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Leading twist amplitudes for exclusive neutrino interactions in the deeply virtual limit

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Neutrino scattering on nucleons in the regime of deeply virtual kinematics is studied both in the charged and the neutral electroweak sectors using a formalism developed by Blümlein, Robaschik, Geyer, and collaborators for the analysis of the virtual Compton amplitude in the generalized Bjorken region. We discuss the structure of the leading twist amplitudes of the process.

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I. INTRODUCTION AND MOTIVATIONS

Exclusive processes mediated by the weak force are an area of investigation which may gather wide interest in the forthcoming years due to the various experimental proposals to detect neutrino oscillations at intermediate energy using neutrino factories and superbeams [1]. These proposals require a study of the neutrino-nucleon interaction over a wide range of energy starting from the elastic/quasielastic domain up to the deep inelastic scattering (DIS) region. However, the discussion of the neutrino-nucleon interaction has, so far, been confined either to the DIS region or to the form factor/nucleon resonance region, while the intermediate energy region, at this time, remains unexplored also theoretically. From our viewpoint, the presence of such a gap in our knowledge well justifies any attempt to improve the current situation.

Together with Amore, we have pointed out [2] that exclusive processes of deeply virtual Compton scattering (DVCS) type could be relevant also in the theoretical study of the exclusive neutrino/nucleon interaction. Thanks to the presence of an on-shell photon emitted in the final state, this particle could be tagged together with the recoiling nucleon in a large underground detector in order to trigger on the process and exclude contamination from other backgrounds. A simple variant of this process, where the initial particle is a generic weakly interacting one, can probably be used to set exclusion limits on the mass of light dark matter, as suggested by various cosmological models [3] (see also [4]), in a suitable experimental environment.

With these motivations, a study of the $\nu N \rightarrow \nu N \gamma$ process has been performed in [2]. The process is mediated by a neutral current and is particularly clean since there is no Bethe-Heitler contribution. It has been termed *deeply virtual neutrino scattering* or DVNS and requires in its partonic description the electroweak analogue of the "nonforward parton distributions," previously introduced in the study of DVCS.

In this work we extend that analysis and provide, in part, a generalization of those results to the charged current case. Our treatment here is purposely short. The method that we use for the study of the charged processes is based on the formalism of the nonlocal operator product expansion and the technique of the harmonic polynomials, which allows to classify the various contributions to the interaction in terms of operators of a definite geometrical twist [5]. We present here a classification of the leading twist amplitudes of the charged process while a detailed phenomenological analysis useful for future experimental searches will be given elsewhere.

II. THE GENERALIZED BJORKEN REGION AND DVCS

Figure 1 illustrates the process that we are going to study, where a neutrino of momentum l scatters off a nucleon of momentum P_1 via a neutral or a charged current interaction; from the final state a photon and a nucleon emerge, of momenta q_2 and P_2, respectively, while the momenta of the final lepton is l'. We recall that Compton scattering has been investigated in the near past by several groups, since the original works [6–8]. A previous study of the virtual Compton process in the generalized Bjorken region, of which DVCS is just a particular case, can be found in [9]. From the hadronic side, the description of the interaction proceeds via new constructs of the parton model termed *generalized parton distributions* (GPD) or also *nonforward parton distributions*. The kinematics for the study of GPD's is characterized by a deep virtuality of the exchanged photon in the initial interaction ($\nu + p \rightarrow \nu + p + \gamma$) ($Q^2 \approx 2\ \mathrm{GeV}^2$), with the final state photon kept on-shell; large energy of the hadronic system ($W^2 > 6\ \mathrm{GeV}^2$) above the resonance domain and small momentum transfers $|t| < 1\ \mathrm{GeV}^2$. In the electroweak case, photon emission can occur from the final state electron (in the case of charged current interactions) and provides an additional contribution to the virtual Compton amplitude. We choose symmetric defining momenta and use as independent variables the average of the hadron and gauge bosons momenta

$$P_{1,2} = \bar{P} \mp \frac{\Delta}{2} \qquad q_{1,2} = q \pm \frac{\Delta}{2}, \tag{1}$$

with $\Delta = P_2 - P_1$ being the momentum transfer. Clearly,

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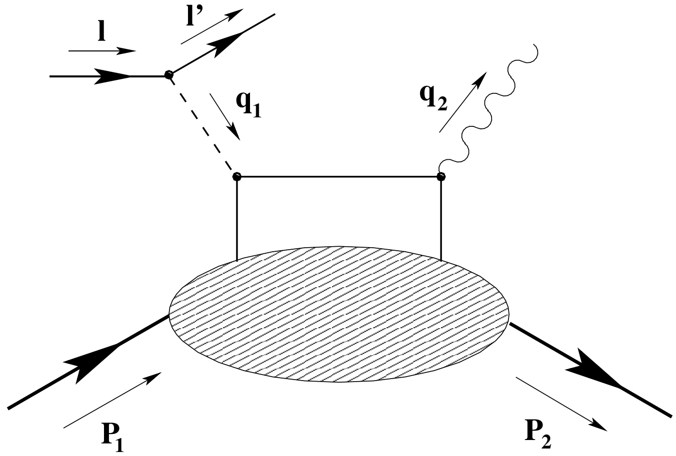

