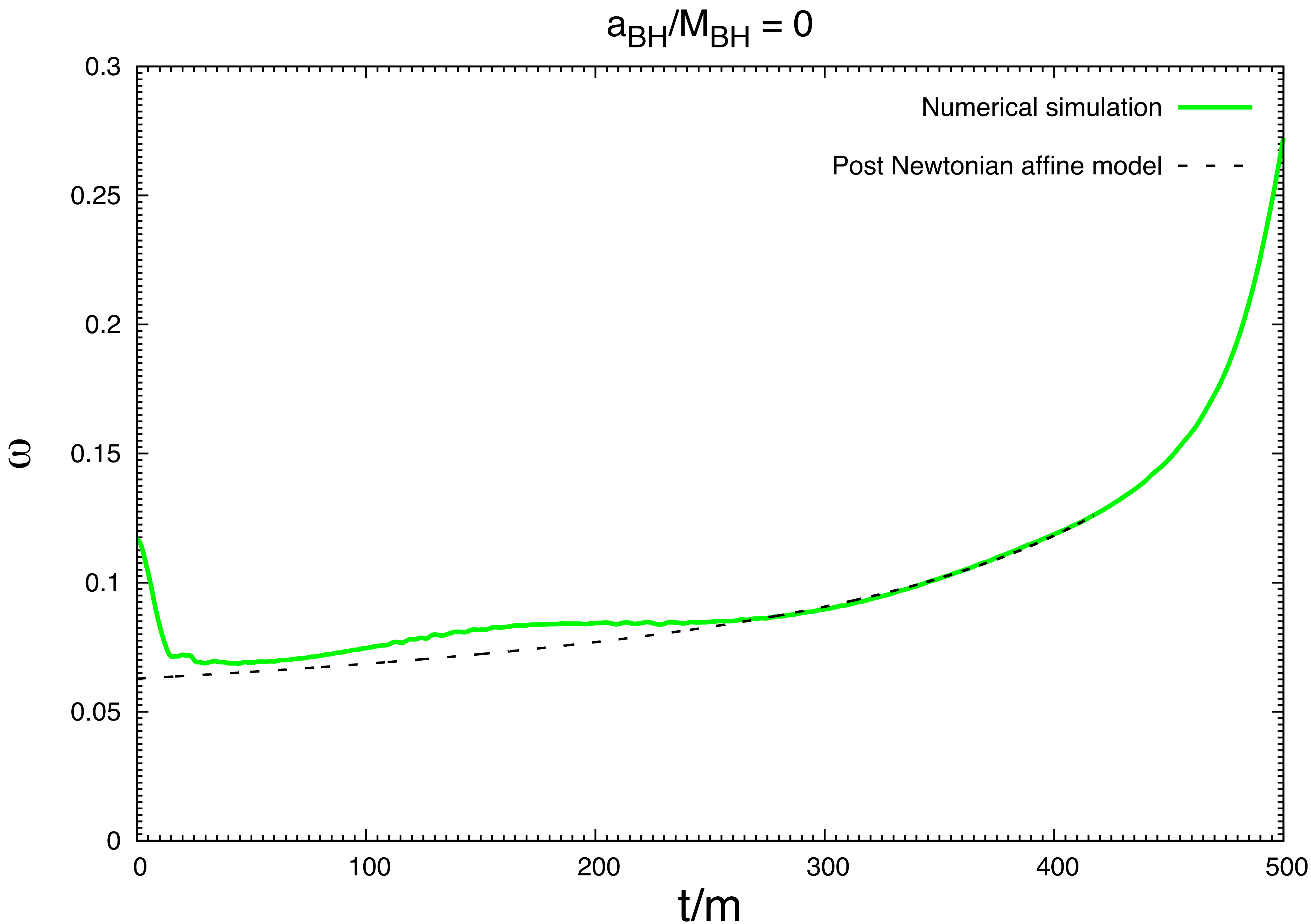
Results

We tuned our model with the numerical results for a BH-NS dynamic simulation

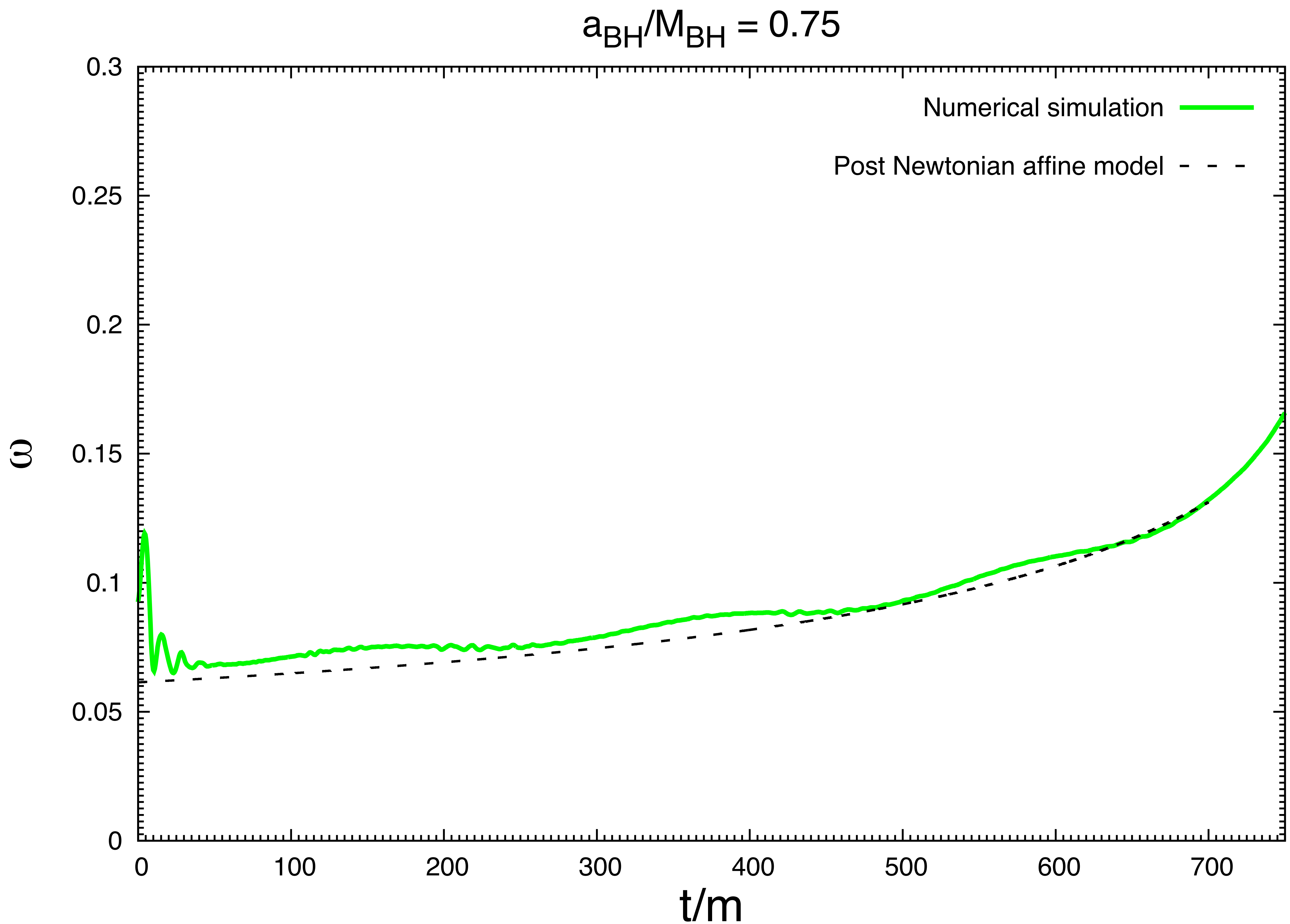
*Z. Etienne, Y. Liu, S. Shapiro and T. Baumgarte, PRD **79**, 044024 (2009)*

- Polytropic EoS
- $M_{NS} = 1.35 M_{\odot}$ and $q = M_{BH}/M_{NS} = 3$
- Black Hole spin values $a_{BH} = \{0, 0.75\} M_{BH}$

Results


$a_{BH}/M_{BH} = 0$
Numerical simulation
Post Newtonian affine model
ω
t/m
0
0.05
0.1
0.15
0.2
0.25
0.3
0
100
200
300
400
500

Results



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700

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Conclusions

We improved a semi-analytic model to describe the last phase of the inspiral in BH-NS binary systems

- ☐ We defined a unique Post-Newtonian framework to study both orbital evolution and tidal interactions

  - ☑ BH-NS dynamic evolution by means of PN equations of motion including gravitational waves dissipation and orbital corrections due to the tidal field

  - ☑ Relativistic tidal tensor up to the $1/c^5$ order, including spin terms

- ☐ We can use realistic equations of state to describe the NS internal structure (numerical simulations use only polytropic EoS)

Future Works

- ☐ Study tidal deformations using different EoS
  - ☑ Sample the parameter space $EoS \times M_{NS} \times q \times a_{BH}$
  - ☑ Investigate NS crust stress
- ☐ Characterize the features of the BH accretion disk for differents EoS
- ☐ Generalize the model in order to describe NS-NS tidal interactions
- ☐ Derive initial data for numerical relativity simulations:
  - ☑ Wide range of mass ratio values can be considered
  - ☑ Differents spin configurations for both bodies

THE END