"Single particle propagator in a many-particle system":

- Gives access to <u>all one-body observables</u>,
  - momentum distributions, $n(k)$
  - optical potential
  - spectroscopic factors
  - spectral functions

- Normally we are interested in knowing how the system reacts to an external probe. A possible perturbation is to add or to take out a particle.

Mathematically: time-ordered propagator

$$g(\alpha,\beta;t,t') = -\frac{i}{\hbar}\frac{\langle\Psi_0^A|\,T\left[a_{\alpha_H}(t)\;a^{\dagger}_{\beta_H}(t')\right]|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle}$$

* Expectation value, in the Heisenberg picture

* $|\alpha\rangle$ are a suitable basis of single particle states.
$|\vec{k}, m_\sigma, m_\tau\rangle$ in the case of a uniform system.

* $|\Psi_0^A\rangle$ is the ground state of the A-particle system which usually we do not know.

* $H|\Psi_0^A\rangle = E_0^A\,|\Psi_0^A\rangle$

* $a_{\alpha_H}(t) = e^{iHt/\hbar}\,a_\alpha\,e^{-iHt/\hbar}$

We separate in two pieces:

$$g_p(\alpha,\beta;t,t') = -\frac{i}{\hbar}\,\frac{\langle\Psi_0^A|\,a_{\alpha H}(t)\;a^+_{\beta H}(t')\,|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle} \qquad t>t'$$

$$g_h(\alpha,\beta;t,t') = \frac{i}{\hbar}\,\frac{\langle\Psi_0^A|\,a^+_{\beta H}(t')\;a_{\alpha H}(t)\,|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle} \qquad t'\geq t$$

Interpretation: $t_2>t_1$

$$g_p(\alpha,\beta;t_2,t_1) = -i\langle\Psi_0^A|\,e^{iHt_2}a_\alpha e^{-iHt_2}\,e^{iHt_1}a^+_\beta e^{-iHt_1}|$$

$e^{-iHt_1}|\Psi_0^A\rangle$ ground state at $t=t_1$

$a^+_\beta e^{-iHt_1}|\Psi_0^A\rangle$ one particle in the state β was added at $t=$

$e^{-iH(t_2-t_1)}a^+_\beta e^{-iHt_1}|\Psi_0^A\rangle$ the state evolves up to $t=$

$$\langle\alpha| \equiv \underbrace{\left(a^+_\alpha e^{-iHt_2}|\Psi_0^A\rangle\right)^+}_{\text{one particle added in the state } \alpha \text{ at } t=t_2} = \langle\Psi_0^A|\,e^{iHt_2}a_\alpha$$

$g_p(\alpha,\beta;t_2,t_1)$ gives the probability amplitude to find the system at $t=t_2$ with an additional particle in the state $|\alpha\rangle$, when at time $t=t_1$ a particle in the state $|\beta\rangle$ was added to the system.

$g(\alpha,\beta;t,t')$ depends on the time difference

$$g(\alpha,\beta;t,t') = -\frac{i}{\hbar}\,\theta(t-t')\,\frac{\langle\Psi_0^A|\,a_{\alpha H}(t)\,a^+_{\beta H}(t')\,|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle} + \frac{i}{\hbar}\,\theta(t'-t)\,\frac{\langle\Psi_0^A|\,a^+_{\beta H}(t')\,a_{\alpha H}(t)\,|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle}$$

$$= -\frac{i}{\hbar}\Big\{\theta(t-t')\,\frac{\langle\Psi_0^A|e^{iHt/\hbar}\,a_\alpha\,e^{-iH(t-t')/\hbar}\,a^+_\beta\,e^{-iHt'/\hbar}|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle} - \theta(t'-t)\,\frac{\langle\Psi_0^A|e^{iHt'/\hbar}\,a^+_\beta\,e^{-iH(t'-t)/\hbar}\,a_\alpha\,e^{iHt/\hbar}|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle}\Big\}$$

$$= -\frac{i}{\hbar}\Big\{\theta(t-t')\,e^{iE_0^A(t-t')/\hbar}\,\frac{\langle\Psi_0^A|a_\alpha\,e^{-iH(t-t')/\hbar}\,a^+_\beta|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle} - \theta(t'-t)\,e^{iE_0^A(t'-t)/\hbar}\,\frac{\langle\Psi_0^A|a^+_\beta\,e^{-iH(t-t')/\hbar}\,a_\alpha|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle}$$