FIG. 1. Leading hand-bag diagrams for the process.

$$\bar{P}\cdot\Delta = 0, \qquad \bar{P}^2 = M^2 - \frac{\Delta^2}{4} \tag{2}$$

and M is the nucleon mass. There are two scaling variables which are identified in the process, since three scalar products can grow large in the generalized Bjorken limit: q^2, $\Delta\cdot q$, $\bar{P}\cdot q$.

The momentum transfer Δ^2 is a small parameter in the process. Momentum asymmetries between the initial and the final state nucleon are measured by two scaling parameters, ξ and η, related to ratios of the former invariants,

$$\xi = -\frac{q^2}{2\bar{P}\cdot q} \qquad \eta = \frac{\Delta\cdot q}{2\bar{P}\cdot q}, \tag{3}$$

where ξ is a variable of Bjorken type, expressed in terms of average momenta rather than nucleon and gauge bosons momenta. The standard Bjorken variable $x = -q_1^2/(2P_1\cdot q_1)$ is trivially related to ξ in the $t = 0$ limit and in the DVCS case $\eta = -\xi$.

Notice also that the parameter ξ measures the ratio between the plus component of the momentum transfer and the average momentum.

ξ, therefore, parametrizes the large component of the momentum transfer Δ, which can be generically described as

$$\Delta = -2\xi\bar{P} - \Delta_\perp, \tag{4}$$

where all the components of $\Delta_\perp$ are $O(\sqrt{|\Delta^2|})$.

III. BETHE-HEITLER CONTRIBUTIONS

Prior to embarking on the discussion of the virtual Compton contribution, we quote the result for the Bethe-Heitler (BH) subprocess, which makes its first appearance in the charged current case, since a real photon can be radiated off the leg of the final state lepton. The amplitude of the BH contribution for a W^+ exchange is as follows:

$$\begin{aligned} T_{\rm BH}^{W^+} &= -|e|\frac{g}{2\sqrt{2}}\frac{g}{\sqrt{2}}\bar{u}(l')\left[\gamma^\mu\frac{(\not{l}-\not{\Delta})}{(l-\Delta)^2+i\epsilon}\gamma^\nu(1-\gamma^5)\right] \\ &\quad\times u(l)\frac{D^{\nu\delta}(q_1)}{\Delta^2-M_W^2+i\epsilon}\epsilon^*_\mu(q_2)\overline{U}(P_2) \\ &\quad\times\Big[[F_1^u(\Delta^2)-F_1^d(\Delta^2)]\gamma^\delta+[F_2^u(\Delta^2) \\ &\quad - F_2^d(\Delta^2)]i\frac{\sigma^{\delta\alpha}\Delta_\alpha}{2M}\Big]U(P_1), \end{aligned} \tag{5}$$

where ϵ is the polarization vector of the photon and

$$\begin{aligned} T_{\rm BH}^{W^-} &= |e|\frac{g}{2\sqrt{2}}\frac{g}{\sqrt{2}}\bar{v}(l)\left[\gamma^\mu\frac{(\not{l}-\not{\Delta})}{(l-\Delta)^2+i\epsilon}\gamma^\nu(1-\gamma^5)\right] \\ &\quad\times v(l')\frac{D^{\nu\delta}(q_1)}{\Delta^2-M_W^2+i\epsilon}\epsilon^*_\mu(q_2)\overline{U}(P_2) \\ &\quad\times\Big[[F_1^u(\Delta^2)-F_1^d(\Delta^2)]\gamma^\delta+[F_2^u(\Delta^2) \\ &\quad - F_2^d(\Delta^2)]i\frac{\sigma^{\delta\alpha}\Delta_\alpha}{2M}\Big]U(P_1), \end{aligned} \tag{6}$$

for the W^- case, with $D^{\nu\delta}(q_1)/(\Delta^2 - M_W^2 + i\epsilon)$ being the propagator of the W's and $F_{1,2}$ the usual nucleon form factors (see also [2]).

