Introduction

Consider an observable R(E) and an integral transform $\Phi(\sigma)$:

$$\Phi(\sigma) = \int dE\ K(\sigma,E)\ R(E)$$

with some kernel $K(\sigma,E)$

Often it is easier to calculate $\Phi(\sigma)$ than R(E). Then the observable R(E) can be obtained via inversion of the integral transform.

In order to make the inversion sufficiently stable the kernel $K(\sigma,E)$ should resemble a kind of energy filter (Lorentzians, Gaussians, ...); best choice would be a δ-function.

For the LIT we consider Lorentzians: $K(\sigma,E) = [(E-\sigma_R)^2 + \sigma_I^2]^{-1}$

Inclusive response functions have the following form

$$R(\omega) = \sum\!\!\!\!\!\!\int_n |\langle n|\Theta|0\rangle|^2 \, \delta(\omega - E_n + E_0)$$

where we have set for q=const: $R(\omega,q) \rightarrow R(\omega)$

$|0\rangle, |n\rangle$ and E_0 , E_n are eigen states and corresponding eigen energies of Hamiltonian H and Θ is transition operator inducing the reaction

main point of the LIT :

Schrödinger-like equation with a source

$$(H - E_0 - \omega_0 + i\Gamma)\,\tilde{\Psi} = S$$

The $\tilde{\Psi}$ solution is unique and has ***bound state like*** asymptotic behavior

→ one can apply ***bound state methods***

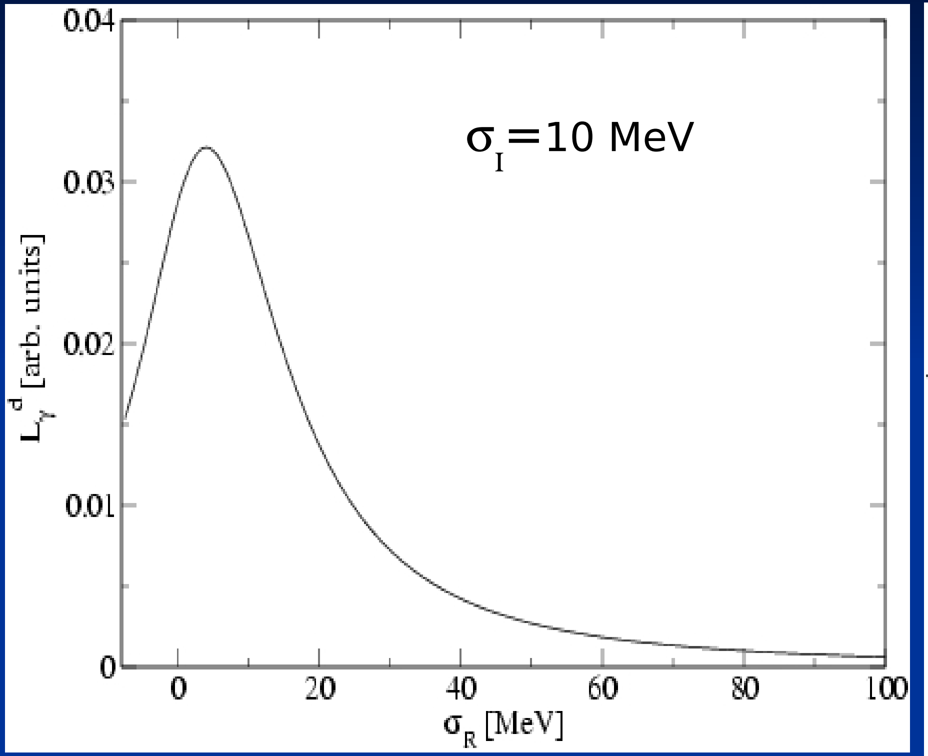
LIT - Example

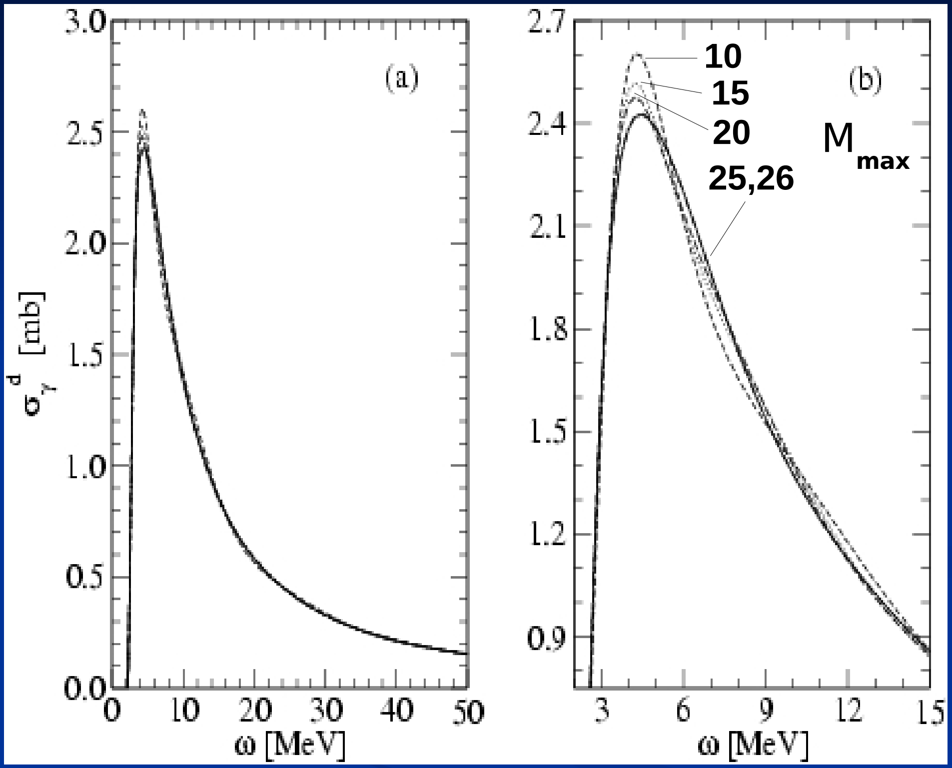
deuteron photodisintegration in unretarded dipole approximation

unretarded dipole approximation $\Rightarrow \Theta = \sum_{i=1}^{A} z_i \frac{1+\tau_{i,z}}{2}$,

z_i , $\tau_{i,z}$: 3rd components of position and isospin coordinates of i-th nucleon

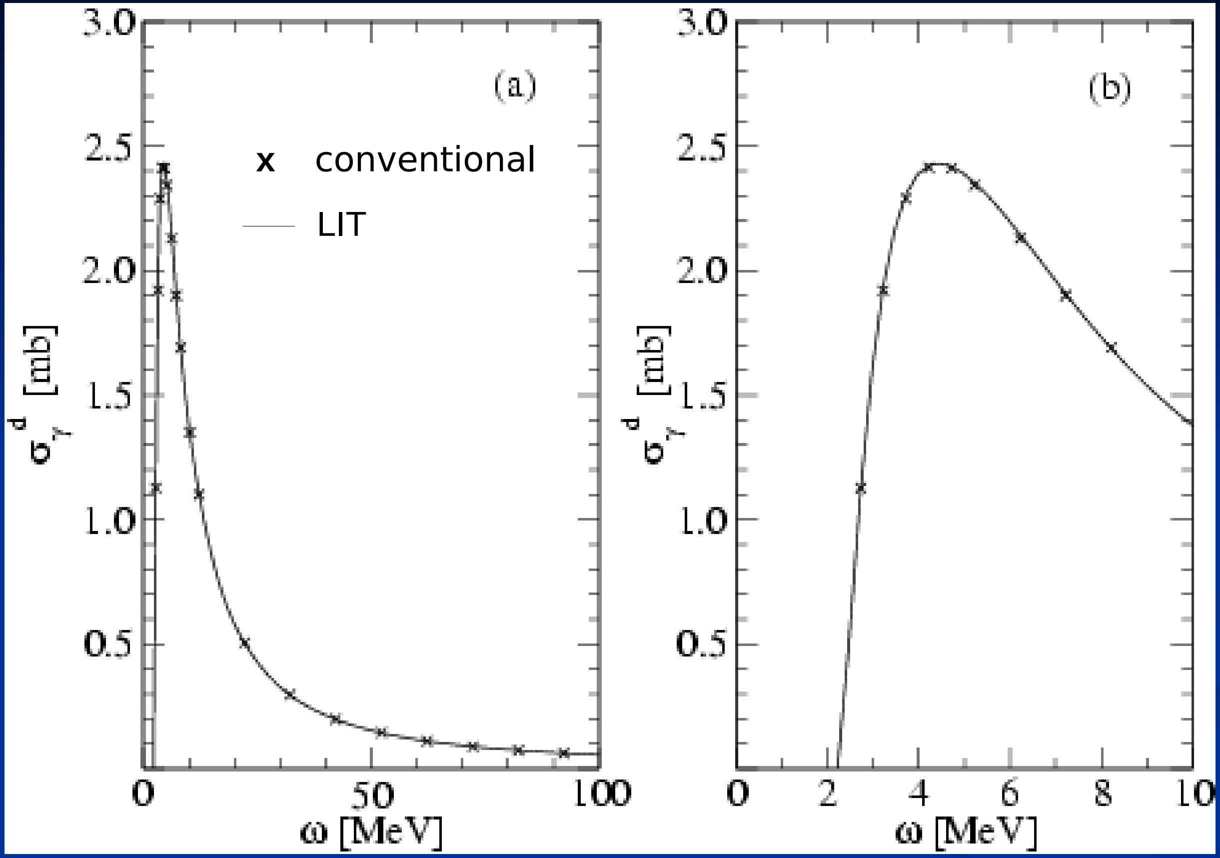
NN interaction: Argonne V14 potential





LIT

$\sigma_\gamma(\omega)$ from inversion with various M_{max}

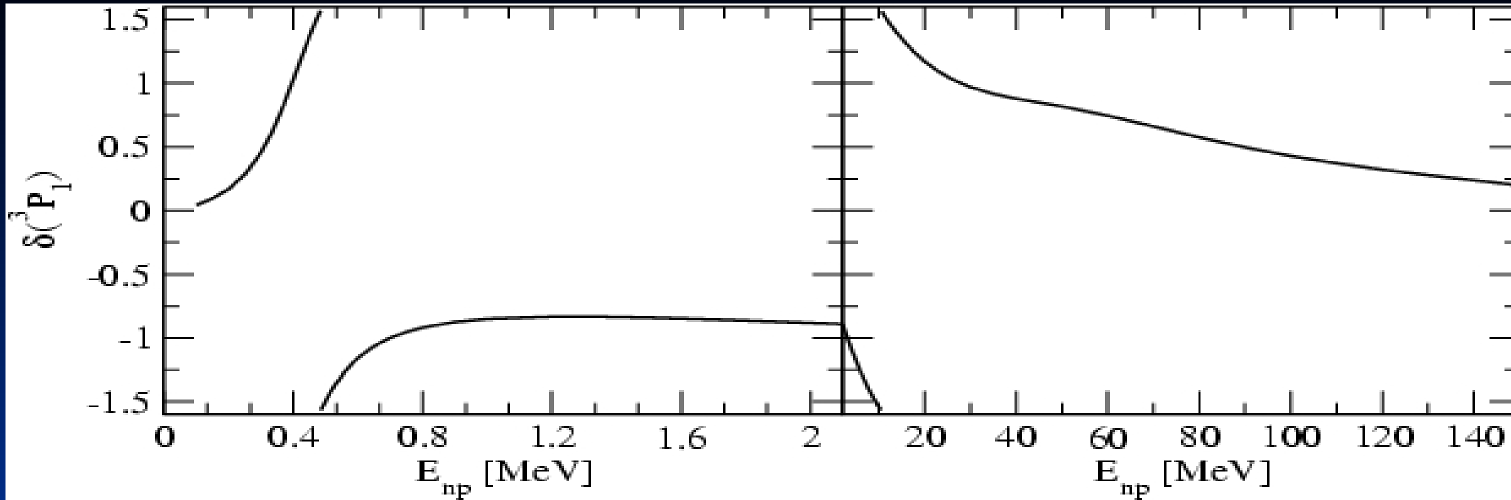


$\sigma_\gamma(\omega)$ from inversion with various M_{max} = 25

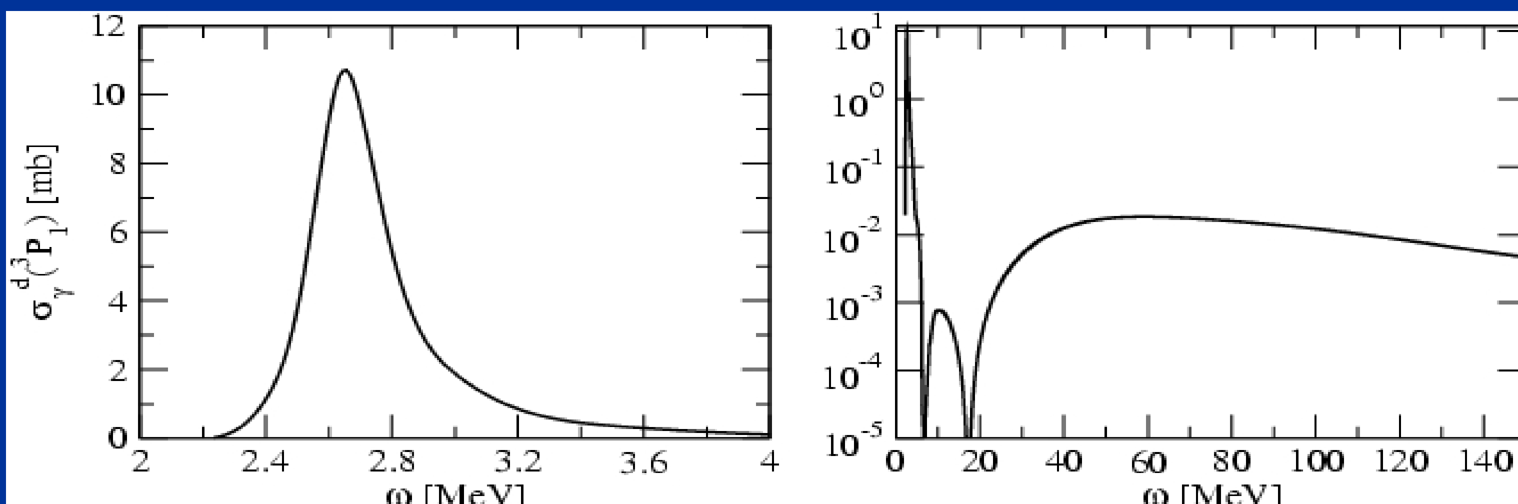
and result from conventional calculation with explicit np continuum wave functions

LIT method and resonances

The LIT: a method with a controlled resolution



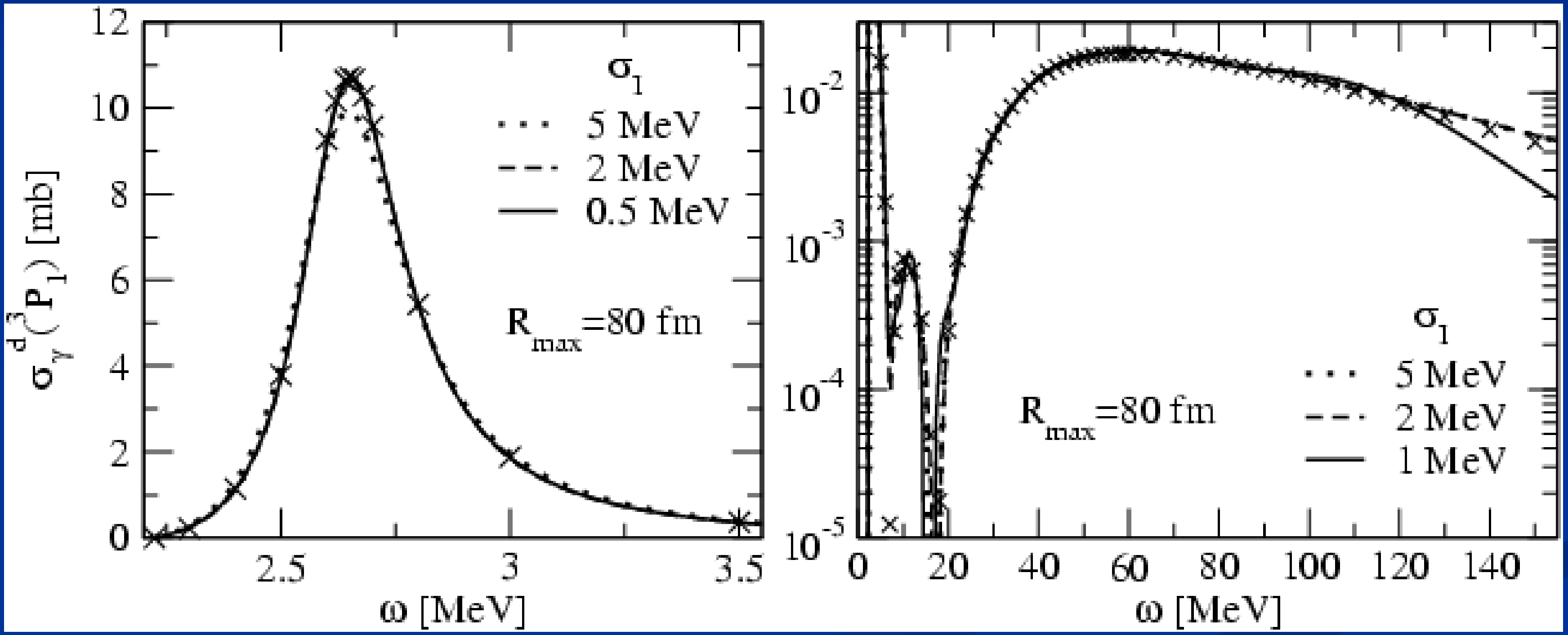
Phase shifts shows two resonances one at $E_{\mathbf{np}}$ = 0.48, 10.5 MeV



$\sigma_{\gamma}(^3P_1)$ shows two corresponding resonances: low-energy resonance very pronounced with small width Γ=270 KeV, the other one is much weaker and has a larger width

Complete inversion with set χ_m defined previously using in addition as new first basis function χ_1^{res}

various σ_I , R_{max} = 80 fm, M_{max} =30



Up to now direct numerical solutions of Schrödinger equation for bound state and LIT equation for $\widetilde{\Psi}$

For A > 2 it is more convenient to use expansions in complete sets using expansions in HH or HO functions

Reformulation of the LIT

$$LIT(\sigma_R,\sigma_I) = -\frac{1}{\sigma_I}\,\mathrm{Im}\left\{\langle\Psi_0|\Theta^\dagger\,(\sigma_R + E_0 - H + i\,\sigma_I)^{-1}\,\Theta\,|\Psi_0\rangle\right\}$$

$$R(E=\sigma_R) = -\frac{1}{\pi}\,\mathrm{Im}\left\{\lim_{\sigma_I\to 0}\langle\Psi_0|\Theta^\dagger\,(\sigma_R + E_0 - H + i\,\sigma_I)^{-1}\,\Theta\,|\Psi_0\rangle\right\}$$

New example:

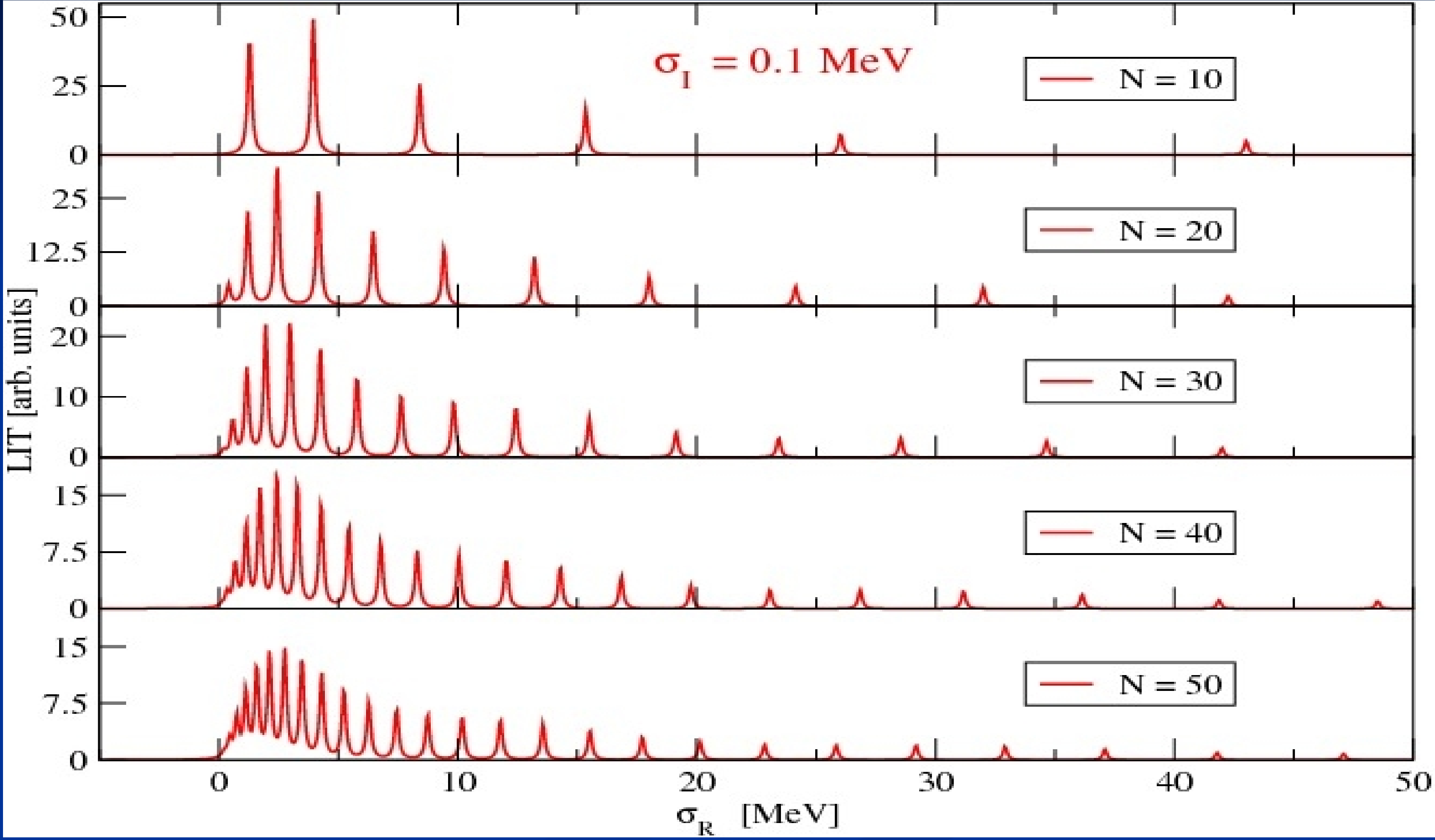
deuteron photodisintegration with the LIT method using expansion techniques

First we use the JISP-6 NN potential which is defined on an HO basis:
<n'| V | n> up n=n'=4 (n=0,1,2,...; HO quantum number, Ω = 40 MeV)

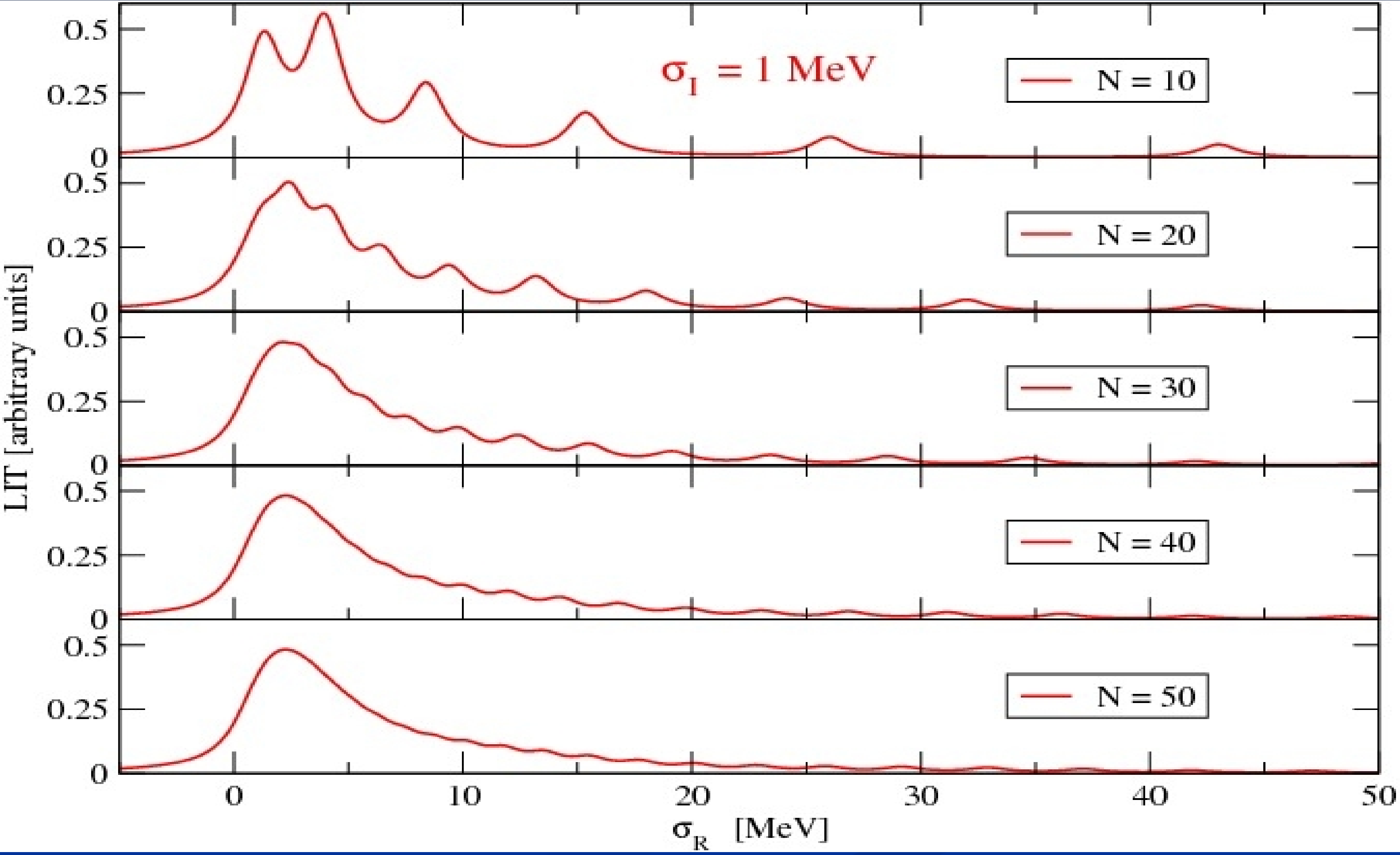
Also deuteron wave function and $\tilde{\Psi}$ are expanded on HO basis
Note: radial parts contain Laguerre polynomials up to order N times Gaussians

Alternatively exponential fall-off exp(-r/b) instead of Gaussians

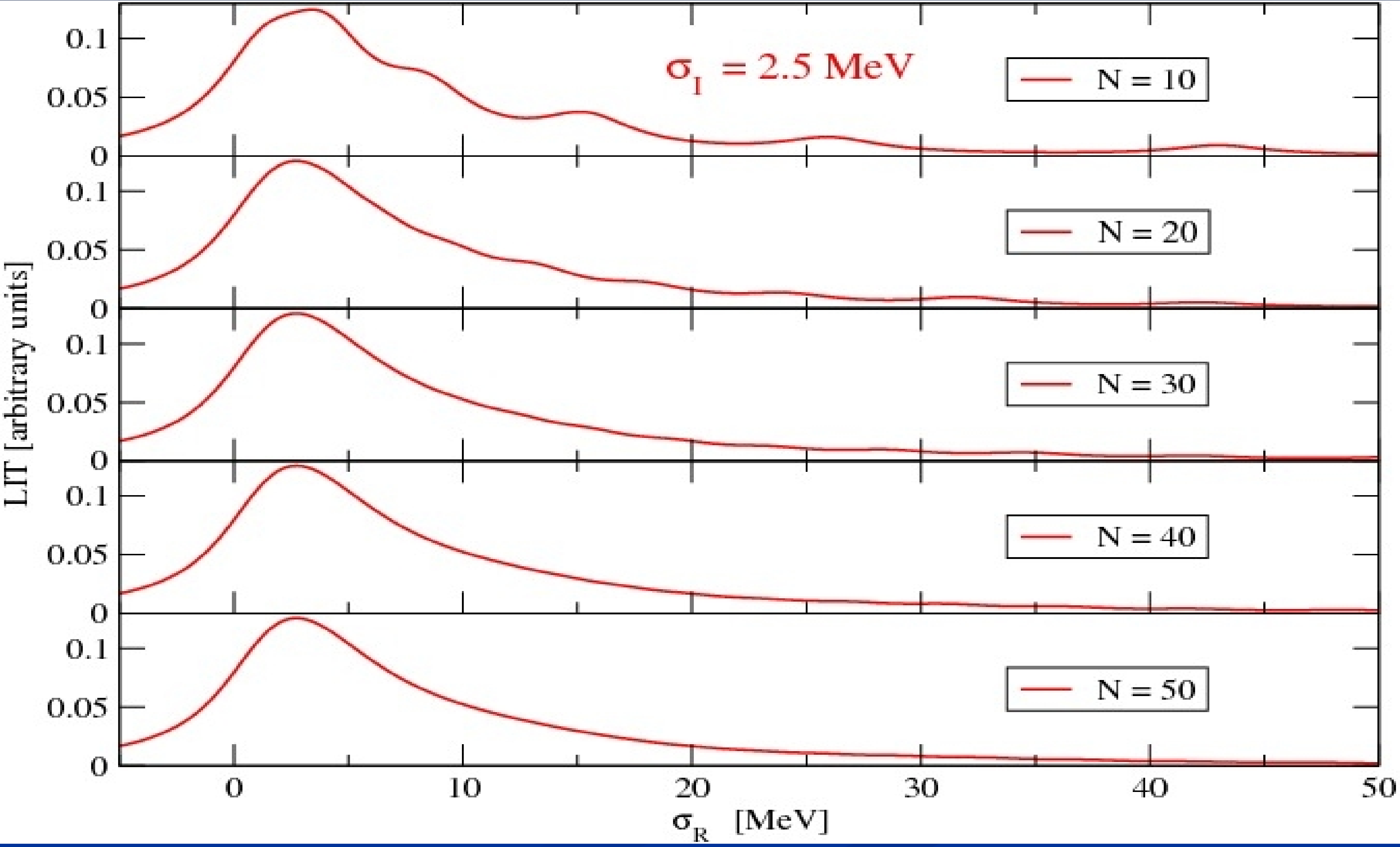
This leads to the following LITs with Laguerre polynomials up to order N with exponential fall-off (b=0.5 fm):



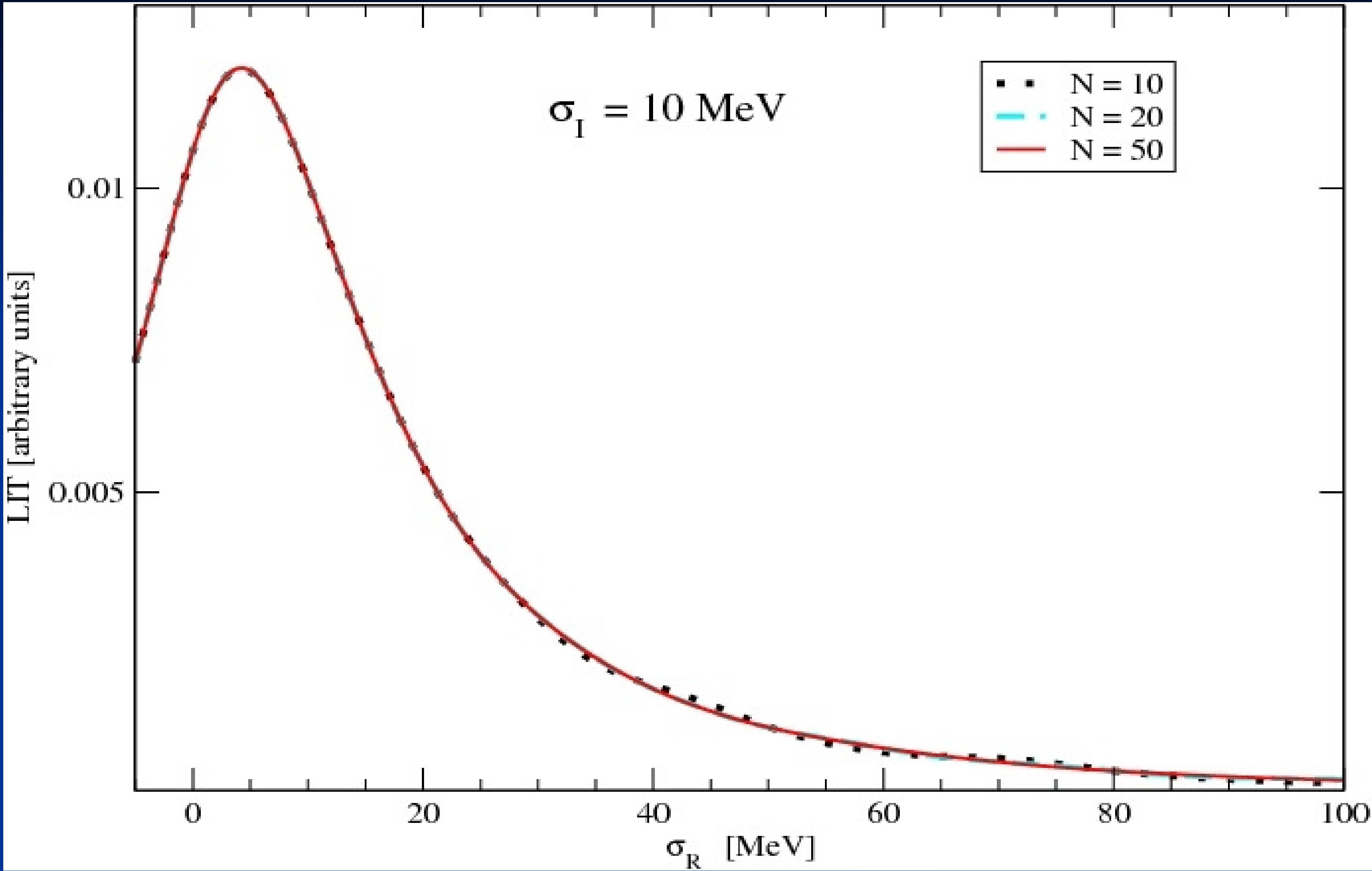
Laguerre polynomials up to order N (exponential fall-off)



Laguerre polynomials up to order N (exponential fall-off)



Laguerre polynomials up to order N (exponential fall-off)



LIT approach is a method with a controlled resolution!

Lanczos response

Since the Lorentzian function is a representation of the δ-function one could think of calculating R(ω) as the limit of $L(\omega,\sigma_R,\sigma_I)$ for $\sigma_I \to 0$.

The extrapolation would give

$$R(\omega) = \sum_{\nu}^{N} r_{\nu}\, \delta(\omega - \varepsilon_{\nu}^{N})$$

Lanczos response

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Lanczos response: δ-function is replaced by Lorentzian with small σ_I

$$R(\omega) = \sum_\nu^N r'_\nu \, L(\omega, \varepsilon_\nu^N, \sigma_I)$$

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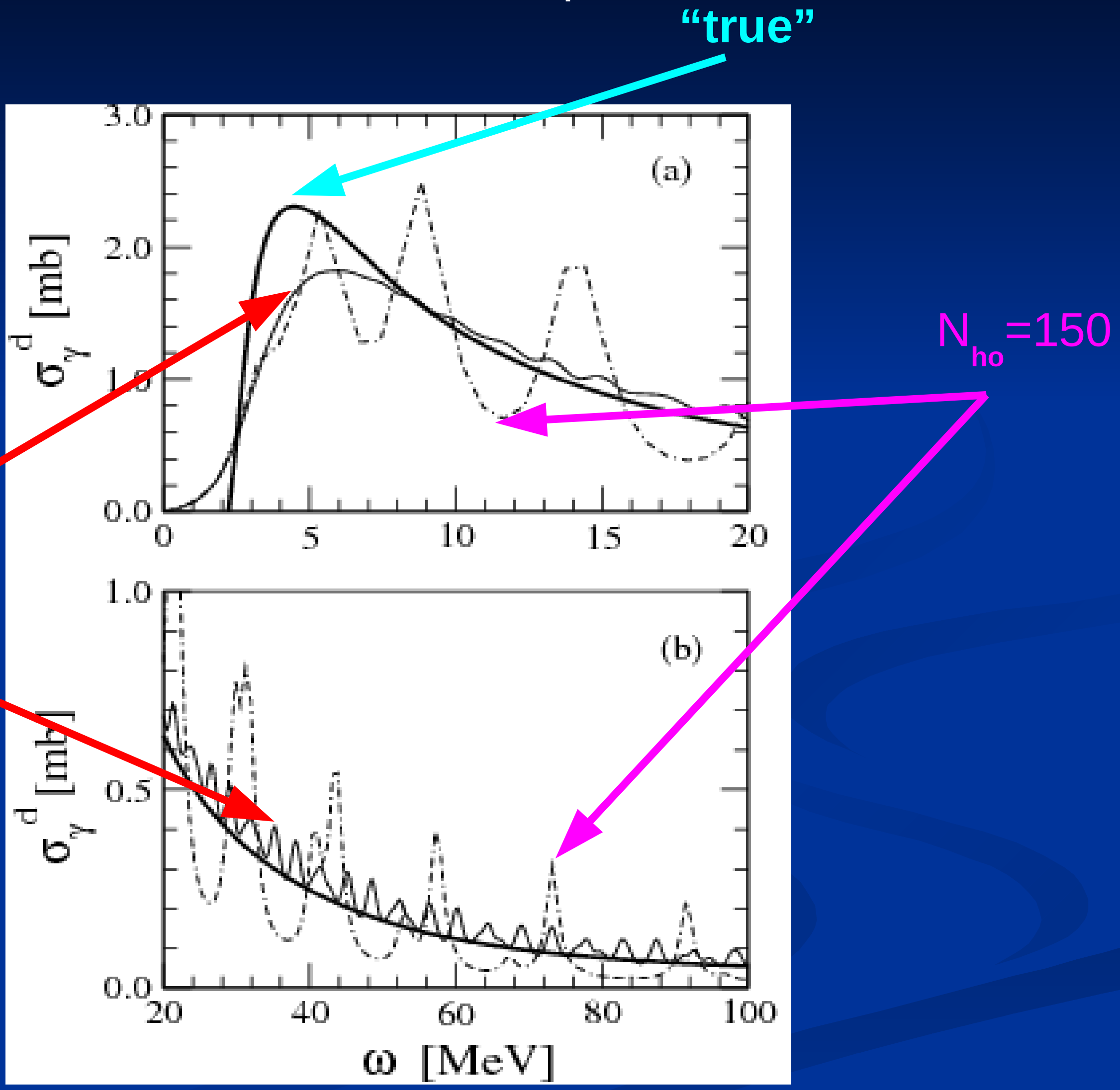
$$R(\omega) = \sum_\nu^N r'_\nu \, L(\omega,\varepsilon_\nu^N,\sigma_I)$$