Introducing the identity : $\sum_m |\Psi_m^{A+1}\rangle\langle\Psi_m^{A+1}|$

$\sum_n |\Psi_n^{A-1}\rangle\langle\Psi_n^{A-1}|$

$$= -\frac{i}{\hbar}\Big\{\theta(t-t')\sum_m e^{i(E_0^A-E_m^{A+1})(t-t')/\hbar}\,\frac{\langle\Psi_0^A|a_\alpha|\Psi_m^{A+1}\rangle\langle\Psi_m^{A+1}|a^+_\beta|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle} - \theta(t'-t)\sum_n e^{i(E_0^A-E_n^{A-1})(t'-t)/\hbar}\,\frac{\langle\Psi_0^A|a^+_\beta|\Psi_n^{A-1}\rangle\langle\Psi_n^{A-1}|a_\alpha|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle}$$

Performing the Fourier transform and introducing the integral representation of the step function:

$$\theta(t-t') = -\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi i} \frac{e^{-i\omega'(t-t')}}{\omega'+i\eta} \rightsquigarrow -\int_{-\infty}^{\infty} \frac{dE'}{2\pi i} \frac{e^{-i\frac{E'}{\hbar}(t-t')}}{E'+i\eta}$$

$$\eta \to 0^+$$

$$g(\alpha,\beta;\overset{E}{\omega}) = \int_{-\infty}^{\infty} d(t-t')\, e^{i\underset{\frac{E}{\hbar}}{\omega}(t-t')}\, g(\alpha,\beta;t-t') \Rightarrow$$

$$\boxed{\begin{aligned} g(\alpha,\beta;\overset{E}{\omega}) = & \sum_m \frac{\langle \Psi_0^A | a_\alpha | \Psi_m^{A+1} \rangle \langle \Psi_m^{A+1} | a_\beta^\dagger | \Psi_0^A \rangle}{\underbrace{\hbar\omega}_{E} - (E_m^{A+1} - E_0^A) + i\eta} \\ & + \sum_n \frac{\langle \Psi_0^A | a_\beta^\dagger | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle}{\underbrace{\hbar\omega}_{E} - (E_0^A - E_n^{A-1}) - i\eta} \end{aligned}}$$

Lehmann representation!

The Green function is an expectation value!

$$\boxed{\begin{aligned} g(\alpha,\beta;\overset{E}{\omega}) = & \langle \Psi_0^A | a_\alpha \frac{1}{\underbrace{\hbar\omega}_{E} - (\hat{H} - E_0^A) + i\eta} a_\beta^\dagger | \Psi_0^A \rangle \\ & + \langle \Psi_0^A | a_\beta^\dagger \frac{1}{\underbrace{\hbar\omega}_{E} - (E_0^A - \hat{H}) - i\eta} a_\alpha | \Psi_0^A \rangle \end{aligned}}$$

Analytical structure: It Has simple poles at the excitation energies.

E-plane
Complex ω-plane

$$g(\alpha,\beta;E) = \int_{-\infty}^{\infty} d(t-t')\, e^{iE(t-t')/\hbar}\, g(\alpha,\beta;t-t') =$$

$$= \int_{-\infty}^{\infty} d(t-t')\, e^{i\frac{E}{\hbar}(t-t')} \Big[-\frac{i}{\hbar}\Big\{\theta(t-t')\sum_m e^{i(E_0^A - E_m^{A+1})(t-t')/\hbar} \frac{\langle\Psi_0^A|a_\alpha|\Psi_m^{A+1}\rangle\langle\Psi_m^{A+1}|a_\beta^\dagger|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle}$$

$$-\theta(t'-t)\sum_n e^{i(E_0^A-E_n^{A-1})(t-t')/\hbar}\frac{\langle\Psi_0^A|a_\beta^\dagger|\Psi_n^{A-1}\rangle\langle\Psi_n^{A+1}|a_\alpha|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle}\Big\}\Big]$$

$$=\int_{-\infty}^{\infty} d(t-t')\, e^{i\frac{E}{\hbar}(t-t')}\Big[-\frac{i}{\hbar}\Big\{-\frac{1}{2\pi i}\int_{-\infty}^{\infty} dE'\, \frac{e^{-iE'(t-t')}}{E'+i\eta^+}\sum_m e^{i(E_0^A-E_m^{A+1})(t-t')/\hbar}\frac{\langle\Psi_0^A|a_\alpha|\Psi_m^{A+1}\rangle\langle\Psi_m^{A+1}|a_\beta^\dagger|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle}$$

$$-\Big(-\frac{1}{2\pi i}\int_{-\infty}^{\infty} dE'\, \frac{e^{-i\frac{E'}{\hbar}(t'-t)}}{E'+i\eta^+}\sum_n e^{i(E_0^A-E_n^{A-1})(t-t')/\hbar}\quad\cdots\Big\}\Big]$$