IV. STRUCTURE OF THE COMPTON AMPLITUDE FOR CHARGED AND NEUTRAL CURRENTS

Moving to the Compton amplitude for charged and neutral currents, this can be expressed in terms of the correlator of currents,

$$T_{\mu\nu} = i\int d^4x e^{iqx}\langle P_2|T[J_\nu^\gamma(x/2)J_\mu^{W^\pm,Z_0}(-x/2)]|P_1\rangle, \tag{7}$$

where for the charged and neutral currents we have the following expressions:

$$J^{\mu Z_0}(-x/2) = \frac{g}{2\cos\theta_W}\overline{\psi}_u(-x/2)\gamma^\mu(g^Z_{uV} + g^Z_{uA}\gamma^5)\psi_u(-x/2) + \overline{\psi}_d(-x/2)\gamma^\mu(g^Z_{dV} + g^Z_{dA}\gamma^5)\psi_d(-x/2),$$

$$J^{\mu W^+}(-x/2) = \frac{g}{2\sqrt{2}}\overline{\psi}_u(-x/2)\gamma^\mu(1-\gamma^5)U^*_{ud}\psi_d(-x/2), \qquad J^{\mu W^-}(-x/2) = \frac{g}{2\sqrt{2}}\overline{\psi}_d(-x/2)\gamma^\mu(1-\gamma^5)U_{du}\psi_u(-x/2),$$

$$J^{\nu,\gamma}(x/2) = \overline{\psi}_d(x/2)\gamma^\nu\left(-\frac{1}{3}e\right)\psi_d(x/2) + \overline{\psi}_u(x/2)\gamma^\nu\left(\frac{2}{3}e\right)\psi_u(x/2). \tag{8}$$

Here we have chosen a simple representation of the flavor mixing matrix $U^*_{ud} = U_{ud} = U_{du} = \cos\theta_C$, where θ_C is the Cabibbo angle.

The coefficients g^Z_V and g^Z_A are

$$\begin{aligned} g^Z_{uV} &= \tfrac{1}{2} + \tfrac{4}{3}\sin^2\theta_W \qquad g^Z_{uA} = -\tfrac{1}{2} \\ g^Z_{dV} &= -\tfrac{1}{2} + \tfrac{2}{3}\sin^2\theta_W \qquad g^Z_{dA} = \tfrac{1}{2}, \end{aligned} \tag{9}$$

and

$$g_u = \tfrac{2}{3}, \qquad g_d = \tfrac{1}{3} \tag{10}$$

are the absolute values of the charges of the up and down quarks in units of the electron charge. A short computation gives

$$\begin{aligned} &\langle P_2|T[J^\gamma_\nu(x/2)J^{Z_0}_\mu(-x/2)]|P_1\rangle \\ &\quad = \langle P_2|\overline{\psi}_u(x/2)g_u\gamma_\nu S(x)\gamma_\mu(g^Z_{uV} + g^Z_{uA}\gamma^5)\psi_u(-x/2) \\ &\qquad - \overline{\psi}_d(x/2)g_d\gamma_\nu S(x)\gamma_\mu(g^Z_{dV} + g^Z_{dA}\gamma^5)\psi_d(-x/2) \\ &\qquad + \overline{\psi}_u(x/2)\gamma_\mu(g^Z_{uV} + g^Z_{uA}\gamma^5)S(-x)g_u\gamma_\nu\psi_u(x/2) \\ &\qquad - \overline{\psi}_d(x/2)\gamma_\mu(g^Z_{dV} + g^Z_{dA}\gamma^5)S(-x)g_d\gamma_\nu\psi_d(x/2)|P_1\rangle, \end{aligned} \tag{11}$$

$$\begin{aligned} &\langle P_2|T[J^\gamma_\nu(x/2)J^{W^+}_\mu(-x/2)]|P_1\rangle \\ &\quad = \langle P_2|\overline{\psi}_u(-x/2)\gamma_\mu(1-\gamma^5)U_{ud}S(-x)\gamma_\nu(-g_d) \\ &\qquad \times \psi_d(x/2) + \overline{\psi}_u(x/2)\gamma_\nu(g_u)S(x)\gamma_\mu(1-\gamma^5) \\ &\qquad \times U_{ud}\psi_d(-x/2)|P_1\rangle, \end{aligned} \tag{12}$$

$$\begin{aligned} &\langle P_2|T[J^\gamma_\nu(x/2)J^{W^-}_\mu(-x/2)]|P_1\rangle \\ &\quad = \langle P_2| - \overline{\psi}_d(x/2)g_d\gamma_\nu S(x)\gamma_\mu(1-\gamma^5)U_{du}\psi_u(-x/2) \\ &\qquad + \overline{\psi}_d(-x/2)\gamma_\mu(1-\gamma^5)S(-x)U_{du}\psi_u(x/2)|P_1\rangle, \end{aligned} \tag{13}$$

where all the factors $g/2\sqrt{2}$ and $g/2\cos\theta_W$, for simplicity, have been suppressed and we have defined

$$S^u(x) = S^d(x) \approx \frac{i\not{x}}{2\pi^2(x^2 - i\epsilon)^2}. \tag{14}$$

Using the following identities,

$$\begin{aligned} \gamma_\mu\gamma_\alpha\gamma_\nu &= S_{\mu\alpha\nu\beta}\gamma^\beta + i\epsilon_{\mu\alpha\nu\beta}\gamma^5\gamma^\beta, \\ \gamma_\mu\gamma_\alpha\gamma_\nu\gamma^5 &= S_{\mu\alpha\nu\beta}\gamma^\beta\gamma^5 - i\epsilon_{\mu\alpha\nu\beta}\gamma^\beta, \\ S_{\mu\alpha\nu\beta} &= (g_{\mu\alpha}g_{\nu\beta} + g_{\nu\alpha}g_{\mu\beta} - g_{\mu\nu}g_{\alpha\beta}), \end{aligned} \tag{15}$$