Deuteron photodisintegration:
Consider all three transitions $^3P_0$, $^3P_1$, $^3P_2 - {}^3F_2$
now expansion of radial LIT part in HO functions
NN potential: JISP6

$\sigma_\gamma(\omega)$ from inversion and Lanczos response

σ_I=1 MeV



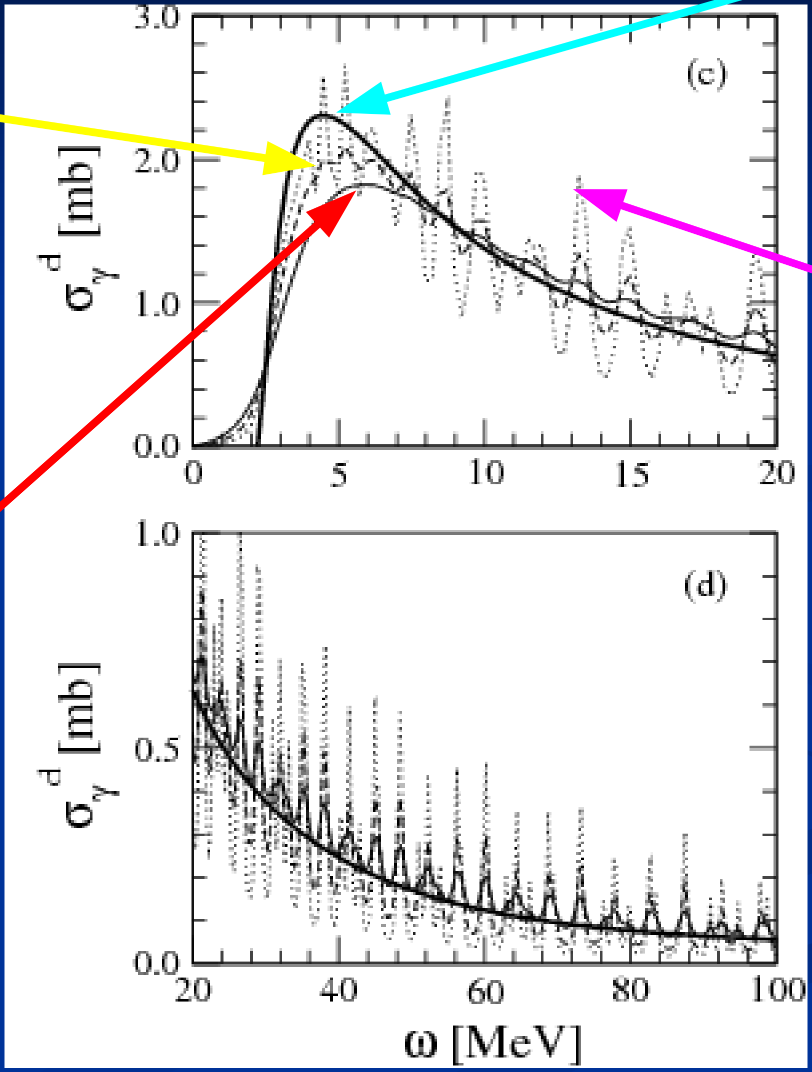
$\sigma_\gamma(\omega)$ from inversion and Lanczos response

Γ= 0.5 MeV

HO basis:

fixed N_{HO}=2400

Γ=1 MeV



Conclusion

Strength for a given discrete state of energy E is not the actual strength for this energy, but can only be interpreted correctly within an integral transform approach.

The correct distribution of strength is obtained via the inversion of the integral transform.

LIT application for inclusive electron scattering

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- 0^+ resonance of ^4He

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- Longitudinal response function $R_L(\omega,q)$ for A = 3 and 4

LIT application for inclusive electron scattering

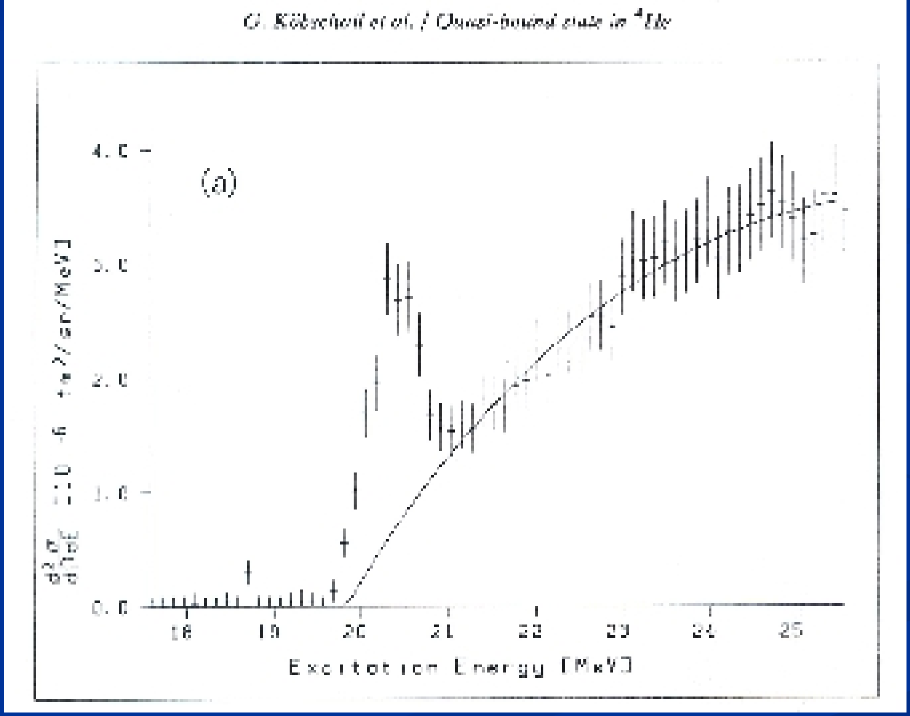
- 0^+ resonance of ^{4}He
- Longitudinal response function $R_L(\omega,q)$ for A = 3 and 4
- Transverse response function $R_T(\omega,q)$ for A = 3
  - Δ degrees of freedom
  - Quasi-elastic response at higher q (q=500-700 MeV/c)

O$^+$ resonance in longitudinal response function R_L in ^{4}He(e,e')

S. Bacca, N. Barnea, WL, G. Orlandini, PRL 110, 042503 (2013)

0^+ Resonance in the ^{4}He compound system

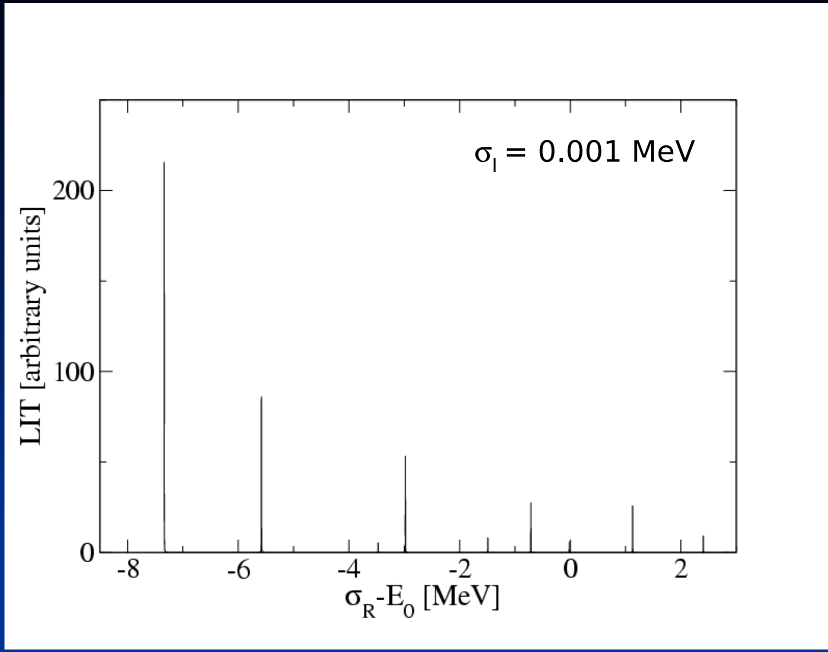
Resonance at E_R = -8.2 MeV, i.e. above the ^{3}H-p threshold. Strong evidence in electron scattering off ^{4}He

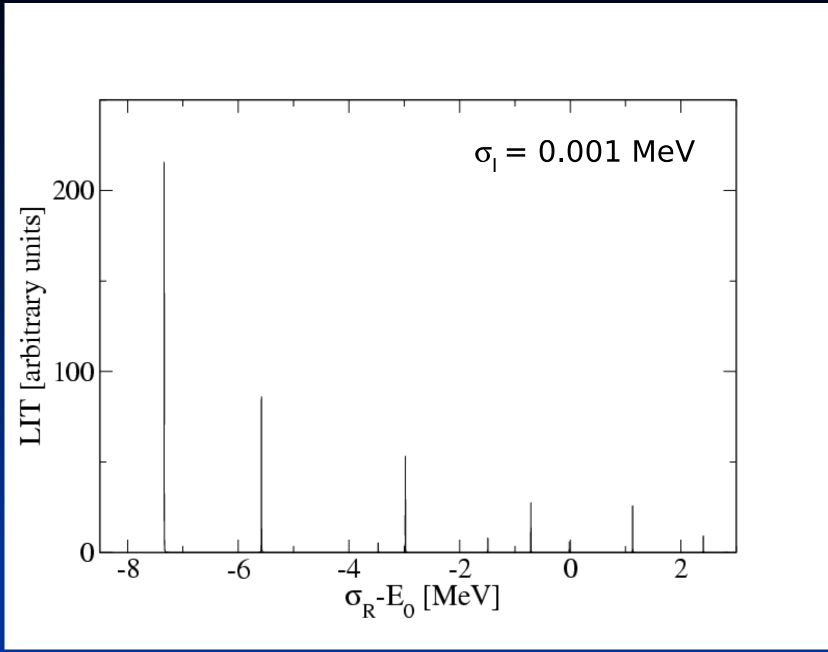


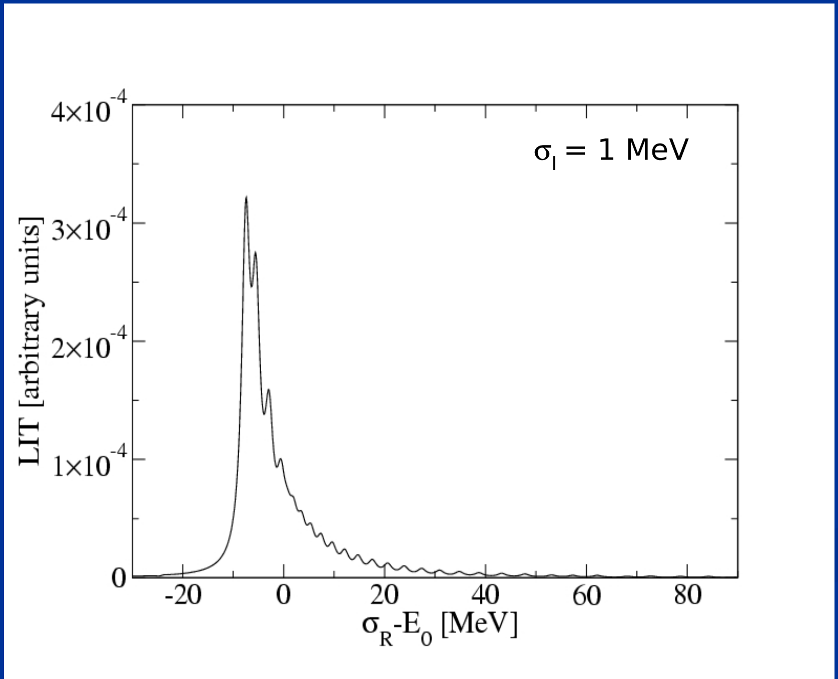
Γ = 270±70 keV

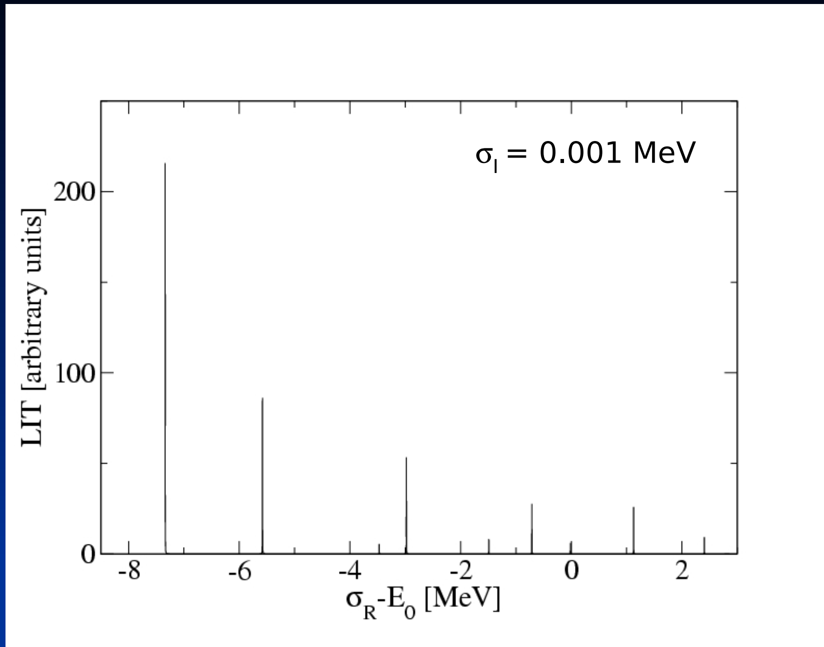
G. Köbschall et al., NPA 405, 648 (1983)

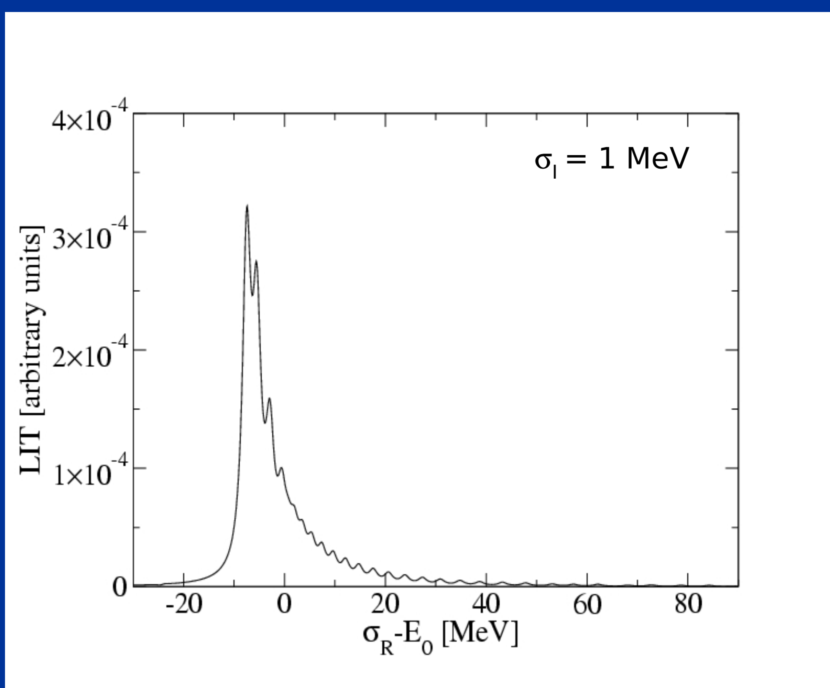
Results of our LIT calculation

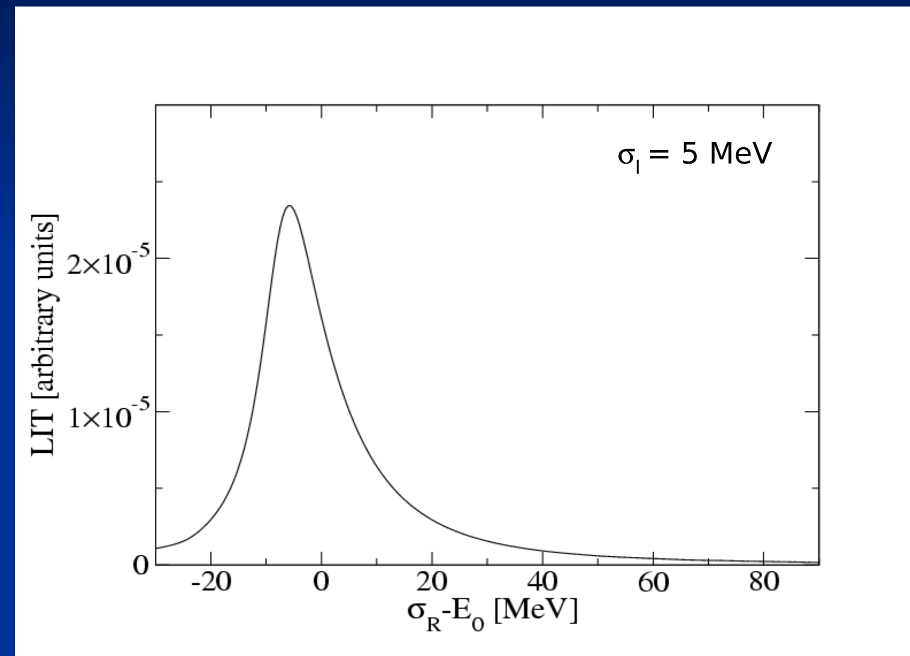

σI = 0.001 MeV
LIT [arbitrary units]
200
100
0
-8
-6
-4
-2
0
2
σR-E0 [MeV]


σ_I = 0.001 MeV
LIT [arbitrary units]
σ_R-E_0 [MeV]
0
100
200
-8
-6
-4
-2
0
2


σ_I = 1 MeV
LIT [arbitrary units]
σ_R-E_0 [MeV]
0
1×10^-4
2×10^-4
3×10^-4
4×10^-4
-20
0
20
40
60
80


σ_I = 0.001 MeV
LIT [arbitrary units]
0
100
200
-8
-6
-4
-2
0
2
σ_R-E_0 [MeV]


σ_I = 1 MeV
LIT [arbitrary units]
0
1×10^-4
2×10^-4
3×10^-4
4×10^-4
-20
0
20
40
60
80
σ_R-E_0 [MeV]


σ_I = 5 MeV
LIT [arbitrary units]
0
1×10^-5
2×10^-5
-20
0
20
40
60
80
σ_R-E_0 [MeV]

The present precision of the calculation does not allow to resolve the shape of the resonance, therefore the width cannot be determined.

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However, the strength of the resonance can be determined!

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However, the strength of the resonance can be determined!

Of course not by taking the strength to the discretized state, but by rearranging the inversion in a suitable way:

The present precision of the calculation does not allow to resolve the shape of the resonance, therefore the width cannot be determined.

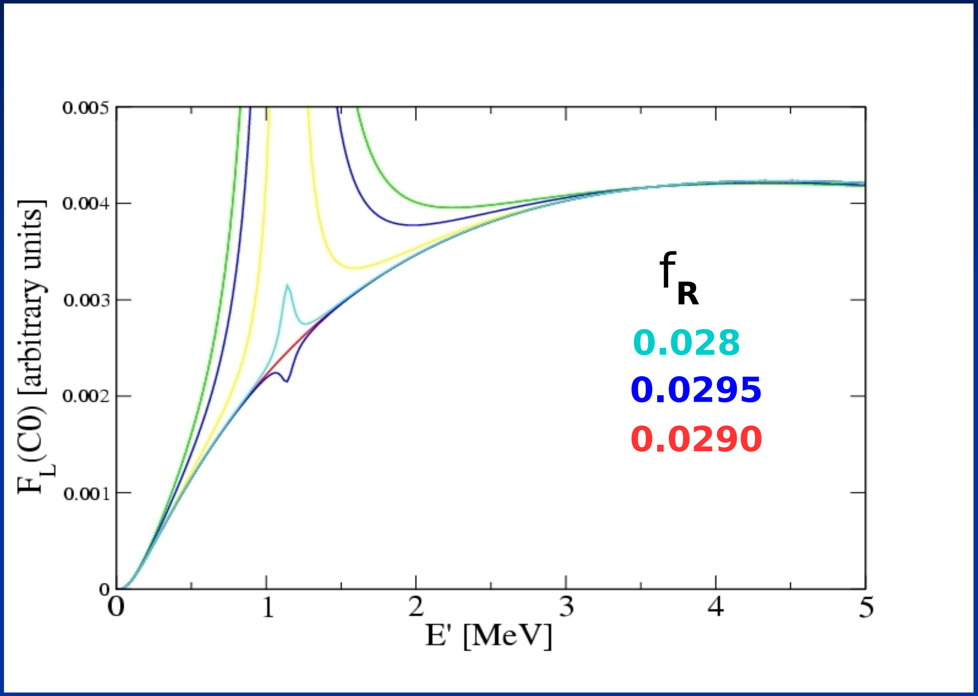
However, the strength of the resonance can be determined!