changing the order of integration →

$$-(-i)\int_{-\infty}^{\infty}\frac{dE'}{2\pi i}\int_{-\infty}^{\infty} d\Big(\frac{t-t'}{\hbar}\Big)\sum_m \frac{e^{-i[E'-(E_0^A-E_m^{A+1})-E]\frac{t-t'}{\hbar}}}{E'+i\eta^+}\,\frac{\langle\Psi_0^A|a_\alpha|\Psi_n^{A+1}\rangle\langle\Psi_n^{A+1}|a_\beta^\dagger|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle}$$

$$+(-i)\int_{-\infty}^{\infty}\frac{dE'}{2\pi i}\int d\Big(\frac{t-t'}{\hbar}\Big)\sum_n \frac{e^{-i[-E'+(E_0^A-E_n^{A-1})-E]\frac{t-t'}{\hbar}}}{E'+i\eta^+}\,\frac{\langle\Psi_0^A|a_\beta^\dagger|\Psi_n^{A-1}\rangle\langle\Psi_n^{A-1}|a_\alpha|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle}$$

$$=-(-i)\int\frac{dE'}{2\pi i}\,2\pi\sum_m\frac{\delta(E'-(E_0^A-E_m^{A+1})-E)}{E'+i\eta^+}\,\frac{\langle\Psi_0^A|a_\alpha|\Psi_m^{A+1}\rangle\langle\Psi_m^{A+1}|a_\beta^\dagger|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle}$$

$$-i\int\frac{dE'}{2\pi i}\,2\pi\sum_n\frac{\delta(-E'+(E_0^A-E_n^{A-1})-E)}{E'+i\eta^+}\,\frac{\langle\Psi_0^A|a_\beta^\dagger|\Psi_n^{A-1}\rangle\langle\Psi_n^{A-1}|a_\alpha|\Psi_0^A\rangle}{\langle\Psi_0^A|\Psi_0^A\rangle}$$

the δ allows to perform the integral:

$$=\sum_m\frac{\langle\Psi_0^A|a_\alpha|\Psi_m^{A+1}\rangle\langle\Psi_m^{A+1}|a_\beta^\dagger|\Psi_0^A\rangle}{E+E_0^A-E_m^{A+1}+i\eta^+} - \sum_n\frac{\langle\Psi_0^A|a_\beta^\dagger|\Psi_n^{A-1}\rangle\langle\Psi_n^{A-1}|a_\alpha|\Psi_0^A\rangle}{-E+E_0^A-E_n^{A-1}+i\eta^+}$$

$$+\sum_n\frac{\langle\Psi_0^A|a_\beta^\dagger|\Psi_n^{A-1}\rangle\langle\Psi_n^{A-1}|a_\alpha|\Psi_0^A\rangle}{E-E_0^A+E_n^{A-1}-i\eta^+}\,.$$

The information is not only in the denominator.

The numerator contains the strength distribution when we pass from one state of A particles to one state with (A-1) or (A+1) particles $\Rightarrow$ This is crucial to understand the single-particle behavior. In a system without correlations, i.e described by one Slater determinant, when we destroy one of the states of the Slater determinant, we can go only to one $|\psi^{A-1}\rangle$ state $\Rightarrow$ all the strength is concentrat

$$S_h(\alpha,\alpha;\overset{E}{\omega}) = \frac{1}{\pi}\,\mathrm{Im}\,g(\alpha,\alpha;\overset{E}{\omega}) \qquad \overset{E<\varepsilon_F}{\hbar\omega<\varepsilon_F}$$