we rewrite the correlators as

$$\begin{aligned} T^{Z_0}_{\mu\nu} = i\int d^4x \frac{e^{iqx}x^\alpha}{2\pi^2(x^2 - i\epsilon)^2}\langle P_2|[g_u g_{uV}(S_{\mu\alpha\nu\beta}O^\beta_u \\ - i\epsilon_{\mu\alpha\nu\beta}O^{5\beta}_u) - g_u g_{uA}(S_{\mu\alpha\nu\beta}\tilde{O}^{5\beta}_u - i\epsilon_{\mu\alpha\nu\beta}\tilde{O}^\beta_u) \\ - g_d g_{dV}(S_{\mu\alpha\nu\beta}O^\beta_d - i\epsilon_{\mu\alpha\nu\beta}O^{5\beta}_d) \\ + g_d g_{dA}(S_{\mu\alpha\nu\beta}\tilde{O}^{5\beta}_d - i\epsilon_{\mu\alpha\nu\beta}\tilde{O}^\beta_d)]|P_1\rangle, \end{aligned} \tag{16}$$

$$\begin{aligned} T^{W^+}_{\mu\nu} = i\int d^4x \frac{e^{iqx}x^\alpha U_{ud}}{2\pi^2(x^2 - i\epsilon)^2}\langle P_2|[iS_{\mu\alpha\nu\beta}(\tilde{O}^\beta_{ud} + O^{5\beta}_{ud}) \\ + \epsilon_{\mu\alpha\nu\beta}(O^\beta_{ud} + \tilde{O}^{5\beta}_{ud})]|P_1\rangle, \end{aligned} \tag{17}$$

$$\begin{aligned} T^{W^-}_{\mu\nu} = i\int d^4x \frac{e^{iqx}x^\alpha U_{du}}{2\pi^2(x^2 - i\epsilon)^2}\langle P_2|[-iS_{\mu\alpha\nu\beta}(\tilde{O}^\beta_{du} + O^{5\beta}_{du}) \\ - \epsilon_{\mu\alpha\nu\beta}(O^\beta_{du} + \tilde{O}^{5\beta}_{du})]|P_1\rangle. \end{aligned} \tag{18}$$

We have suppressed the x dependence of the operators in the former equations. The relevant operators are denoted by

$$
\begin{aligned}
\tilde{O}_a^{\beta}(x/2,-x/2) &= \overline{\psi}_a(x/2)\gamma^{\beta}\psi_a(-x/2) + \overline{\psi}_a(-x/2)\gamma^{\beta}\psi_a(x/2),\\
\tilde{O}_a^{5\beta}(x/2,-x/2) &= \overline{\psi}_a(x/2)\gamma^5\gamma^{\beta}\psi_a(-x/2) - \overline{\psi}_a(-x/2)\gamma^5\gamma^{\beta}\psi_a(x/2),\\
O_a^{\beta}(x/2,-x/2) &= \overline{\psi}_a(x/2)\gamma^{\beta}\psi_a(-x/2) - \overline{\psi}_a(-x/2)\gamma^{\beta}\psi_a(x/2),\\
O_a^{5\beta}(x/2,-x/2) &= \overline{\psi}_a(x/2)\gamma^5\gamma^{\beta}\psi_a(-x/2) + \overline{\psi}_a(-x/2)\gamma^5\gamma^{\beta}\psi_a(x/2),
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\tilde{O}_{ud}^{\beta}(x/2,-x/2) &= g_u\overline{\psi}_u(x/2)\gamma^{\beta}\psi_d(-x/2) + g_d\overline{\psi}_u(-x/2)\gamma^{\beta}\psi_d(x/2),\\
\tilde{O}_{ud}^{5\beta}(x/2,-x/2) &= g_u\overline{\psi}_u(x/2)\gamma^5\gamma^{\beta}\psi_d(-x/2) - g_d\overline{\psi}_u(-x/2)\gamma^5\gamma^{\beta}\psi_d(x/2),\\
O_{ud}^{\beta}(x/2,-x/2) &= g_u\overline{\psi}_u(x/2)\gamma^{\beta}\psi_d(-x/2) - g_d\overline{\psi}_u(-x/2)\gamma^{\beta}\psi_d(x/2),\\
O_{ud}^{5\beta}(x/2,-x/2) &= g_u\overline{\psi}_u(x/2)\gamma^5\gamma^{\beta}\psi_d(-x/2) + g_d\overline{\psi}_u(-x/2)\gamma^5\gamma^{\beta}\psi_d(x/2),
\end{aligned} \tag{20}
$$

and similar ones with $u \leftrightarrow d$ interchanged.