Of course not by taking the strength to the discretized state, but by rearranging the inversion in a suitable way:

Reduce strength to the state up to the point that the inversion does not show any resonant structure at the resonance energy E_R:

$$\mathrm{LIT}(\sigma_R, \sigma_I) \rightarrow \mathrm{LIT}(\sigma_R, \sigma_I) - f_R \,/\, [(E_R - \sigma_R)^2 + \sigma_I^2] \equiv \mathrm{LIT}(\sigma_R, \sigma_I, f_R)$$

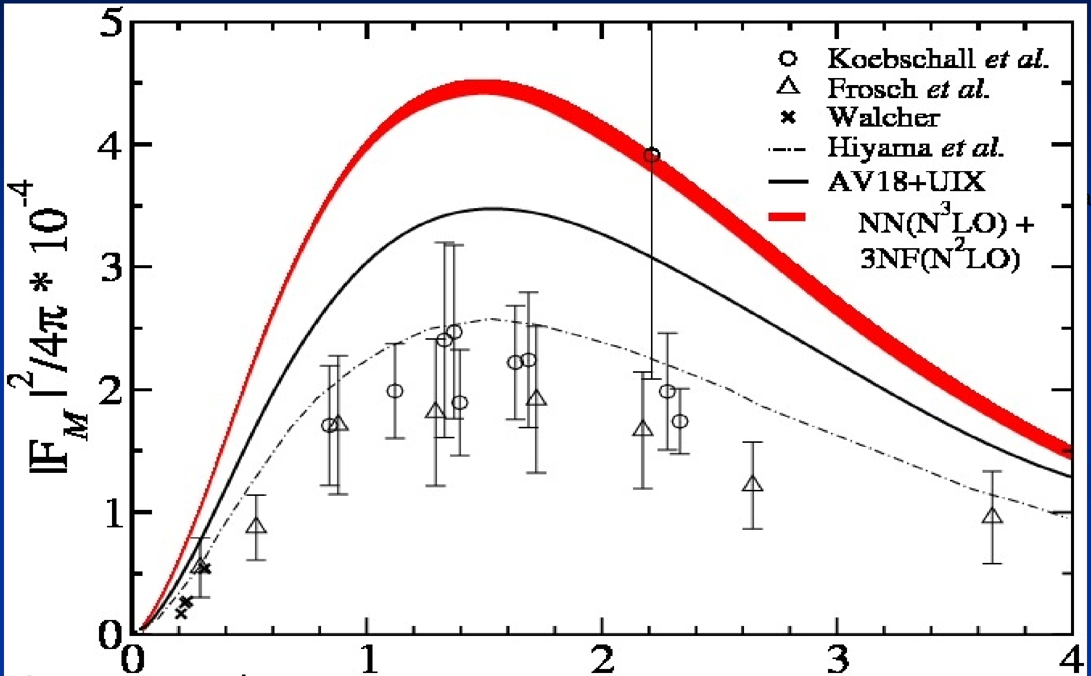
with resonance strength f_R



Inversion results with different f_R values

AV18+UIX, q=300 MeV/c

Comparison to experimental results



S. Bacca et al.
PRL 110, 042503 (2013)

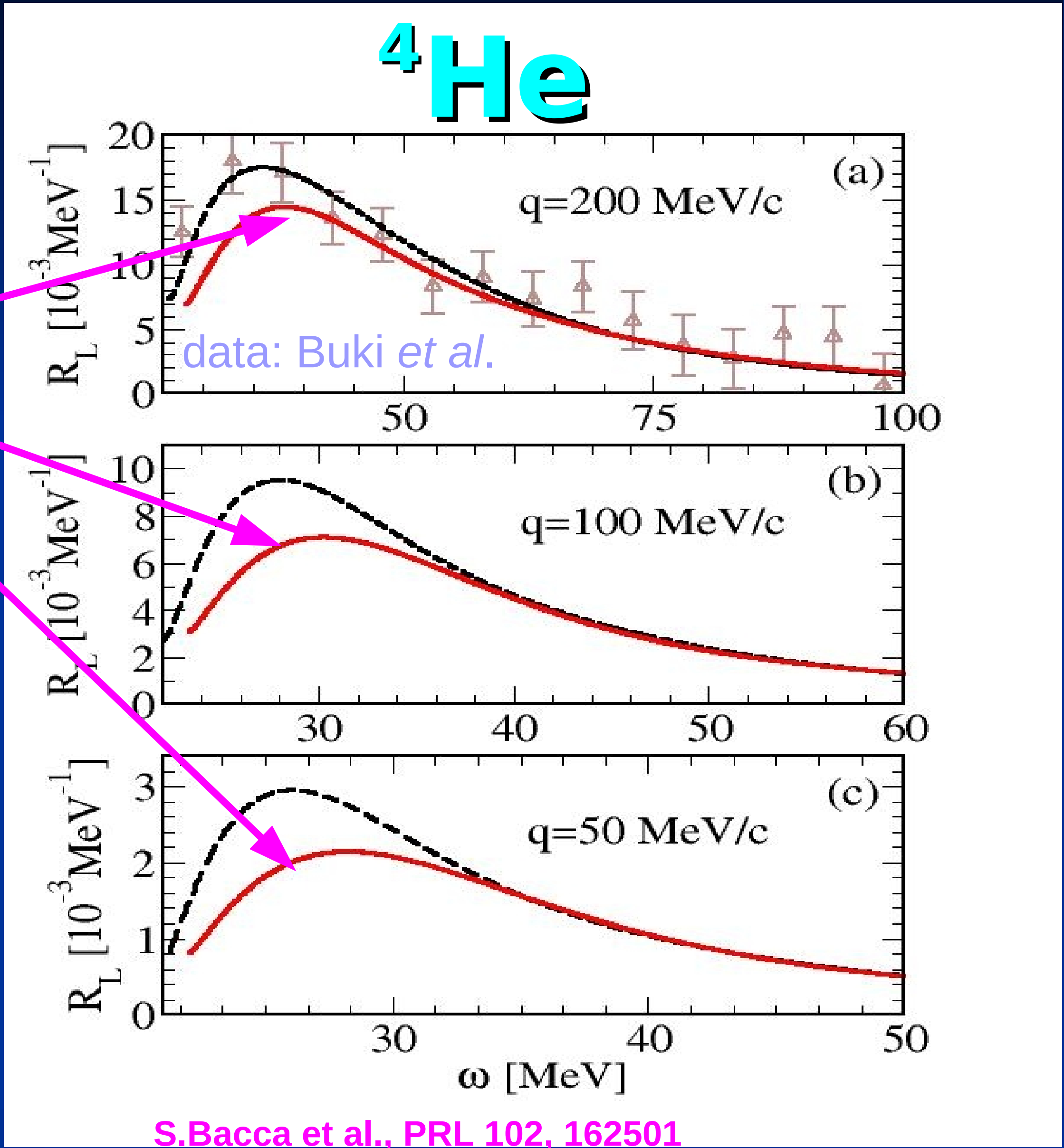
LIT/EIHH Calculation for AV18+UIX and Idaho-N3LO+N2LO

Dotted: AV8' + central 3NF (Hiyama et al.)

(e,e') Longitudinal Response

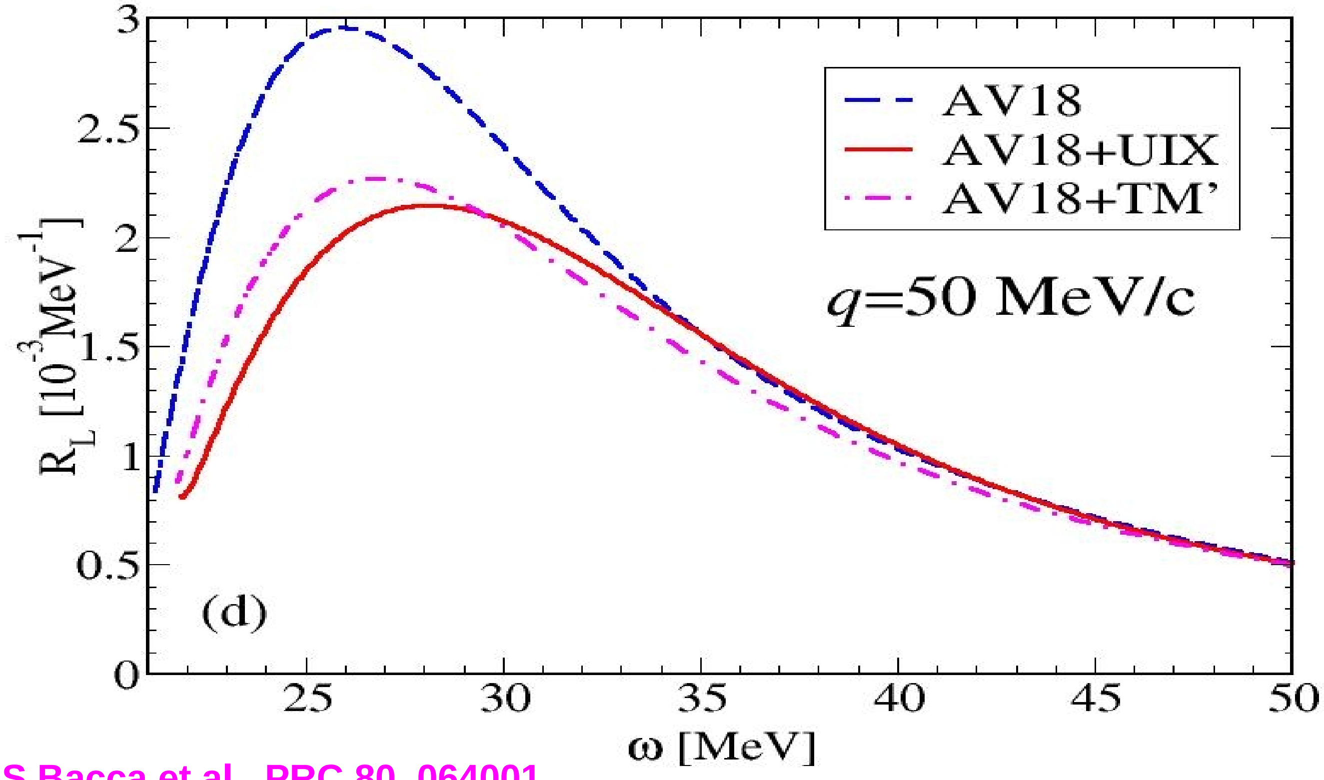
SURPRISE:
LARGE EFFECT OF 3-BODY FORCE
AT LOW q

Calculation via EIHH with force model: AV18 + UIX



S.Bacca et al., PRL 102, 162501

Dependence on different 3-nucleon forces

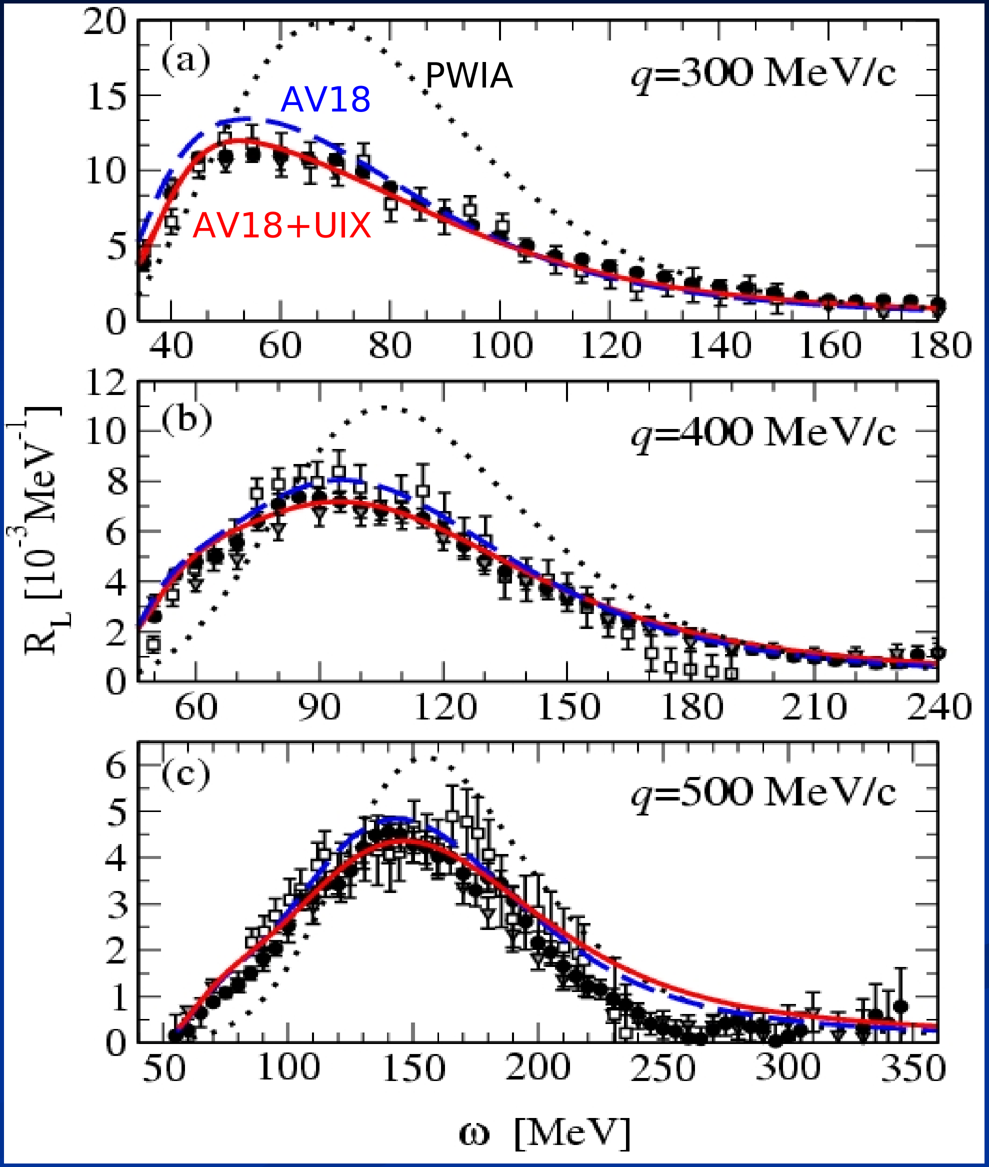


S.Bacca et al., PRC 80, 064001

^{4}He (e,e') Longitudinal Response

SMALL EFFECT OF 3-BODY FORCE AT HIGH q

Exp.: Saclay
Bates
world data (J. Carlson et *al*.)



3-Body inclusive electrodisintegration
Role of 3-Nucleon force

LONGITUDINAL
RESPONSE

"low" q

------ AV18
____ AV18 + UIX

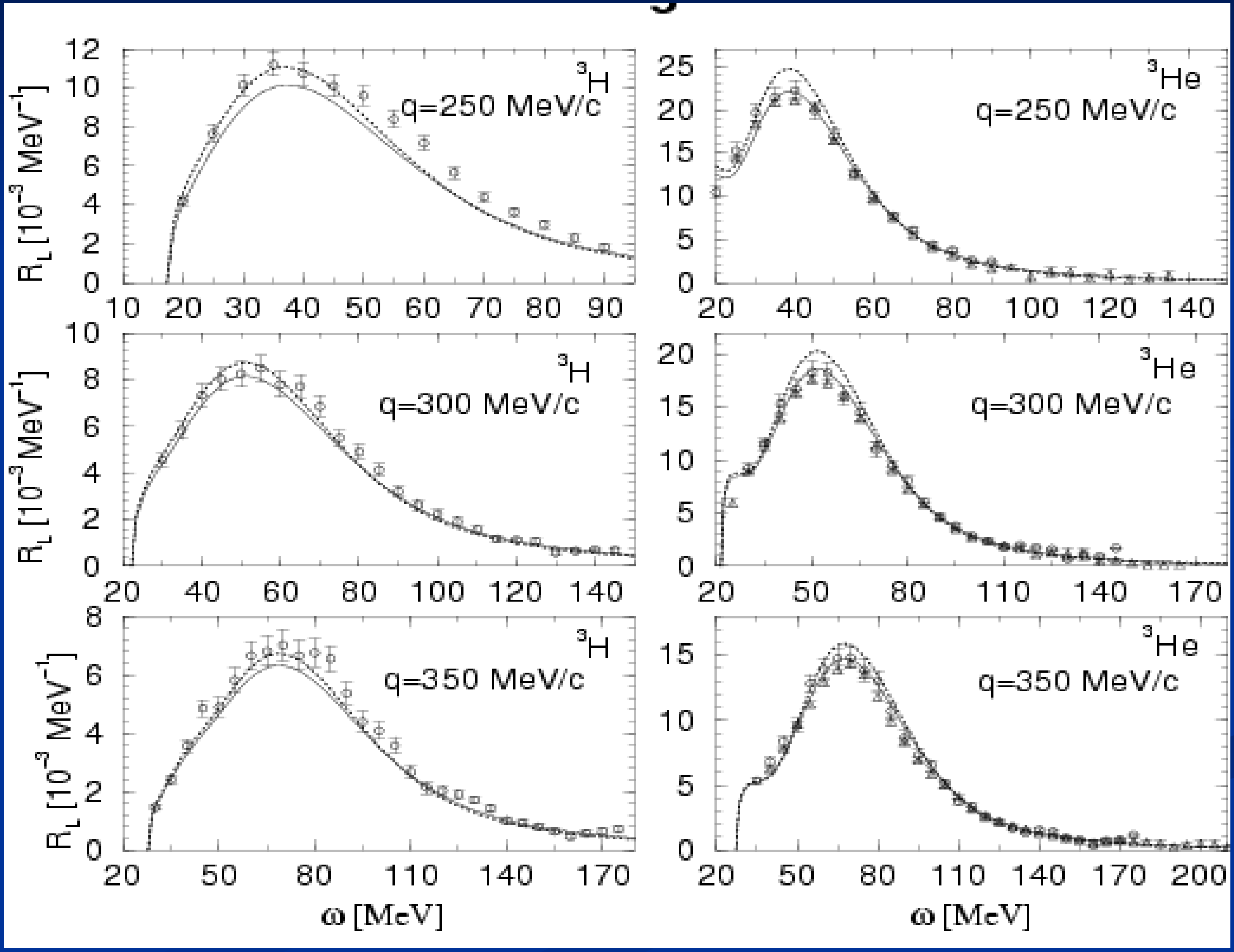
CHH

V. Efros, W.L., G. Orlandini
E. Tomusiak
PRC69, 044001 (2004)

Exp:

ɸ Dow

Marchand



Transverse response function $R_T(\omega,q)$

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Subnuclear degrees of freedom can become important

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Subnuclear degrees of freedom can become important

- Meson exchange currents (MEC)

MEC with LIT method: S. Della Monaca, V.D. Efros, A. Khugaev, WL, G. Orlandini, E.L. Tomusiak, L. Yuan, PRC 77, 044007 (2008)