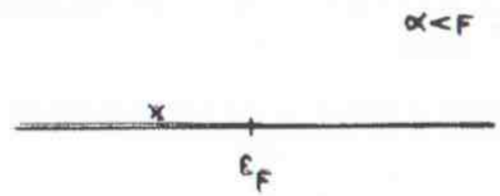
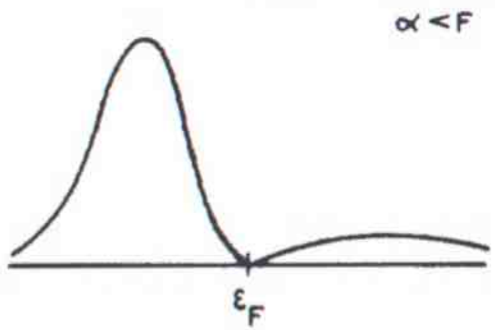
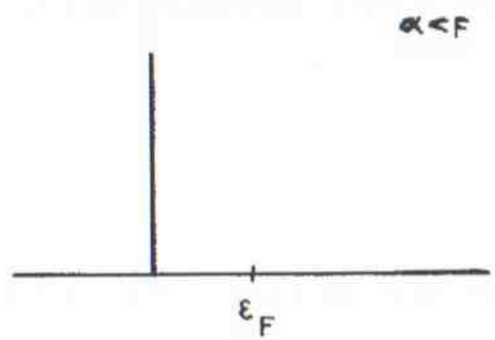
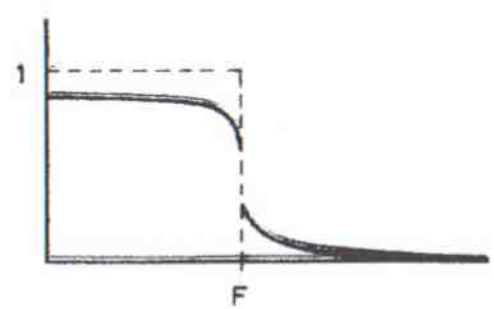
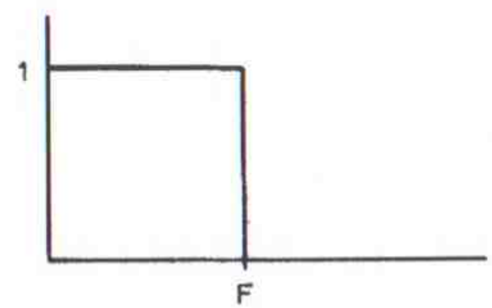
$$S_p(\alpha,\alpha;\overset{E}{\omega}) = -\frac{1}{\pi}\,\mathrm{Im}\,g(\alpha,\alpha;\overset{E}{\omega}) \qquad \overset{E>\varepsilon_F}{\hbar\omega>\varepsilon_F}$$

The spectral functions are contained in the single-particle Green function!

Using: $$\frac{1}{\omega\pm i\eta} = \mathcal{P}\frac{1}{\omega} \mp i\pi\,\delta(\omega) \qquad \text{we get.}$$

$$\frac{1}{\pi}\,\mathrm{Im}\,g(\alpha,\alpha;E) = \sum_n |\langle\Psi_n^{A-1}|\,a_\alpha\,|\Psi_0^A\rangle\,\delta(E-(E_0^A-E_n^{A-1}))$$

$$\boxed{\frac{1}{E\pm i\eta} = \mathcal{P}\frac{1}{E} \mp i\pi\,\delta(E)}$$

sp propagator $g(k,\omega)$	$\int_{\epsilon_F}^{\infty} d\omega' \frac{S_p(\alpha,\omega')}{\omega-\omega'+i\eta} + \int_{-\infty}^{\epsilon_F} d\omega' \frac{S_h(\alpha,\omega')}{\omega-\omega'-i\eta}$	$\frac{\theta(\alpha-F)}{\omega-\epsilon_\alpha+i\eta} + \frac{\theta(F-\alpha)}{\omega-\epsilon_\alpha-i\eta}$
analytic structure		
spectral functions		
momentum distribution		

$$g(\alpha,\alpha,E) = \sum_m \frac{\langle\Psi_0^A|a_\alpha|\Psi_m^{A+1}\rangle\langle\Psi_m^{A+1}|a_\beta^\dagger|\Psi_0^A\rangle}{E+E_0^A-E_m^{A+1}+i\eta^+} + \sum_n \frac{\langle\Psi_0^A|a_\beta^\dagger|\Psi_n^{A-1}\rangle\langle\Psi_n^{A-1}|a_\alpha|\Psi_0^A\rangle}{E-E_0^A+E_n^{A-1}-i\eta^+}$$

$$\frac{1}{E\pm i\eta} = P\frac{1}{E} \mp i\pi\delta(E)$$

$$\boxed{\begin{aligned} S_h(\alpha,E) &= \frac{1}{\pi}\,\mathrm{Im}\, g(\alpha,\alpha,E) \qquad\qquad E<\mu \\ &= \sum_n |\langle\Psi_n^{A-1}|a_\alpha|\Psi_0^A\rangle|^2\,\delta\left(E-(E_0^A-E_n^{A-1})\right)\end{aligned}}$$

gives the probability to take out a particle in the state α from the ground state leaving the resulting A-1 system with energy $E_n^{A-1} = E_0^A - E$

$$\boxed{\begin{aligned} S_p(\alpha,E) &= -\frac{1}{\pi}\,\mathrm{Im}\, g(\alpha,\alpha,E) \qquad\qquad E>\mu \\ &= \sum_m |\langle\Psi_m^{A+1}|a_\alpha^\dagger|\Psi_0^A\rangle|^2\,\delta\left(E-(E_m^{A+1}-E_0^A)\right)\end{aligned}}$$

gives the probability to add a particle in the state α to the ground state of a system leaving the resulting A+1 system with energy $E_m^{A+1} = E_0^A + E$

Occupation of the state $|\alpha\rangle$.