We use isospin symmetry to relate flavor nondiagonal operators ($\hat{O}_{ff'}$) to flavor diagonal ones ($\hat{O}_{ff}$):

$$
\begin{aligned}
\langle p|\hat{O}^{ud}(x)|n\rangle &= \langle p|\hat{O}^{ud}(x)\tau^-|n\rangle = \langle p|[\hat{O}^{ud}(x),\tau^-]|n\rangle\\
&= \langle p|\hat{O}^{uu}(x)|p\rangle - \langle p|\hat{O}^{dd}(x)|p\rangle,\\
\langle p|\hat{O}^{ud}(x)|n\rangle &= \langle n|\hat{O}^{dd}(x)|n\rangle - \langle n|\hat{O}^{uu}(x)|n\rangle,\\
\langle n|\hat{O}^{du}(x)|p\rangle &= \langle p|\hat{O}^{uu}(x)|p\rangle - \langle p|\hat{O}^{dd}(x)|p\rangle,\\
\langle n|\hat{O}^{du}(x)|p\rangle &= \langle n|\hat{O}^{dd}(x)|n\rangle - \langle n|\hat{O}^{uu}(x)|n\rangle,
\end{aligned} \tag{21}
$$

where

$$\tau^{\pm} = \tau^x \pm \tau^y \tag{22}$$

are isospin raising/lowering operators expressed in terms of Pauli matrices.

V. PARAMETRIZATION OF NONFORWARD MATRIX ELEMENTS

The extraction of the leading twist contribution to the hand-bag diagram is performed using the geometrical twist expansion, as developed in [10–12], adapted to our case. We set the twist-2 expansions on the light cone (with $x^2 = 0$) and we choose the light-cone gauge to remove the gauge link,

$$
\begin{aligned}
&\langle P_2|\overline{\psi}_a(-kx)\gamma^{\mu}\psi_a(kx)|P_1\rangle^{\text{twist-2}}\\
&\quad= \int Dz e^{-ik(x\cdot P_z)}F^{a(\nu)}(z_1,z_2,P_i\cdot P_j x^2,P_i\cdot P_j)\\
&\qquad\times \overline{U}(P_2)[\gamma^{\mu} - ikP_z^{\mu}\not{x}]U(P_1)\\
&\qquad+ \int Dz e^{-ik(x\cdot P_z)}G^{a(\nu)}(z_1,z_2,P_i\cdot P_j x^2,P_i\cdot P_j)\\
&\qquad\times \overline{U}(P_2)\left[\frac{(i\sigma^{\mu\alpha}\Delta_\alpha)}{M} - ikP_z^{\mu}\frac{(i\sigma^{\alpha\beta}x_\alpha\Delta_\beta)}{M}\right]U(P_1),
\end{aligned} \tag{23}
$$

with $0 < k < 1$ a scalar parameter, with

$$P_z = P_1 z_1 + P_2 z_2, \tag{24}$$

and

$$
\begin{aligned}
&\langle P_2|\overline{\psi}_a(-kx)\gamma^5\gamma^{\mu}\psi_a(kx)|P_1\rangle^{\text{twist-2}}\\
&\quad= \int Dz e^{-ik(x\cdot P_z)}F^{5a(\nu)}(z_1,z_2,P_i\cdot P_j x^2,P_i\cdot P_j)\\
&\qquad\times \overline{U}(P_2)[\gamma^5\gamma^{\mu} - ikP_z^{\mu}\gamma^5\not{x}]U(P_1)\\
&\qquad+ \int Dz e^{-ik(x\cdot P_z)}G^{5a(\nu)}(z_1,z_2,P_i\cdot P_j x^2,P_i\cdot P_j)\\
&\qquad\times \overline{U}(P_2)\gamma^5\left[\frac{(i\sigma^{\mu\alpha}\Delta_\alpha)}{M} - ikP_z^{\mu}\frac{(i\sigma^{\alpha\beta}x_\alpha\Delta_\beta)}{M}\right]U(P_1).
\end{aligned} \tag{25}
$$

The index (ν) in the expressions of the distribution functions F, G has been introduced in order to distinguish them from the parametrization given in [9,10], which is related to linear combinations of electromagnetic correlators. In the expressions above a is a flavor index and we have introduced both a vector (Dirac) and a Pauli-type form factor contribution with nucleon wave functions [$U(P)$]. The product $P_i \cdot P_j$ denotes all the possible products of the two momenta P_1 and P_2, and the measure of integration is defined by [10]

$$Dz = \tfrac{1}{2}dz_1 dz_2\theta(1-z_1)\theta(1+z_1)\theta(1-z_2)\theta(1+z_2). \tag{26}$$

In our parametrization of the correlators, we are omitting the so-called "trace terms" (see Ref. [9]), since these terms vanish on-shell. In order to arrive at a partonic interpretation, one introduces variables z_+ and z_- conjugated to $2\bar{P}$ and Δ and defined as

$$
\begin{aligned}
&z_+ = 1/2(z_1+z_2), \qquad z_- = 1/2(z_2-z_1),\\
&Dz = dz_+dz_-\theta(1+z_++z_-)\theta(1+z_+-z_-)\\
&\qquad\times\theta(1-z_++z_-)\theta(1-z_+-z_-).
\end{aligned} \tag{27}
$$

In terms of these new variables $P_z = 2\bar{P}z_+ + \Delta z_-$, which will be used below.