Transverse response function $R_T(\omega,q)$

Subnuclear degrees of freedom can become important

- Meson exchange currents (MEC)

  MEC with LIT method: S. Della Monaca, V.D. Efros, A. Khugaev, WL, G. Orlandini, E.L. Tomusiak, L. Yuan, PRC 77, 044007 (2008)

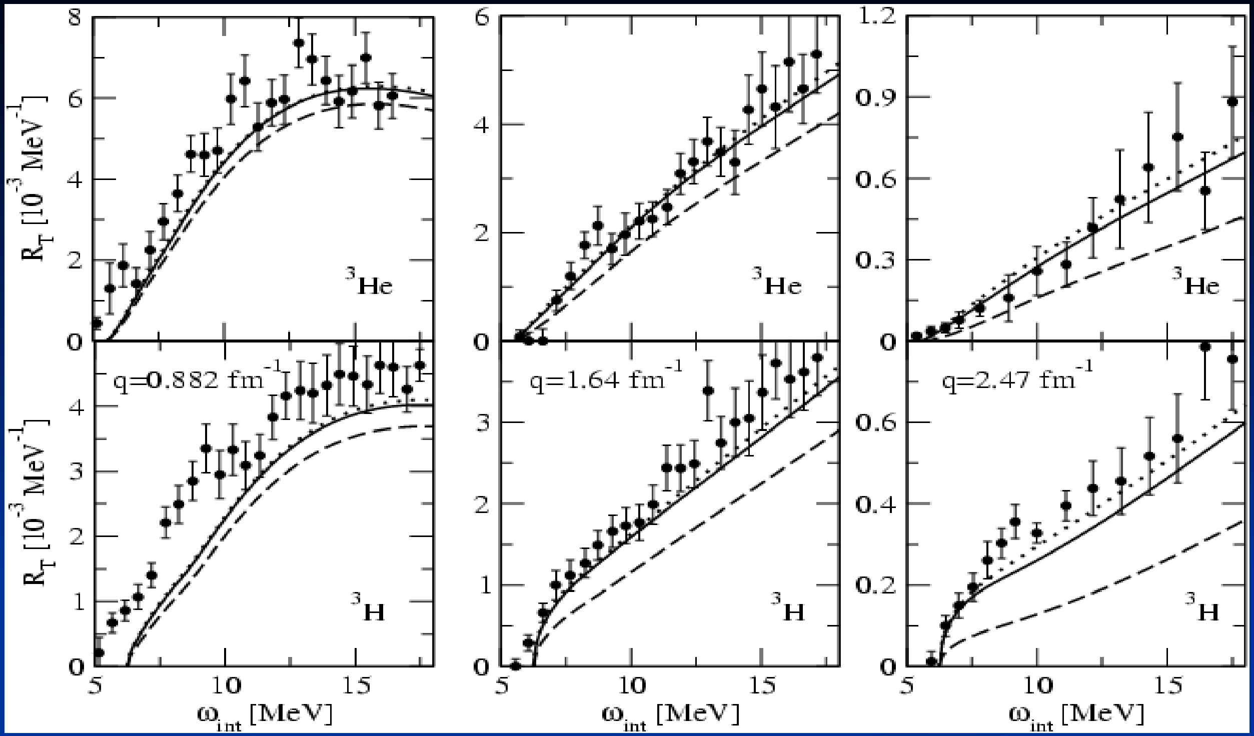
- Δ isobar currents (Δ-IC)

  Δ-IC with LIT method: L. Yuan, V.D. Efros, WL, E.L. Tomusiak, PRC 81, 064001 (2010)

NR: dashed

NR+MEC: dotted

Rel.+MEC: full



q = 174 MeV/c q = 324 MeV/c q = 487 MeV/c

R_T close to break-up threshold

(V.D. Efros, WL, G. Orlandini, E.L. Tomusiak, Few-Body Syst. 47, 157 (2010))

Δ degrees of freedom

Schrödinger equation with Δ degrees of freedom

$$\Psi = \Psi_N + \Psi_\Delta$$

$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta$$

$$(\delta m + T_\Delta + V_{N\Delta} - E)\,\Psi_\Delta = -V_{N\Delta,NN}\,\Psi_N$$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ $(V_{N\Delta})$ transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

Schrödinger equation with Δ degrees of freedom

$$\Psi = \Psi_N + \Psi_\Delta$$

$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta$$ coupled channel calculation

$$(\delta m + T_\Delta + V_{N\Delta} - E)\,\Psi_\Delta = -V_{N\Delta,NN}\,\Psi_N$$ solve eqs. simultaneously

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

Schrödinger equation with Δ degrees of freedom

$$\Psi = \Psi_N + \Psi_\Delta$$

$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta$$ Impulse approximation

$$(\delta m + T_\Delta + V_{N\Delta} - E)\,\Psi_\Delta = -V_{N\Delta,NN}\,\Psi_N$$ Solve formally for Ψ_Δ

$= H_\Delta$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

Schrödinger equation with Δ degrees of freedom

$$\Psi = \Psi_N + \Psi_\Delta$$

$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta$$

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$$= H_\Delta$$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

$$\Psi_\Delta = -(H_\Delta - E)^{-1}\,V_{N\Delta,NN}\,\Psi_N$$

Schrödinger equation with Δ degrees of freedom

$$\Psi = \Psi_N + \Psi_\Delta$$

$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta \quad (*)$$

$$(\delta m + T_\Delta + V_{N\Delta} - E)\,\Psi_\Delta = -V_{N\Delta,NN}\,\Psi_N$$

$$= H_\Delta$$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

$$\Psi_\Delta = -(H_\Delta - E)^{-1}\,V_{N\Delta,NN}\,\Psi_N$$

Insert formal solution in (*)

$$(T_N + V_{NN} - V_{NN,N\Delta}(H_\Delta - E)^{-1}\,V_{N\Delta,NN} - E)\,\Psi_N = 0$$

$$\cong V_{NN}^{realistic}$$

Schrödinger equation with Δ degrees of freedom

$$\Psi = \Psi_N + \Psi_\Delta$$

$$(T_N + V_{NN} - E)\,\Psi_N = -V_{NN,N\Delta}\,\Psi_\Delta \quad (*)$$

$$(\delta m + T_\Delta + V_{N\Delta} - E)\,\Psi_\Delta = -V_{N\Delta,NN}\,\Psi_N$$

$$= H_\Delta$$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ $(V_{N\Delta})$ transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

$$\Psi_\Delta = -(H_\Delta - E)^{-1}\,V_{N\Delta,NN}\,\Psi_N \quad \textbf{(IA)}$$

$$(T_N + V_{NN} - V_{NN,N\Delta}(H_\Delta - E)^{-1}\,V_{N\Delta,NN} - E)\,\Psi_N = 0 \quad \textbf{(**)}$$

$$\cong V^{realistic}$$

Step 1: solve (**) with realistic V_{NN} +3NF

Step 2: solve Ψ_Δ in IA

LIT equation with Δ degrees of freedom

$$\tilde{\Psi} = \tilde{\Psi}_N + \tilde{\Psi}_\Delta$$

$$(T_N + V_{NN} - \sigma)\, \tilde{\Psi}_N = - V_{NN,N\Delta}\, \tilde{\Psi}_\Delta + O_{NN}\, \Psi_{0,N} + O_{N\Delta}\, \Psi_{0,\Delta}$$

$$(\delta m + T_\Delta + V_{N\Delta} - \sigma)\, \tilde{\Psi}_\Delta = - V_{N\Delta,NN}\, \tilde{\Psi}_N + O_{\Delta N}\, \Psi_{0,N} + O_{\Delta\Delta}\, \Psi_{0,\Delta}$$

$= H_\Delta$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

LIT equation with Δ degrees of freedom

$$\widetilde{\Psi} = \widetilde{\Psi}_N + \widetilde{\Psi}_\Delta$$

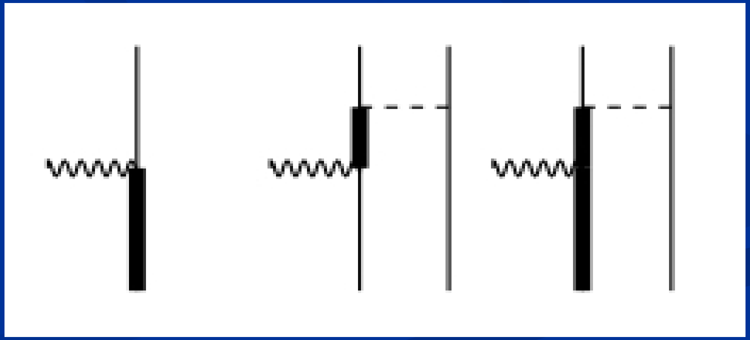
$$(T_N + V_{NN} - \sigma)\,\widetilde{\Psi}_N = -V_{NN,N\Delta}\,\widetilde{\Psi}_\Delta + O_{NN}\,\Psi_{0,N} + O_{N\Delta}\,\Psi_{0,\Delta}$$

$$(\delta m + T_\Delta + V_{N\Delta} - \sigma)\,\widetilde{\Psi}_\Delta = -V_{N\Delta,NN}\,\widetilde{\Psi}_N + O_{\Delta N}\,\Psi_{0,N} + O_{\Delta\Delta}\,\Psi_{0,\Delta}$$

$= H_\Delta$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

We take into account electromagnetic operators with the Δ (Δ-IC) represented by the following graphs



LIT equation with Δ degrees of freedom

$$\widetilde{\Psi} = \widetilde{\Psi}_N + \widetilde{\Psi}_\Delta$$

$$(T_N + V_{NN} - \sigma)\, \widetilde{\Psi}_N = -V_{NN,N\Delta}\, \widetilde{\Psi}_\Delta + O_{NN}\, \Psi_{0,N} + O_{N\Delta}\, \Psi_{0,\Delta}$$

$$(\underbrace{\delta m + T_\Delta + V_{N\Delta}}_{= H_\Delta} - \sigma)\, \widetilde{\Psi}_\Delta = -V_{N\Delta,NN}\, \widetilde{\Psi}_N + O_{\Delta N}\, \Psi_{0,N} + O_{\Delta\Delta}\, \Psi_{0,\Delta}$$

$V_{NN,N\Delta}$ (V_{NN}) and $V_{N\Delta,NN}$ ($V_{N\Delta}$) transition (diagonal) potentials between NNN and NNΔ spaces (A=3), $\delta m = M_\Delta - M_N$

$$(T_N + V^{realistic} - \sigma)\, \widetilde{\Psi}_{\mathbf{N}} = -V_{NN,N\Delta}(H_\Delta - \sigma)^{-1}(O_{\Delta N}\, \Psi_{0,N} + O_{\Delta\Delta}\, \Psi_{0,\Delta})$$

$$+ O_{NN}\, \Psi_{0,N} + O_{N\Delta}\, \Psi_{0,\Delta}$$

^{3}He (e,e') Response Functions in the Quasielastic Region

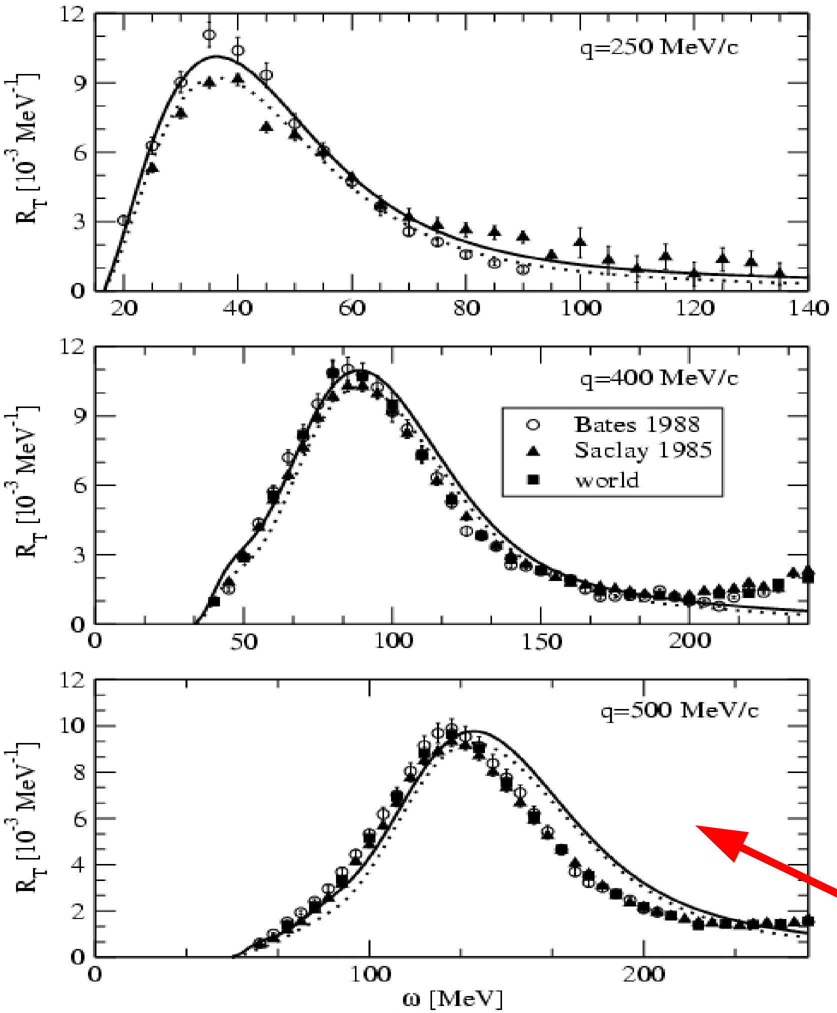
The quasielastic region is dominated by the one-body parts of ρ and **J**, but relativistic contributions become increasingly important with growing momentum transfer q

Our aim: non-rel. calculation + rel. corrections

with realistic nuclear forces

Motivation

$R_T(\omega,q)$ at various q



Potential: BonnRA +TM'

one-body current: dashed
one+two-body current: full

(S. Della Monaca et al., PRC 77, 044007 (2008))

Bad agreement between theory and experiment because of non considered relativistic effects

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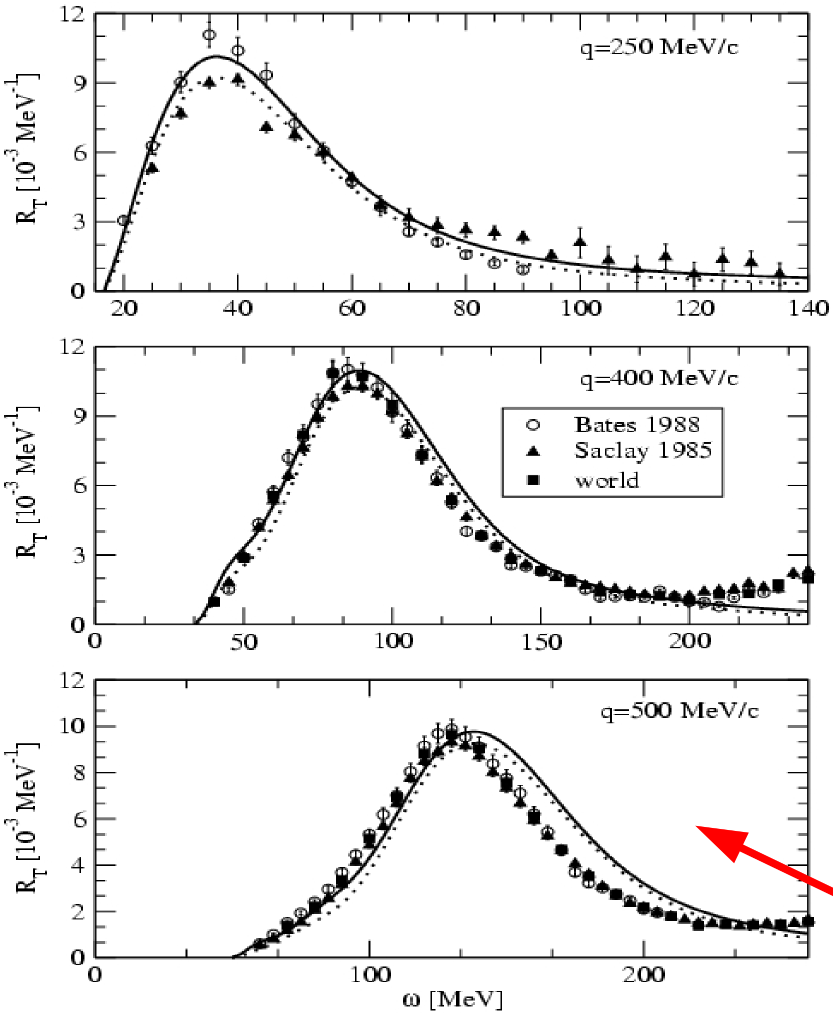


Potential: BonnRA +TM'

one-body current: dashed
one+two-body current: full

Quasi-elastic kinematics (q=500 MeV/c),
Kinetic energy of outgoing nucleon:

non-rel. : $T = q^2/2m = 133$ MeV
rel.: $T= (m^2+q^2)^{1/2} - m = 125$ MeV

Bad agreement between theory and experiment because of non considered relativistic effects

We already considered this problem for R_L and studied R_L in various reference frames:

Laboratory: $P_T = 0$

Breit: $P_T = -q/2$

Anti-Lab: $P_T = -q$

Active Nucleon Breit: $P_T = -Aq/2$

$R_L(\omega,q)$ at higher q

Frame dependence

calculation in various frames:

Laboratory: $P_T = 0$

Breit: $P_T = -q/2$

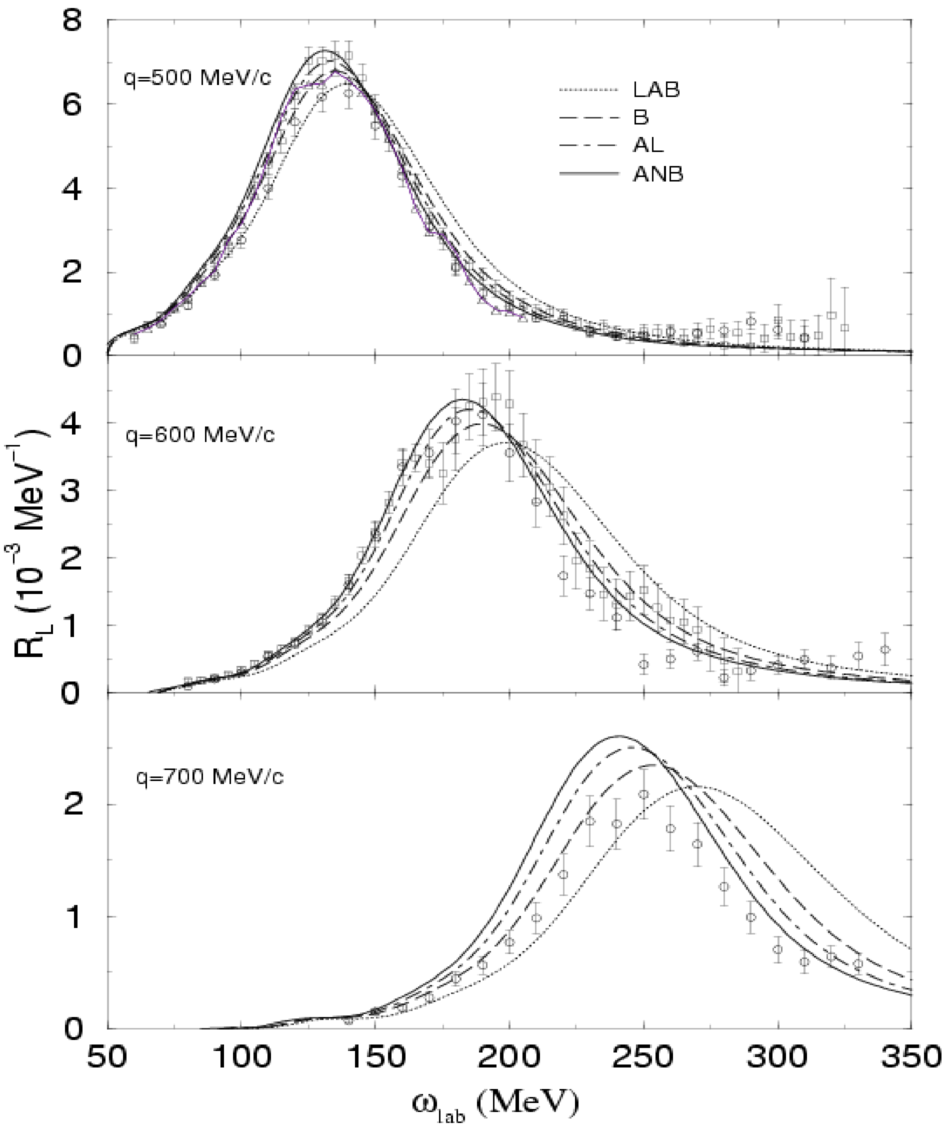
Anti-Lab: $P_T = -q$

Active Nucleon Breit: $P_T = -Aq/2$

Potential: AV18+UIX

Result in LAB frame

$$R_L(\omega,q) = \frac{q^2}{(q_{fr})^2}\frac{E_T^{fr}}{M_T} R_L^{fr}(\omega^{fr},q^{fr})$$



Exp: Marchand 1985, Dow 1988, Carlson 2002

V. Efros, W.L., G. Orlandini, E. Tomusiak
PRC 72 (2005) 011002(R)

How to get more frame independent results?