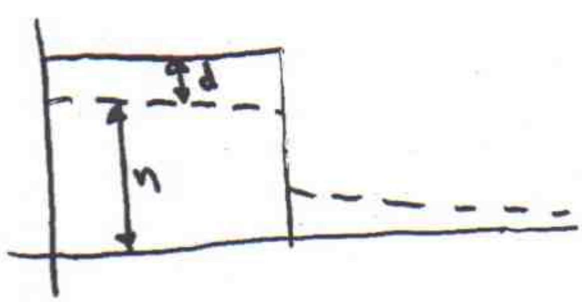
$$n(\alpha) = \langle \Psi_0^A | a_\alpha^+ a_\alpha | \Psi_0^A \rangle$$
$$= \sum_n | \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle |^2$$
$$= \int_{-\infty}^{\varepsilon_F} dE \sum_n | \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle |^2 \delta(E - (E_0^A - E_n^{A-1}))$$
$$= \int_{-\infty}^{\varepsilon_F} dE \; S_h(\alpha, E)$$

The same for desoccupation!

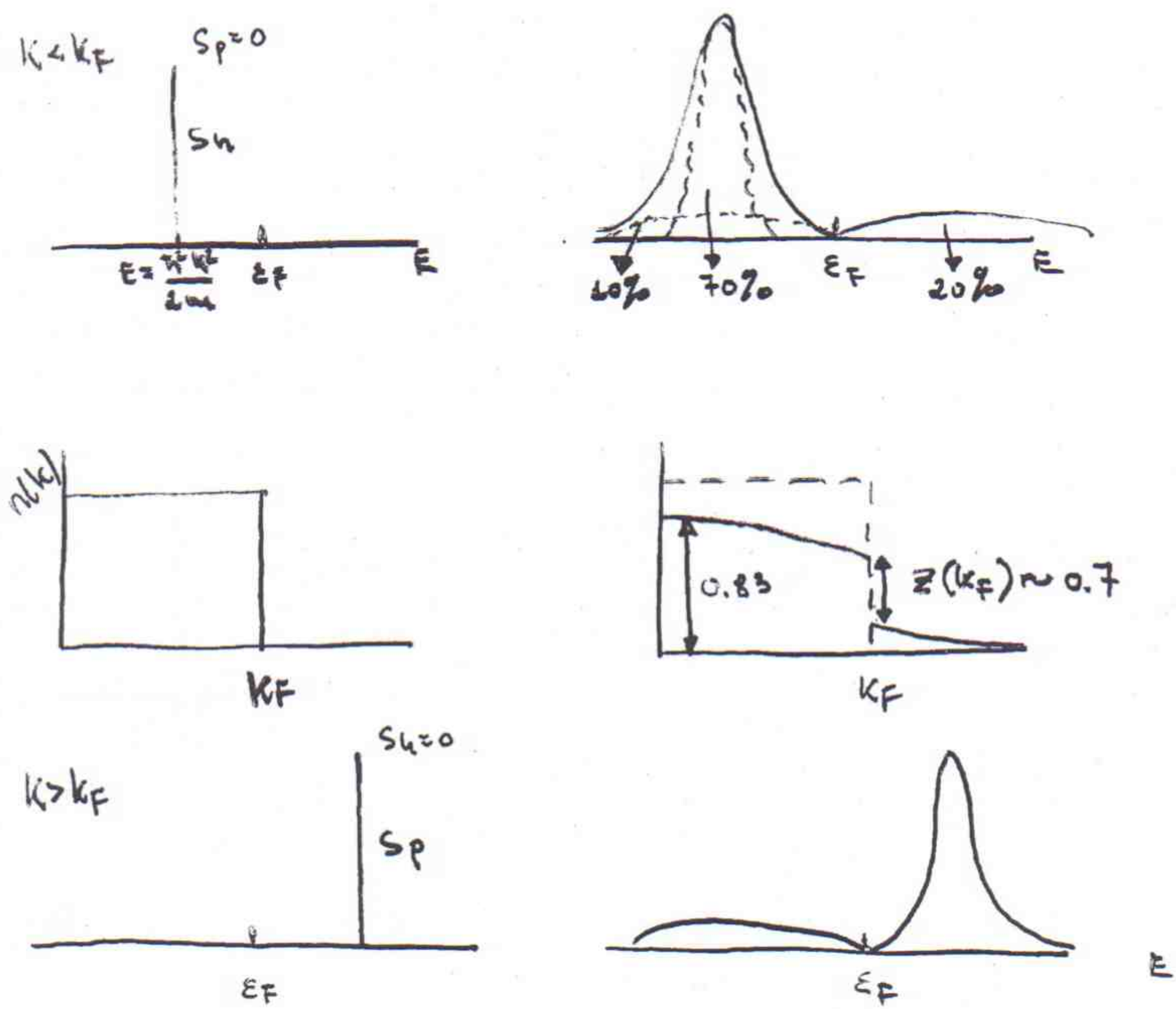
$$d(\alpha) = \langle \Psi_0^A | a_\alpha a_\alpha^+ | \Psi_0^A \rangle = \int_{\varepsilon_F}^{\infty} dE \; S_p(\alpha, E)$$