At this stage, we can proceed to calculate the hadronic tensor by performing the x-space integrations. This will be illustrated in the case of the W^+ current, the others being similar. We define

$$\begin{aligned}
&\int dx^4 \frac{e^{iqx}x^\alpha}{2\pi^2(x^2-i\epsilon)^2}\langle P_2|S_{\mu\alpha\nu\beta}\tilde{O}^{a\beta}|P_1\rangle = \tilde{S}^a_{\mu\nu},\\
&\int dx^4 \frac{e^{iqx}x^\alpha}{2\pi^2(x^2-i\epsilon)^2}\langle P_2|S_{\mu\alpha\nu\beta}O^{5a\beta}|P_1\rangle = S^{5a}_{\mu\nu},\\
&\int dx^4 \frac{e^{iqx}x^\alpha}{2\pi^2(x^2-i\epsilon)^2}\langle P_2|\epsilon_{\mu\alpha\nu\beta}O^{a\beta}|P_1\rangle = \varepsilon^a_{\mu\nu},\\
&\int dx^4 \frac{e^{iqx}x^\alpha}{2\pi^2(x^2-i\epsilon)^2}\langle P_2|\epsilon_{\mu\alpha\nu\beta}\tilde{O}^{5a\beta}|P_1\rangle = \tilde{\varepsilon}^{5a}_{\mu\nu},
\end{aligned} \tag{28}$$

and introduce the variables

$$Q_1^\alpha(z) = q^\alpha + \tfrac{1}{2}P_z^\alpha, \qquad Q_2^\alpha(z) = q^\alpha - \tfrac{1}{2}P_z^\alpha, \tag{29}$$

where (z) is now meant to denote both variables (z_+, z_-). The presence of a new variable Q_2, compared to [10], is related to the fact that we are parametrizing each single bilinear covariant rather than linear combinations of them, as in the electromagnetic case.

After some rearrangements we get

$$\begin{aligned}
\tilde{S}^a_{\mu\nu} = {}& g_u \int Dz \frac{F^{a(\nu)}(z)}{(Q_1^2+i\epsilon)}\bigg\{[-g_{\mu\nu}\not{q} + q_\nu\gamma_\mu + q_\mu\gamma_\nu] + [P_{z\mu}\gamma_\nu + P_{z\nu}\gamma_\mu] - \frac{\not{q}}{(Q_1^2+i\epsilon)}[P_{z\mu}P_{z\nu} + P_{z\mu}q_\nu + P_{z\nu}q_\mu\\
&- g_{\mu\nu}(P_z\cdot q)]\bigg\} + g_d \int Dz \frac{F^{a(\nu)}(z)}{(Q_2^2+i\epsilon)}\bigg\{[-g_{\mu\nu}\not{q} + q_\nu\gamma_\mu + q_\mu\gamma_\nu] - [P_{z\mu}\gamma_\nu + P_{z\nu}\gamma_\mu]\\
&+ \frac{\not{q}}{(Q_2^2+i\epsilon)}[-P_{z\mu}P_{z\nu} + P_{z\mu}q_\nu + P_{z\nu}q_\mu - g_{\mu\nu}(P_z\cdot q)]\bigg\} + g_u \int Dz \frac{G^{a(\nu)}(z)}{(Q_1^2+i\epsilon)}\\
&\times \bigg\{\bigg[-g_{\mu\nu}\frac{i\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M} + q_\nu\frac{i\sigma^{\mu\beta}\Delta_\beta}{M} + q_\mu\frac{i\sigma^{\nu\beta}\Delta_\beta}{M}\bigg] + \bigg[P_{z\mu}\frac{i\sigma^{\nu\beta}\Delta_\beta}{M} + P_{z\nu}\frac{i\sigma^{\mu\beta}\Delta_\beta}{M}\bigg]\\
&- \frac{i\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M(Q_1^2+i\epsilon)}[P_{z\mu}P_{z\nu} + P_{z\mu}q_\nu + P_{z\nu}q_\mu - g_{\mu\nu}(P_z\cdot q)]\bigg\} + g_d \int Dz \frac{G^{a(\nu)}(z)}{(Q_2^2+i\epsilon)}\\
&\times \bigg\{\bigg[-g_{\mu\nu}\frac{i\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M} + q_\nu\frac{i\sigma^{\mu\beta}\Delta_\beta}{M} + q_\mu\frac{i\sigma^{\nu\beta}\Delta_\beta}{M}\bigg] - \bigg[P_{z\mu}\frac{i\sigma^{\nu\beta}\Delta_\beta}{M} + P_{z\nu}\frac{i\sigma^{\mu\beta}\Delta_\beta}{M}\bigg]\\
&+ \frac{i\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M(Q_2^2+i\epsilon)}[-P_{z\mu}P_{z\nu} + P_{z\mu}q_\nu + P_{z\nu}q_\mu - g_{\mu\nu}(P_z\cdot q)]\bigg\},
\end{aligned} \tag{30}$$