Assume quasi-elastic kinematics:

whole energy and momentum transfer taken by the knocked out nucleon (residual two-body system is in its lowest energy state)

$\Longrightarrow$ Effective two-body problem
Treat kinematics relativistically correct

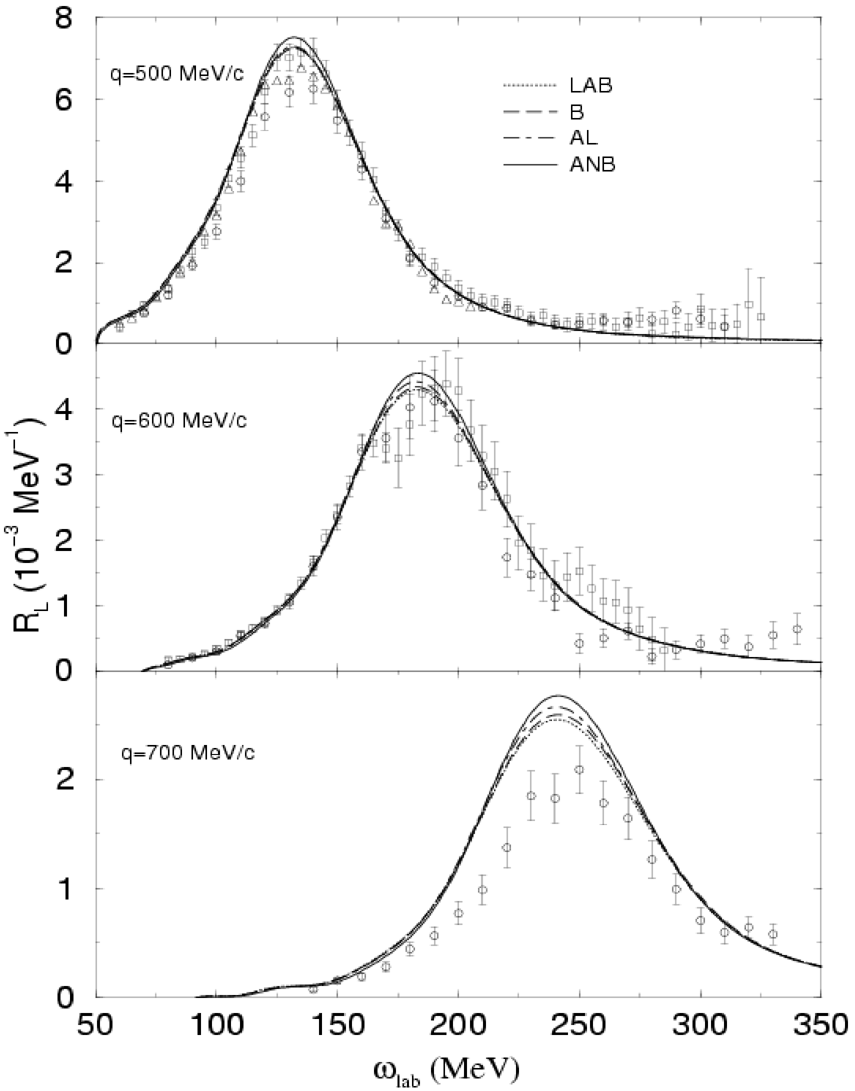
Take the correct relativistic relative momentum k_{rel} and calculate the corresponding non-relativistic relative energy

$$E_{nr} = (k_{rel})^2/2\mu$$

with reduced mass μ of nucleon and residual system

use E_{nr} as internal excitation energy in your calculation

$R_L(\omega,q)$ at higher q



Quasielastic region: assume two-body break-up and use the correct relativistic relative momentum

Transverse response function $R_T(q,\omega)$ of ^3He in the quasi-elastic region

Nuclear current operator includes besides the usual non-relativistic one-body currents also meson exchange currents and Δ-isobar currents as well as relativistic corrections for the one-body current

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Calculation of bound state wave function and solution of LIT equation with the help of expansions in correlated hyperspherical harmonics

Nuclear force model: Argonne v18 NN potential and Urbana 3NF

Further calculation details

The current operator $\mathbf{J}$

$\mathbf{J} = \mathbf{J}^{(1)} + \mathbf{J}^{(2)}$

$\mathbf{J}^{(1)} = \mathbf{J}^{(1)}(q,\omega,P_T) = \mathbf{J}_{spin} + \mathbf{J}_p + \mathbf{J}_q + (\omega/M)\, \mathbf{J}_\omega$

for instance spin current

$\mathbf{J}_{spin} = \exp(i\mathbf{q}\cdot\mathbf{r})\; i\, \boldsymbol{\sigma}\times\mathbf{q}/2M\, [G_M(1\text{-}q^2/8M^2) - G_E\, \kappa^2 q^2/8M^2]$

with $\kappa=1+2P_T/Aq$

Further calculation details

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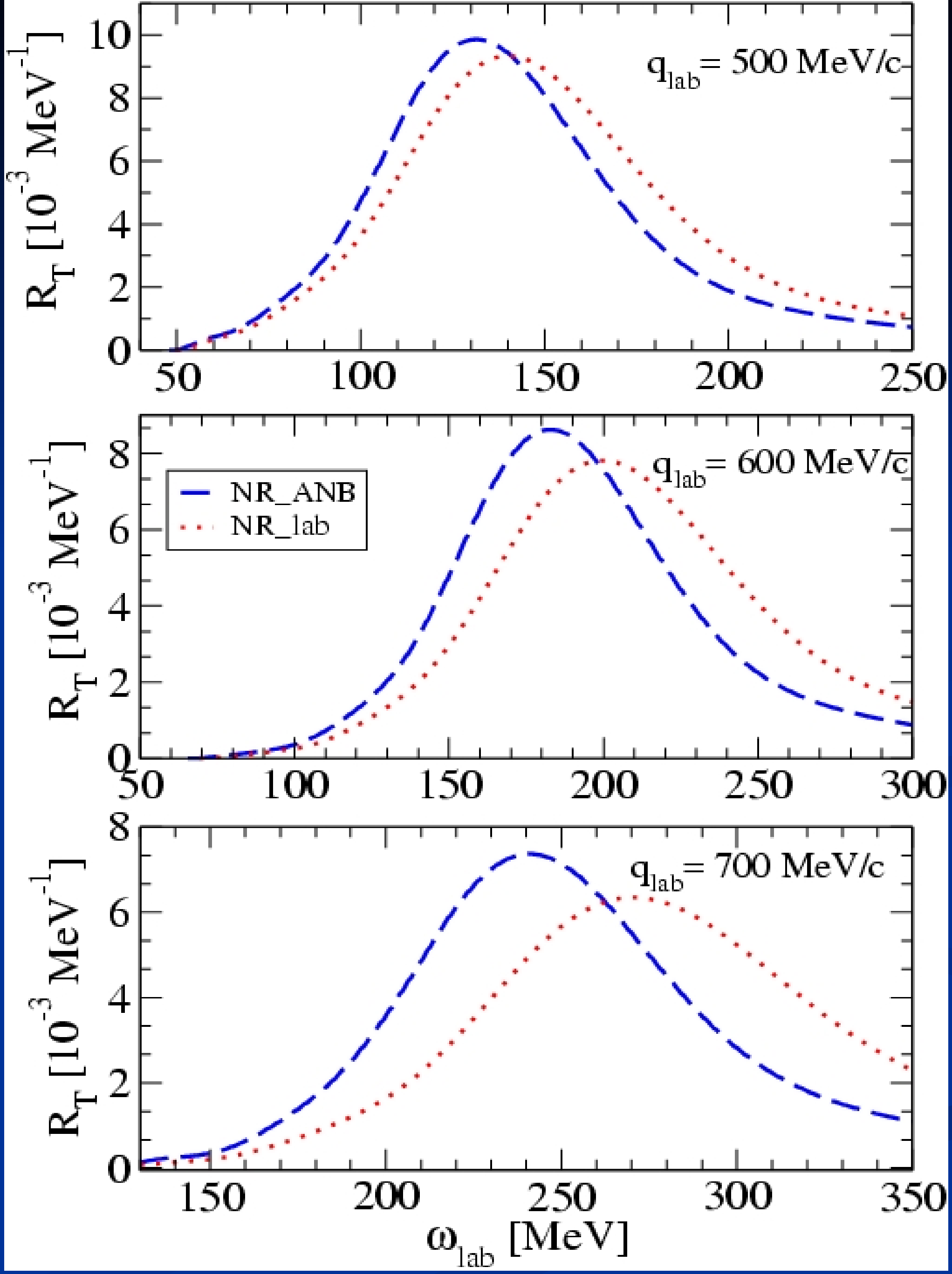
with $\kappa=1+2P_T/Aq$

Transformation from ANB frame to LAB frame

$R_T^{LAB}(\omega^{LAB},q^{LAB}) = R_T^{ANB}(\omega^{ANB},q^{ANB})\;\; E_T^{ANB}/M_T$

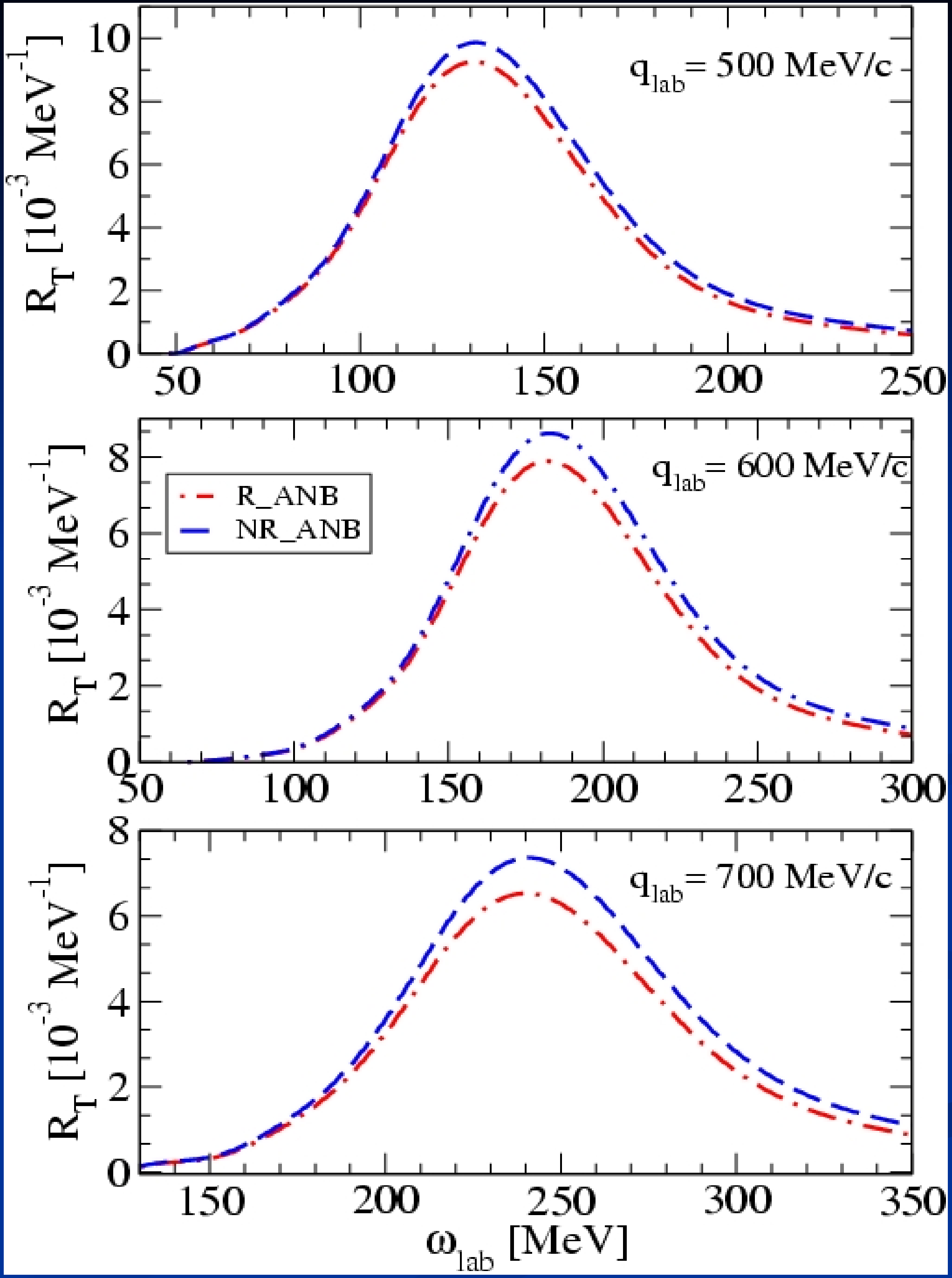
Results

- Comparison of ANB and LAB calculation: strong shift of peak to lower energies! (8.7, 16.7, 29.3 MeV at q=500, 600, 700 MeV/c)



Results

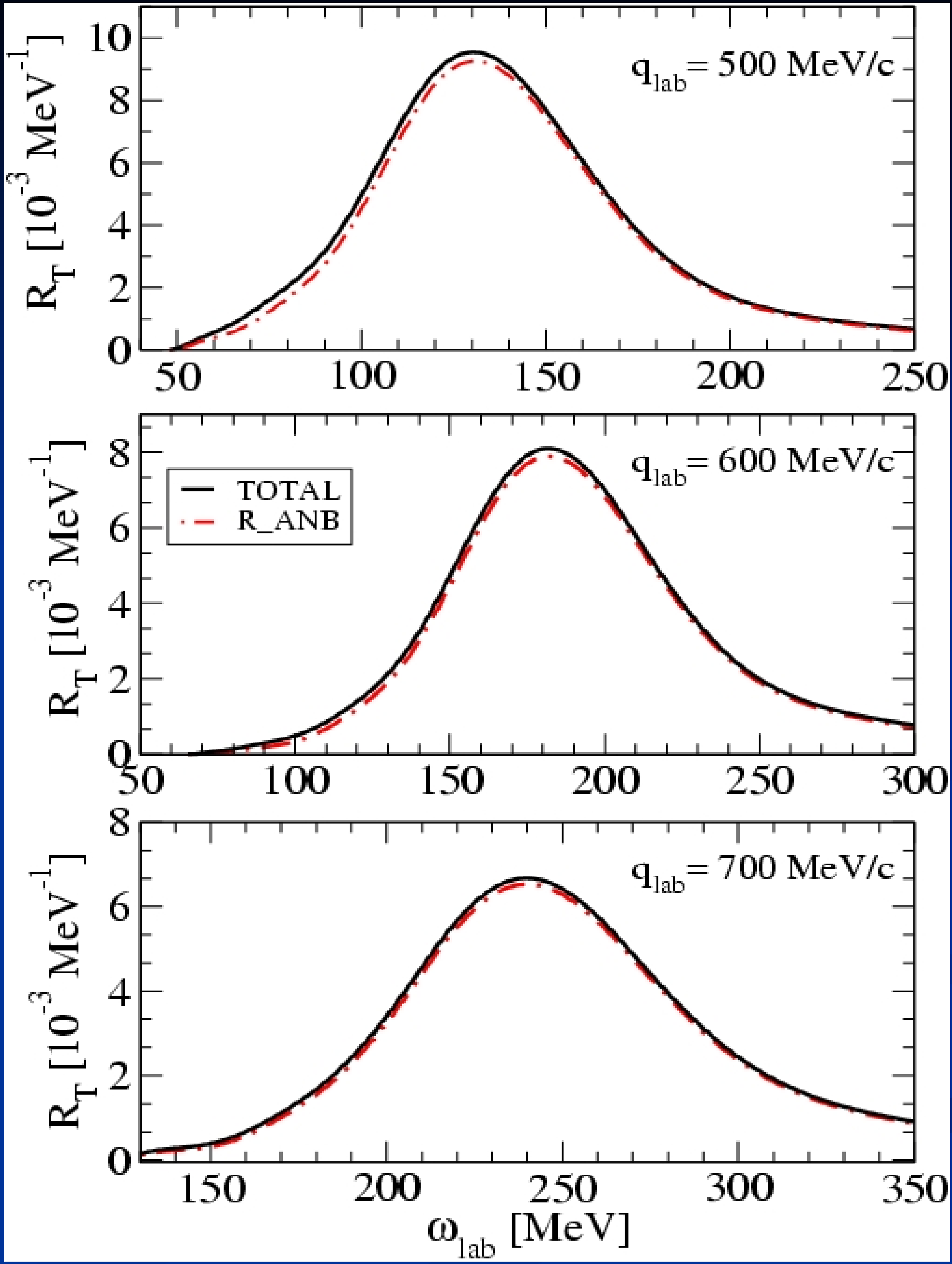
- Rel. contribution: reduction of peak height (6.2%, 8.5%, 11.3 % at q=500, 600, 700 MeV/c)