$$n(\alpha) + d(\alpha) = 1 = \langle \Psi_0^A | a_\alpha^+ a_\alpha + a_\alpha^+ a_\alpha | \Psi_0^A \rangle.$$

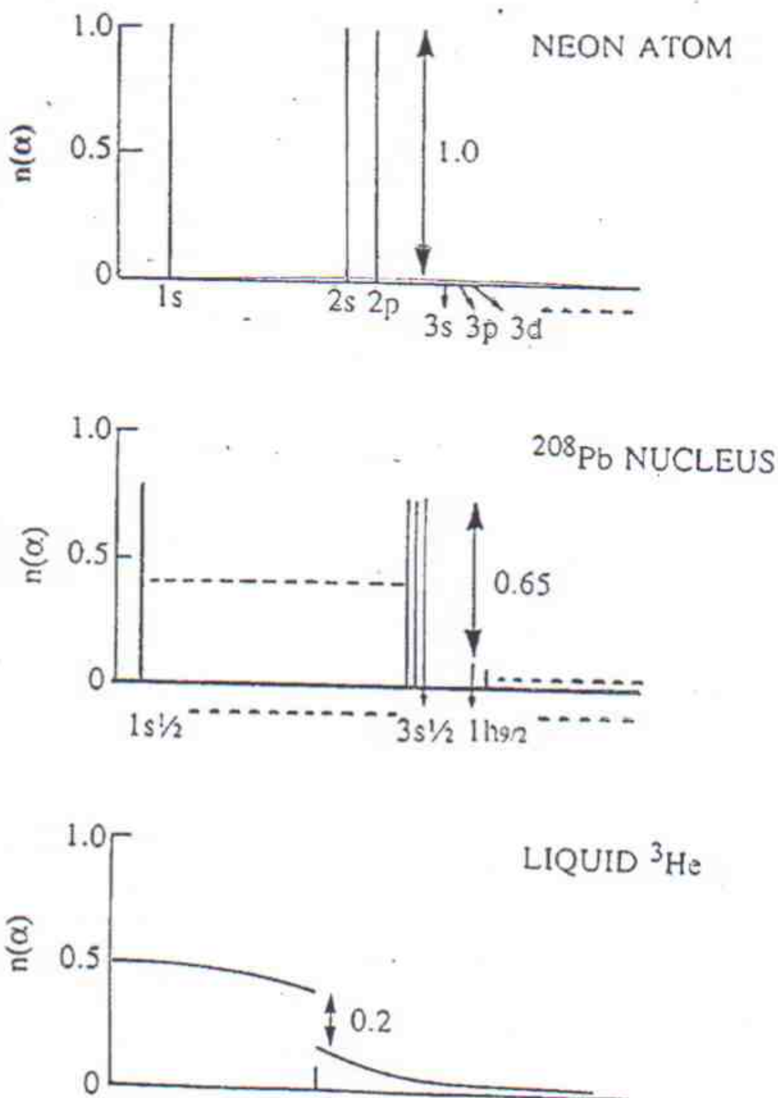
$$\{a_\alpha^+, a_\alpha\} = 1$$



• microscopic calculations have been performed mainly for: nuclear matter



Nuclear matter: $\begin{cases} e \sim -16 \text{ MeV} & \rho_0 = 0.17 \text{ nucl/fm}^3 \\ \frac{1}{\rho_0} = 5.88 \text{ fm}^3 & \langle T \rangle \sim 46 \text{ MeV} \\ \frac{\hbar^2}{2m} \sim 20 \text{ MeV}\cdot\text{fm}^2 & \langle E_{FS} \rangle \sim 23. \text{ MeV} \\ & k_F = 1.36 \text{ fm}^{-1} \end{cases}$



^{3}He : $\begin{cases} \text{the interaction is simpler} \\ \text{it is denser than nuclear matter.} \end{cases}$