with an analogous expression for $S^{5a}_{\mu\nu}$, that we omit, since it can be recovered by performing the substitutions

$$\gamma_\mu \to \gamma^5\gamma_\mu \qquad \sigma^{\mu\nu} \to \gamma^5\sigma^{\mu\nu}, \qquad F^{a(\nu)}, G^{a(\nu)} \to F^{5a(\nu)}, G^{5a(\nu)} \tag{31}$$

in (30).

Similarly, for $\varepsilon^a_{\mu\nu}$ we get

$$\begin{aligned}
\varepsilon^a_{\mu\nu} = {}& g_u \int Dz F^{a(\nu)}(z)\bigg\{\frac{1}{(Q_1^2+i\epsilon)}\epsilon_{\mu\alpha\nu\beta}\bigg[q^\alpha\gamma^\beta - \frac{P_z^\beta q^\alpha\not{q}}{(Q_1^2+i\epsilon)}\bigg]\bigg\} - g_d \int Dz F^{a(\nu)}(z)\bigg\{\frac{1}{(Q_2^2+i\epsilon)}\epsilon_{\mu\alpha\nu\beta}\bigg[q^\alpha\gamma^\beta\\
&+ \frac{P_z^\beta q^\alpha\not{q}}{(Q_2^2+i\epsilon)}\bigg]\bigg\} + g_u \int Dz G^{a(\nu)}(z)\bigg\{\frac{1}{(Q_1^2+i\epsilon)}\epsilon_{\mu\alpha\nu\beta}\bigg[q^\alpha\frac{i\sigma^{\beta\delta}\Delta_\delta}{M} - \frac{P_z^\beta q^\alpha(i\sigma^{\lambda\delta}q_\lambda\Delta_\delta)}{M(Q_1^2+i\epsilon)}\bigg]\bigg\} - g_d \int Dz G^{a(\nu)}(z)\\
&\times\bigg\{\frac{1}{(Q_2^2+i\epsilon)}\epsilon_{\mu\alpha\nu\beta}\bigg[q^\alpha\frac{i\sigma^{\beta\delta}\Delta_\delta}{M} + \frac{P_z^\beta q^\alpha(i\sigma^{\lambda\delta}q_\lambda\Delta_\delta)}{M(Q_2^2+i\epsilon)}\bigg]\bigg\}.
\end{aligned} \tag{32}$$

The expression of $\tilde{\varepsilon}^{5a}_{\mu\nu}$ can be obtained in a similar way.

To compute the $T^{Z_0}_{\mu\nu}$ tensor, we need to include the following operators, which are related to the previous ones by $g_u, g_d \to 1$:

$$\begin{aligned}
&\int dx^4 \frac{e^{iqx}x^\alpha}{2\pi^2(x^2-i\epsilon)^2}\langle P_2|S_{\mu\alpha\nu\beta}O^{a\beta}|P_1\rangle = S^a_{\mu\nu}, \qquad \int dx^4 \frac{e^{iqx}x^\alpha}{2\pi^2(x^2-i\epsilon)^2}\langle P_2|S_{\mu\alpha\nu\beta}\tilde{O}^{5a\beta}|P_1\rangle = \tilde{S}^{5a}_{\mu\nu},\\
&\int dx^4 \frac{e^{iqx}x^\alpha}{2\pi^2(x^2-i\epsilon)^2}\langle P_2|\epsilon_{\mu\alpha\nu\beta}\tilde{O}^{a\beta}|P_1\rangle = \tilde{\varepsilon}^a_{\mu\nu}, \qquad \int dx^4 \frac{e^{iqx}x^\alpha}{2\pi^2(x^2-i\epsilon)^2}\langle P_2|\epsilon_{\mu\alpha\nu\beta}O^{5a\beta}|P_1\rangle = \varepsilon^{5a}_{\mu\nu}.
\end{aligned} \tag{33}$$