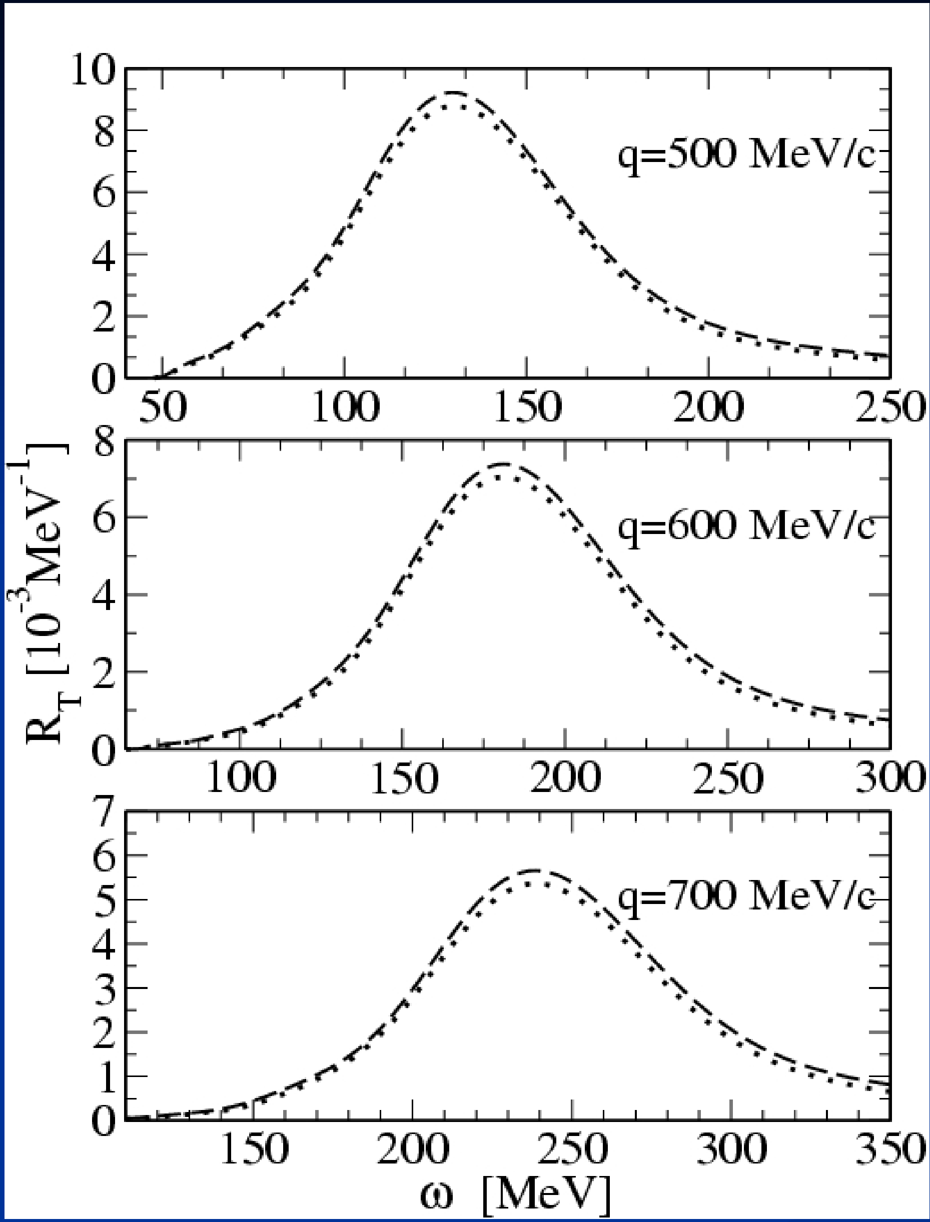
Results

- MEC:

small increase of peak height (3.2%, 2.7%, 2.2% at q=500, 600, 700 MeV/c)



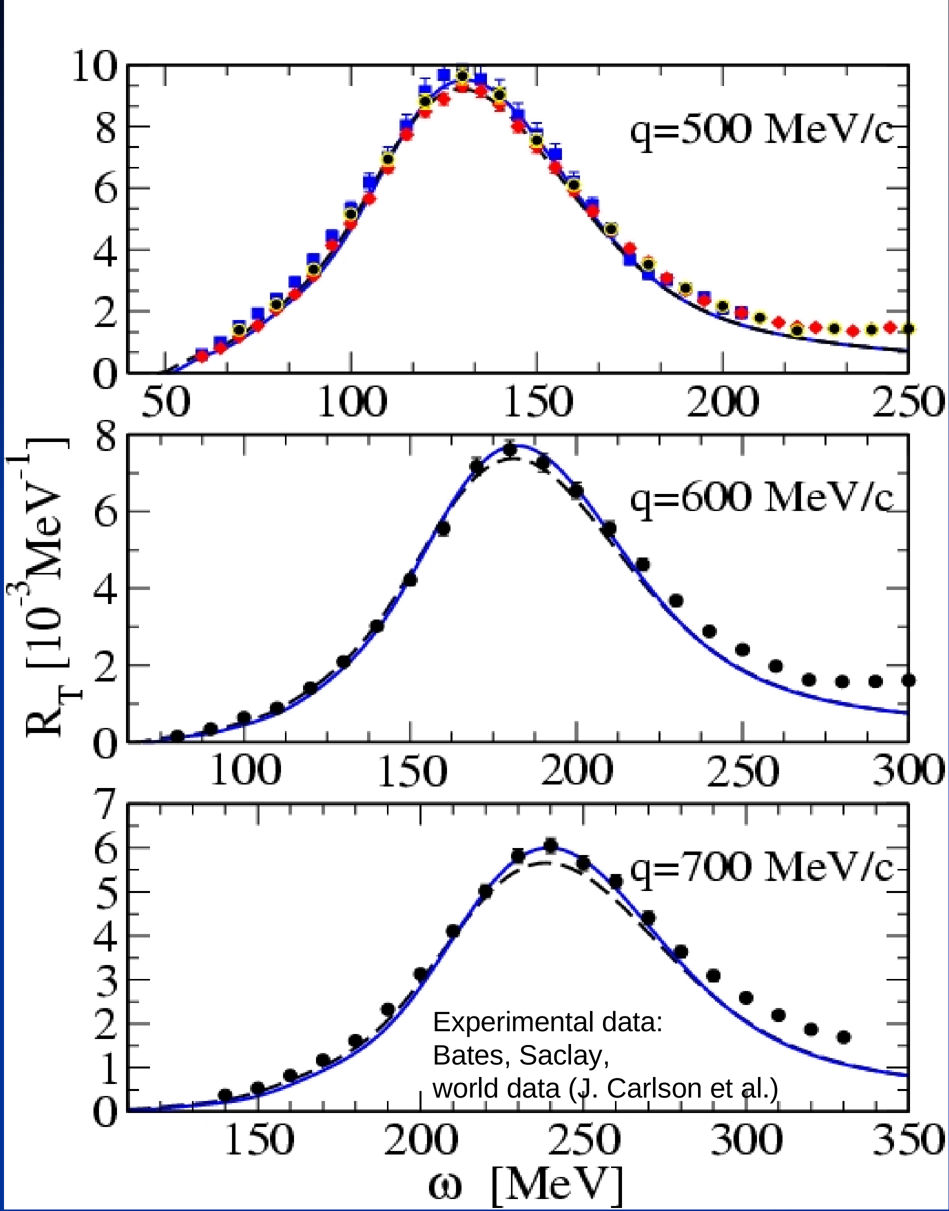
Δ-IC contribution



Dotted: without Δ

Dashed with Δ

Effect of two-fragment model



Dashed: with Δ (as before)

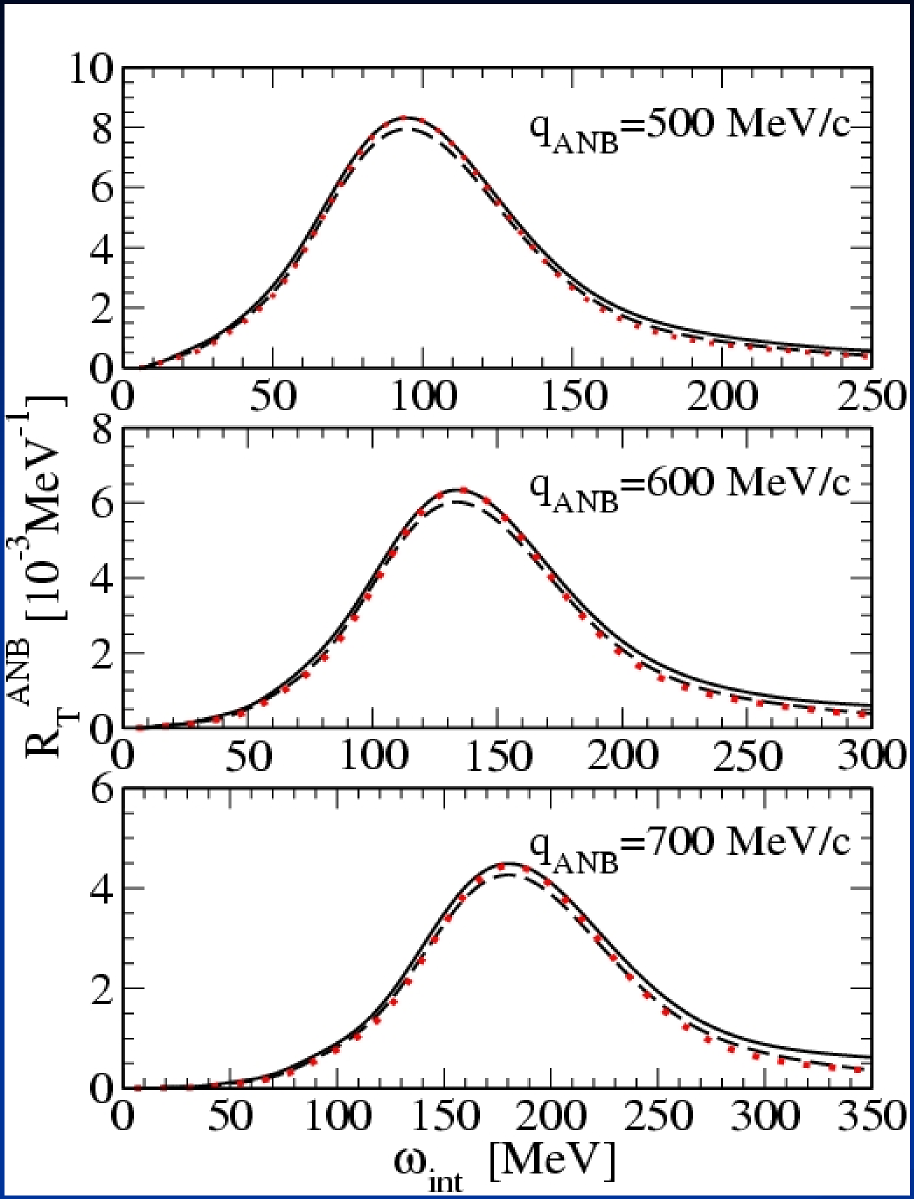
Solid: same but with two-fragment model

L. Yuan et al., PLB 706, 90 (2011)

Deltuva et al. (PRC70, 034004,2004):

Calculation of R_T of ^{3}He with CDBonn and CDBonn+Δ:

no Δ effects in peak region!



Partial compensation of Δ-IC and 3NF

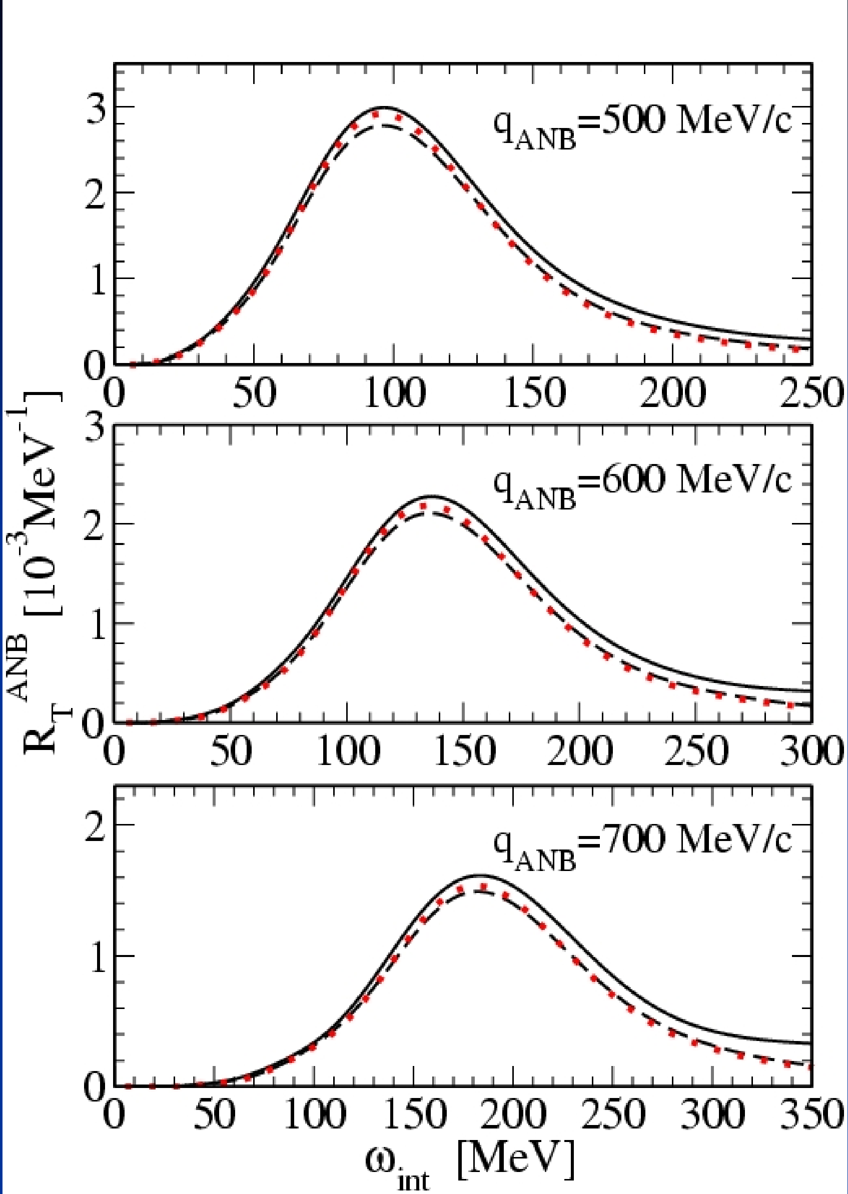
Dotted: no Δ and no 3NF

Dashed: no Δ but with 3NF

Solid: with Δ and with 3NF

No Δ effect in peak region
In a CC calculation!

Only Isospin channel T=3/2

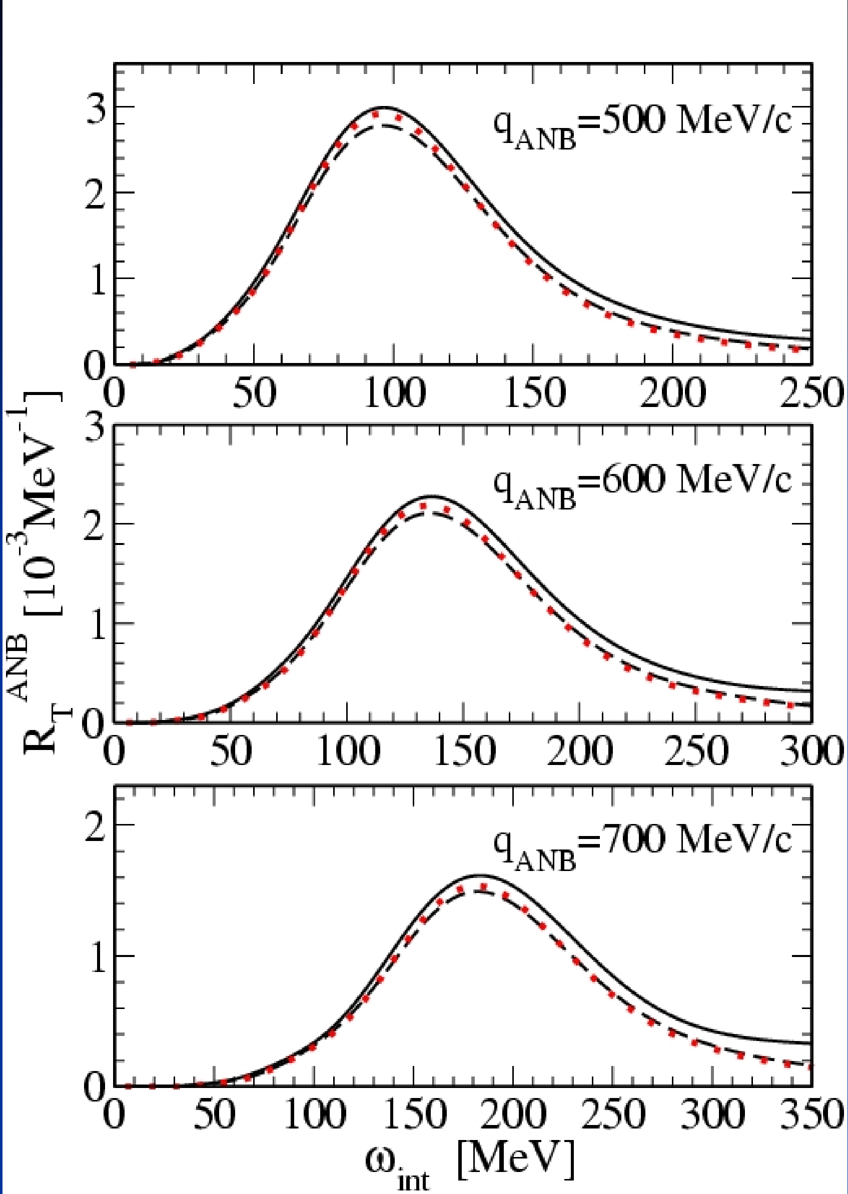


Dotted: no Δ and no 3NF

Dashed: no Δ but with 3NF

Solid: with Δ and with 3NF

Δ-IC contribution larger than 3NF effect in peak region!



Only Isospin channel T=3/2

Dotted: no Δ and no 3NF

Dashed: no Δ but with 3NF

Solid: with Δ and with 3NF

Strong Δ-IC effect also beyond peak

⇒ for this kinematics Δ-IC are important in 3-body breakup reactions

Conclusions

- the LIT metod opens up the possibility to carry out ab-initio calculations of reactions into the A-body continuum for A > 2
- only **bound states** techniques are needed
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We have discussed quite a few applications, there are still more (Compton scattering, pion production, weak nuclear responses)

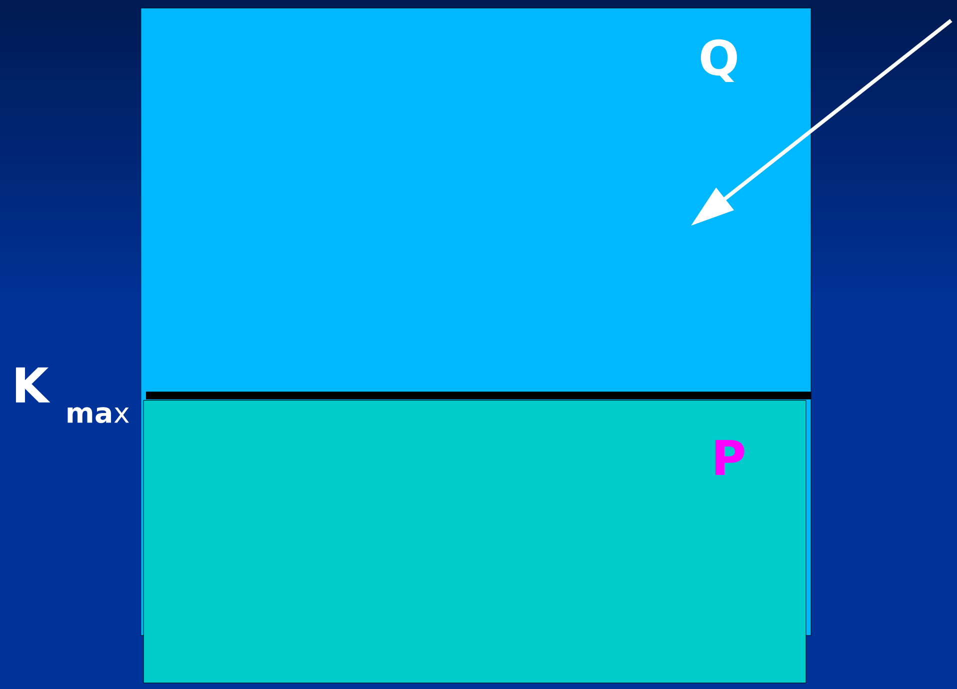
HOW TO SPEED UP THE CONVERGENCE?