$e_0 = -2.47$ K $\qquad \frac{\hbar^2}{2m_3} \sim 8.03$ K·Å^2

$\rho_0 = 0.0164 \frac{\text{Atoms}}{\text{Å}^3} \qquad \langle T \rangle \sim 12$ K $\qquad \langle E_{FS} \rangle \sim 3$ K

$k_F \sim 0.78$ Å^{-1}

$\frac{1}{\rho_0} = 61$ Å^3

Expectation value of any one-body operator.

$$\hat{O} = \sum_{\alpha\beta} \langle \alpha | O | \beta \rangle \, a_\alpha^+ a_\beta$$

$$\langle \Psi_0^A | \hat{O} | \Psi_0^A \rangle = \sum_{\alpha\beta} \langle \alpha | O | \beta \rangle \, n_{\alpha\beta}$$

$$n_{\alpha\beta} = \langle \Psi_0^A | a_\alpha^+ a_\beta | \Psi_0^A \rangle$$

$$n_{\alpha\beta} = \oint \frac{dE}{2\pi i} \, g(\alpha, \beta; E)$$

I will catch only the poles of the upper plane.

Remember:

$$g(\alpha,\beta;E) = \boxed{\sum_m \frac{\langle \Psi_0^A | a_\alpha | \Psi_m^{A+1} \rangle \langle \Psi_m^{A+1} | a_\beta^+ | \Psi_0^A \rangle}{E - (E_m^{A+1} - E_0^A) + i\eta}}$$

→ does not contribute (poles below)

$$+ \sum_n \frac{\langle \Psi_0^A | a_\beta^+ | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle}{E - (E_0^A - E_n^{A-1}) - i\eta}$$

→ poles ~~do~~ in the upper part.

Residous:

$$\lim_{E \to (E_0^A - E_n^{A-1}) + i\eta} g(\alpha,\beta;E) \left(E - (E_0^A - E_n^{A-1}) - i\eta\right)$$

$$= \langle \Psi_0^A | a_\beta^+ | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle$$

$$\Rightarrow \oint \frac{dE}{2\pi i} \, g(\alpha,\beta;E) = \cancel{2\pi} i \frac{1}{\cancel{2\pi i}} \sum \mathrm{Res} =$$

$$= \sum_n \langle \Psi_0^A | a_\beta^+ | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle =$$

$$= \langle \Psi_0^A | a_\beta^+ a_\alpha | \Psi_0^A \rangle = n_{\beta\alpha}$$

One can also calculate special two-body operator: H

$$\hat{H} = \sum_{rs} \langle r|t|s\rangle a_r^{+} a_s + \frac{1}{4} \sum_{rstu} \langle rs|V|tu\rangle a_r^{+} a_s^{+} a_u a_t$$

$$\langle rs|V|tu\rangle = \langle rs|V|tu - ut)$$ antisymmetrized two-body matrix element.

$$\boxed{\oint \frac{dE}{2\pi i} E\, g(\alpha,\alpha,E)}\ ?$$

$$\oint \frac{dE}{2\pi i} E\, g(\alpha,\alpha;E) = \oint \frac{dE}{2\pi i} E \Bigg[\sum_n \frac{\langle \Psi_0^N | a_\alpha | \Psi_n^{N+1}\rangle \langle \Psi_n^{N+1} | a_\alpha^{+} | \Psi_0^N\rangle}{E - (E_n^{N+1} - E_0^N) + i\eta}$$

$$+ \sum_m \frac{\langle \Psi_0^N | a_\alpha^{+} | \Psi_m^{N-1}\rangle \langle \Psi_m^{N-1} | a_\alpha | \Psi_0^N\rangle}{E - (E_0^N - E_m^{N-1}) - i\eta} \Bigg] = 2\pi i \sum R$$

$$R_j = \lim_{\substack{E \to (E_0^N - E_j^{N-1}) + i\eta \\ \eta \to 0}} \frac{E}{2\pi i} \frac{\langle \Psi_0^N | a_\alpha^{+} | \Psi_j^{N-1}\rangle \langle \Psi_j^{N-1} | a_\alpha | \Psi_0^N\rangle}{\cancel{E - (E_0^N - E_j^{N-1}) - i\eta}} \cancel{(E - (E_0^N - E_j^{N-1}) - i\eta)}$$

$$= \frac{E_0^N - E_j^{N-1}}{2\pi i} \langle \Psi_0^N | a_\alpha^{+} | \Psi_j^{N-1}\rangle \langle \Psi_j^{N-1} | a_\alpha | \Psi_0^N\rangle$$

$$\boxed{\oint \frac{dE}{2\pi i} E\, g(\alpha,\alpha;E) = \sum_m (E_0^N - E_m^{N-1}) \langle \Psi_0^N | a_\alpha^{+} | \Psi_m^{N-1}\rangle \langle \Psi_m^{N-1} | a_\alpha | \Psi_0^N\rangle}$$

$$\to \langle \Psi_0^N | a_\alpha^{+} a_\alpha E_0^N | \Psi_0^N\rangle - \sum_m \langle \Psi_0^N | a_\alpha^{+} E_m^{N-1} | \Psi_m^{N-1}\rangle \langle \Psi_m^{N-1} | a_\alpha | \Psi_0^N\rangle$$

$$= \langle \Psi_0^N | a_\alpha^{+} a_\alpha \hat{H} | \Psi_0^N\rangle - \sum_m \langle \Psi_0^N | a_\alpha^{+} \hat{H} | \Psi_m^{N-1}\rangle \langle \Psi_m^{N-1} | a_\alpha | \Psi_0^N\rangle$$

$$= \langle \Psi_0^N | a_\alpha^+ a_\alpha \hat{H} | \Psi_0^N \rangle - \langle \Psi_0^N | a_\alpha^+ \hat{H} a_\alpha | \Psi_0^N \rangle =$$

$$= \langle \Psi_0^N | a_\alpha^+ [a_\alpha . \hat{H}] | \Psi_0^N \rangle$$

$$\boxed{\oint \frac{dE}{2\pi i} E \, g(\alpha, \alpha, E) = \langle \Psi_0^N | a_\alpha^+ [a_\alpha, \hat{H}] | \Psi_0^N \rangle}$$

$$[a_\alpha, \hat{T}] = \sum_\beta \langle \alpha | T | \beta \rangle a_\beta$$

$$[a_\alpha, \hat{V}] = \frac{1}{2} \sum_{\beta, \gamma, \delta} (\alpha \beta | V | \gamma \delta) a_\beta^+ a_\delta a_\gamma$$

$$\boxed{\begin{aligned} \langle \Psi_0^N | a_\alpha^+ [a_\alpha, \hat{H}] | \Psi_0^N \rangle &= \sum_\beta \langle \Psi_0^N | a_\alpha^+ a_\beta | \Psi_0^N \rangle \langle \alpha | T | \beta \rangle \\ &+ \frac{1}{2} \sum_{\beta \gamma \delta} \langle \Psi_0^N | a_\alpha^+ a_\beta^+ a_\delta a_\gamma | \Psi_0^N \rangle (\alpha \beta | V | \gamma \delta) \end{aligned}}$$

$$\sum_\alpha \oint \frac{dE}{2\pi i} E \, g(\alpha, \alpha; E) = \langle T \rangle + 2 \langle V \rangle$$

$$\oint \frac{dE}{2\pi i} \sum_{\alpha \beta} \langle \alpha | T | \beta \rangle g(\beta, \alpha; E) = \langle T \rangle$$

$$\boxed{\langle \hat{H} \rangle = \langle \hat{T} + \hat{V} \rangle = \frac{1}{2} \oint \frac{dE}{2\pi i} \sum_{\alpha \beta} \left[\langle \alpha | T | \beta \rangle + E \, \delta_{\alpha \beta} \right] g(\beta, \alpha; E)}$$

$$\oint \frac{dE}{2\pi i} E \, g(\alpha,\alpha; E) = \sum_n (E_0^N - E_n^{N-1}) \, |\langle \Psi_n^{N-1} | a_\alpha | \Psi_0^N \rangle|^2$$

$$= \int_{-\infty}^{\varepsilon_F} dE \; E \underbrace{\sum_n |\langle \Psi_n^{N-1} | a_\alpha | \Psi_0^N \rangle|^2 \, \delta\left(E - (E_0^N - E_n^{N-1})\right)}_{S_h(\alpha, E)}$$

$$= \int_{-\infty}^{\varepsilon_F} dE \; E \, S_h(\alpha, E)$$

For a uniform system $\Rightarrow$ single particle wave-functio[n]
= plane waves

$$\langle \hat{H} \rangle = \frac{1}{2} \sum_k \int_{-\infty}^{\varepsilon_F} dE \left(\frac{\hbar^2 k^2}{2m} + E \right) S_h(k, E)$$

$\rightarrow$ energy per particle $\frac{1}{N} \sum_k \longrightarrow \frac{1}{(2\pi)^3 \rho} \int d^3k$

$$\boxed{\frac{1}{N} \langle \hat{H} \rangle = \frac{1}{2} \frac{1}{(2\pi)^3 \rho} \int d^3k \int_{-\infty}^{\varepsilon_F} dE \left(\frac{\hbar^2 k^2}{2m} + E \right) S_h(k, E)}$$

no spin-isospin included.