SOLUTION

Here comes the idea of **EFFECTIVE INTERACTION**

same idea as for No Core Shell Model.
there the many particle basis is HO
here the many particle basis is HH

What is the main idea of an **effective interaction?**



whole Hilbert space

P and Q are projection operators

$$P + Q = 1$$

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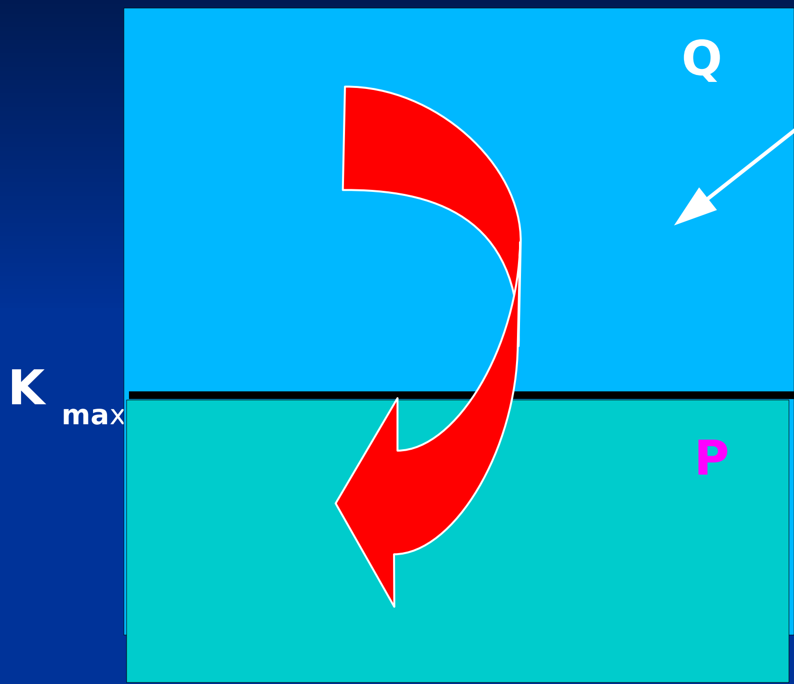

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Find a transformation $V \xrightarrow{T} V_{eff}$ such that

$$\langle \Psi | \, P H_{eff} P \, | \Psi \rangle = \langle \Psi | H | \Psi \rangle$$

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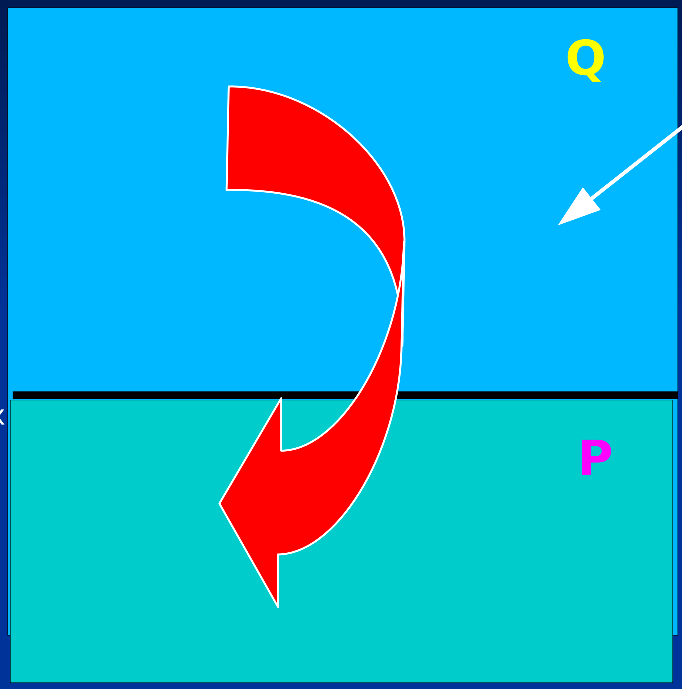

K_{max}

whole Hilbert space

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$$< \Psi \,|\, P H_{eff} P \,|\, \Psi > = < \Psi \,|\, H \,|\, \Psi >$$

formally this transformation exists (Bloch-Horowitz, **Lee-Suzuki**), however,

1) V_{eff} becomes an **A-body** operator
2) T is written in function of **Q**

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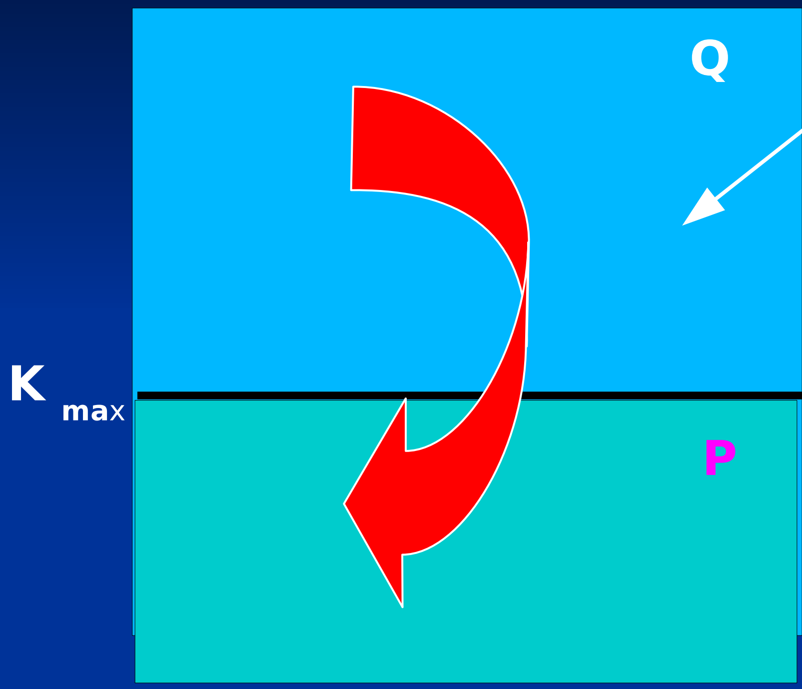

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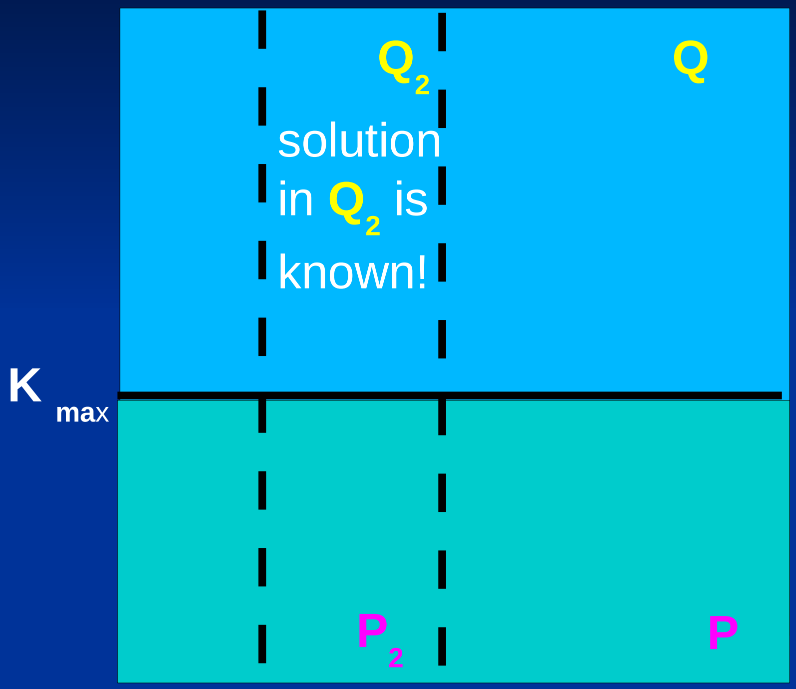
formally this transformation exists (Bloch-Horowitz, **Lee-Suzuki**), however,

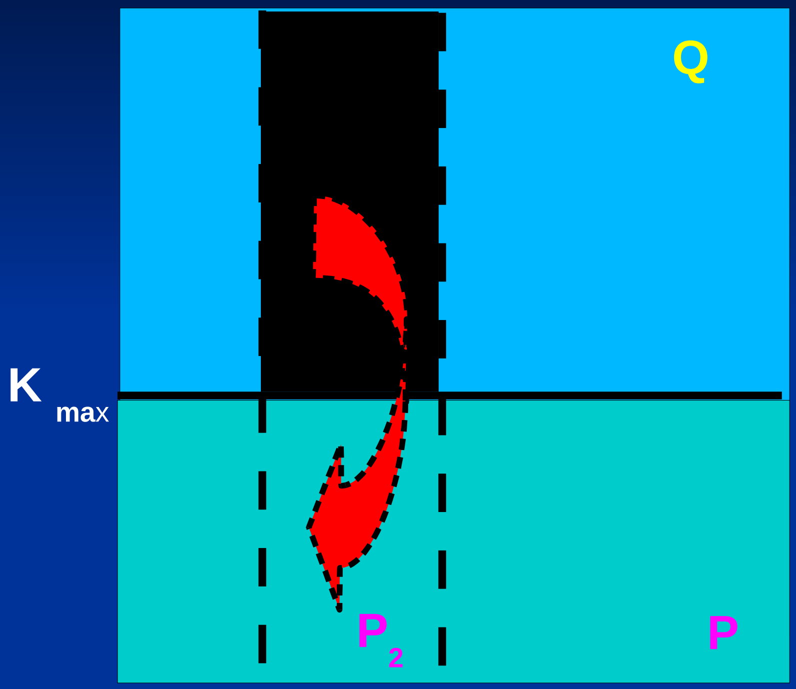
1) V_{eff} becomes an **A-body** operator $V_{eff}^{[A]}$
2) T is written in function of **Q**

Useless for practical purposes, the same as solving the full problem

PRACTICALLY:

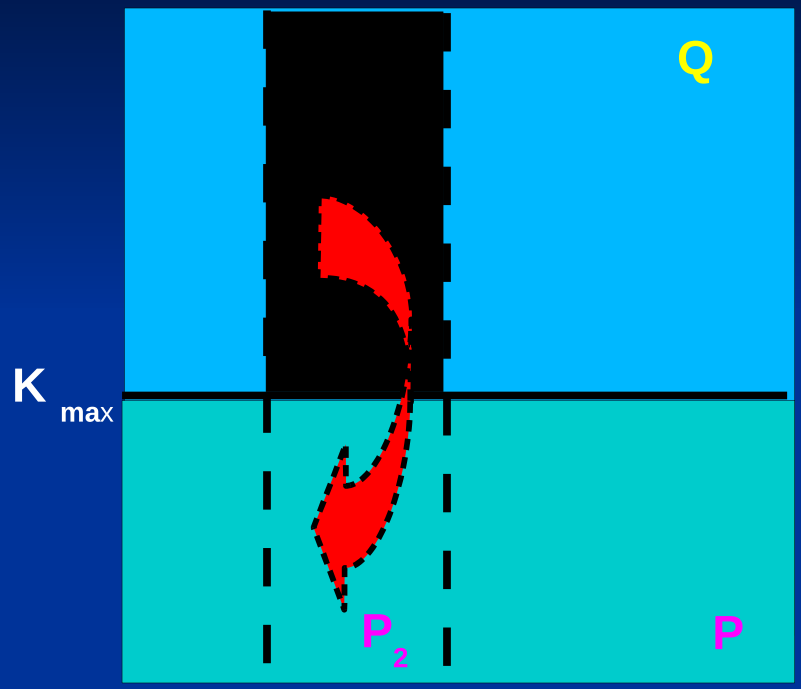


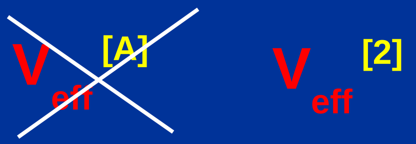
PRACTICALLY:



~~V_{eff} [A]~~ V_{eff} [2]

PRACTICALLY:





PRICE: I have to increase P (i.e. K_{max}) up to convergence

GAIN: what is missing is **less** than before -------> **faster convergence!**

Where, in the full H, is the two-body H_2 which I have to solve ?

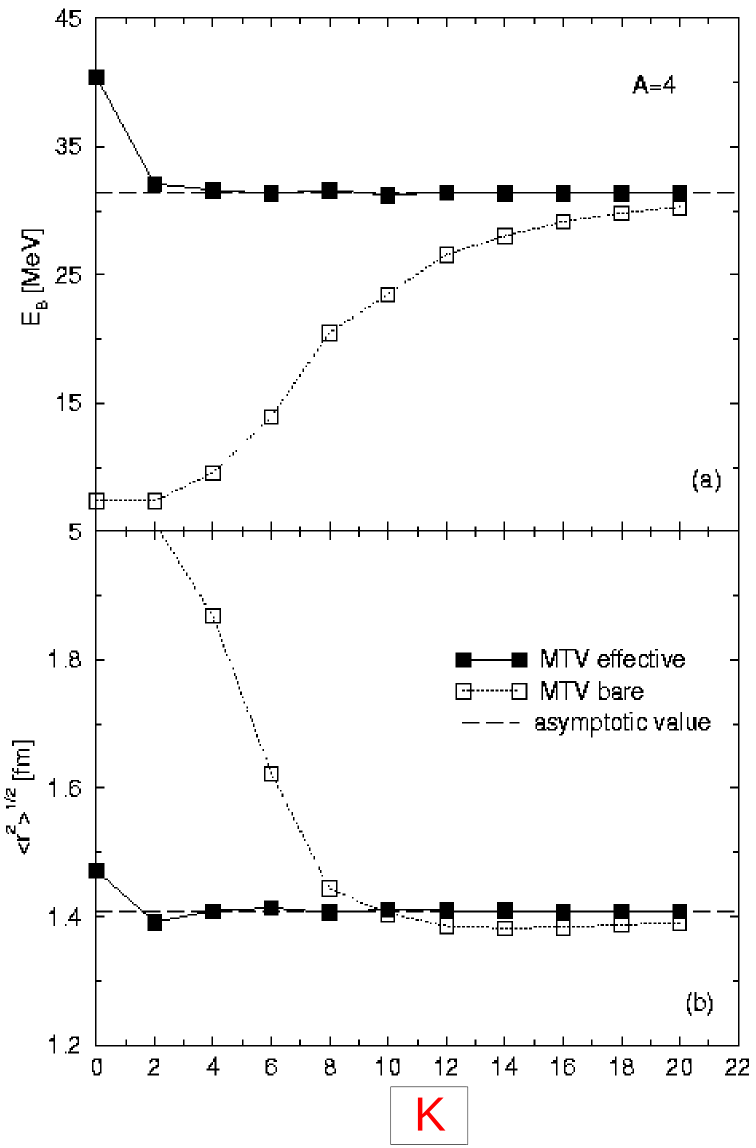
$$H_{NCSM} = \Sigma_k^{A-1} \; h_k^{ho} + (V_{12} - V_{12}^{HO}) + (V_{13} - V_{13}^{HO}) + \ldots..$$

$$= h^{ho}(\xi_1) + h^{ho}(\xi_2) + \ldots \quad + V(\vec{\xi}_1) - V^{HO}(\xi_1^2) + \ldots$$

$$H_{EIHH} = T + V_{12} + V_{13} + \ldots..$$

$$= 1/\mu\,(\Delta_\rho - K^2/\rho^2) + V(\xi_1) + V(\xi_1, \xi_2, \ldots.\, \xi_{A-1})$$

convergence:



^{4}He with MTV NN Potential

^{4}He

TABLE I. Convergence of the HH expansion for the ^{4}He ground-state energy (in MeV) and the ^{4}He ground-state energy for the ^{4}He radius root-mean-square radius (in fm) with the bare nonlocal Idaho-N3LO potential

K_{max}	Bare		Effective	
	$\langle H \rangle$	$\sqrt{\langle r^2 \rangle}$	$\langle H \rangle$	$\sqrt{\langle r^2 \rangle}$
2	−3.507	1.935	−17.773	1.620
4	−13.356	1.523	−22.188	1.533
6	−20.135	1.446	−24.228	1.496
8	−23.721	1.451	−25.445	1.498
10	−24.617	1.470	−25.363	1.506
12	−25.115	1.491	−25.439	1.515
14	−25.259	1.501	−25.398	1.516
16	−25.310	1.509	−25.390	1.518
18	−25.359	1.513	−25.385	1.518
20	−25.370	1.515	−25.381	1.518
	−25.37(2)	1.515(4)	−25.38(1)	1.518(1)
HH [20]	−25.38	1.516		
FY [20,21]	−25.37	—		
NCSM [22]	−25.39(1)	1.515(2)		

^{4}He

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6 -body JISP potential

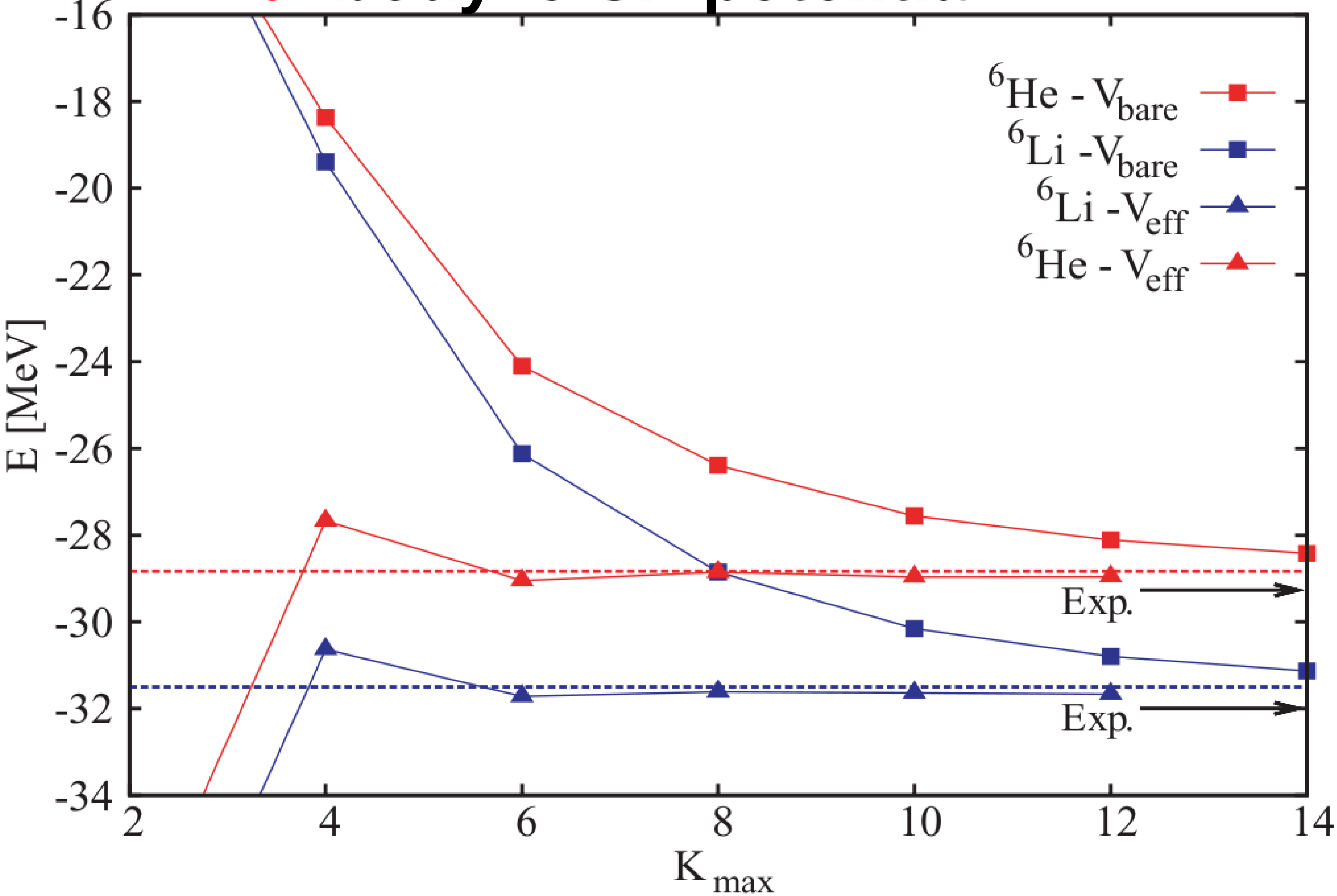

FIG. 2. (Color online) The ground-state energies of ^{6}He and ^{6}Li