In this case a simple manipulation of (30) gives

$$\begin{aligned}
S^a_{\mu\nu} = &\int Dz \frac{F^{a(\nu)}(z)}{(Q_1^2 + i\epsilon)}\bigg\{[-g_{\mu\nu}\not{q} + q_\nu\gamma_\mu + q_\mu\gamma_\nu] + [P_{z\mu}\gamma_\nu + P_{z\nu}\gamma_\mu] - \frac{\not{q}}{(Q_1^2 + i\epsilon)}[P_{z\mu}P_{z\nu} + P_{z\mu}q_\nu + P_{z\nu}q_\mu \\
&- g_{\mu\nu}(P_z\cdot q)]\bigg\} - \int Dz \frac{F^{a(\nu)}(z)}{(Q_2^2 + i\epsilon)}\bigg\{[-g_{\mu\nu}\not{q} + q_\nu\gamma_\mu + q_\mu\gamma_\nu] - [P_{z\mu}\gamma_\nu + P_{z\nu}\gamma_\mu] + \frac{\not{q}}{(Q_2^2 + i\epsilon)}[-P_{z\mu}P_{z\nu} \\
&+ P_{z\mu}q_\nu + P_{z\nu}q_\mu - g_{\mu\nu}(P_z\cdot q)]\bigg\} + \int Dz \frac{G^{a(\nu)}(z)}{(Q_1^2 + i\epsilon)}\bigg\{\bigg[-g_{\mu\nu}\frac{i\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M} + q_\nu\frac{i\sigma^{\mu\beta}\Delta_\beta}{M} + q_\mu\frac{i\sigma^{\nu\beta}\Delta_\beta}{M}\bigg] \\
&+ \bigg[P_{z\mu}\frac{i\sigma^{\nu\beta}\Delta_\beta}{M} + P_{z\nu}\frac{i\sigma^{\mu\beta}\Delta_\beta}{M}\bigg] - \frac{i\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M(Q_1^2 + i\epsilon)}[P_{z\mu}P_{z\nu} + P_{z\mu}q_\nu + P_{z\nu}q_\mu - g_{\mu\nu}(P_z\cdot q)]\bigg\} - \int Dz\frac{G^{a(\nu)}(z)}{(Q_2^2 + i\epsilon)} \\
&\times\bigg\{\bigg[-g_{\mu\nu}\frac{i\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M} + q_\nu\frac{i\sigma^{\mu\beta}\Delta_\beta}{M} + q_\mu\frac{i\sigma^{\nu\beta}\Delta_\beta}{M}\bigg] - \bigg[P_{z\mu}\frac{i\sigma^{\nu\beta}\Delta_\beta}{M} + P_{z\nu}\frac{i\sigma^{\mu\beta}\Delta_\beta}{M}\bigg] \\
&+ \frac{i\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M(Q_2^2 + i\epsilon)}[-P_{z\mu}P_{z\nu} + P_{z\mu}q_\nu + P_{z\nu}q_\mu - g_{\mu\nu}(P_z\cdot q)]\bigg\}. \qquad (34)
\end{aligned}$$

The expression of $\tilde{S}^{5a}_{\mu\nu}$ is obtained from (34) by the replacements (31).

For the $\tilde{\varepsilon}^a_{\mu\nu}$ case, a rearrangement of (32) gives

$$\begin{aligned}
\tilde{\varepsilon}^a_{\mu\nu} = &\int DzF^{a(\nu)}(z)\bigg\{\frac{1}{(Q_1^2 + i\epsilon)}\epsilon_{\mu\alpha\nu\beta}\bigg[q^\alpha\gamma^\beta - \frac{P_z^\beta q^\alpha\not{q}}{(Q_1^2 + i\epsilon)}\bigg]\bigg\} + \int DzF^{a(\nu)}(z)\bigg\{\frac{1}{(Q_2^2 + i\epsilon)}\epsilon_{\mu\alpha\nu\beta}\bigg[q^\alpha\gamma^\beta + \frac{P_z^\beta q^\alpha\not{q}}{(Q_2^2 + i\epsilon)}\bigg]\bigg\} \\
&+ \int DzG^{a(\nu)}(z)\bigg\{\frac{1}{(Q_1^2 + i\epsilon)}\epsilon_{\mu\alpha\nu\beta}\bigg[q^\alpha\frac{i\sigma^{\beta\delta}\Delta_\delta}{M} - \frac{P_z^\beta q^\alpha(i\sigma^{\lambda\delta}q_\lambda\Delta_\delta)}{M(Q_1^2 + i\epsilon)}\bigg]\bigg\} \\
&+ \int DzG^{a(\nu)}(z)\bigg\{\frac{1}{(Q_2^2 + i\epsilon)}\epsilon_{\mu\alpha\nu\beta}\bigg[q^\alpha\frac{i\sigma^{\beta\delta}\Delta_\delta}{M} + \frac{P_z^\beta q^\alpha(i\sigma^{\lambda\delta}q_\lambda\Delta_\delta)}{M(Q_2^2 + i\epsilon)}\bigg]\bigg\}. \qquad (35)
\end{aligned}$$

Also in this case, the expression of the $\varepsilon^{5a}_{\mu\nu}$ tensor is obtained by the replacements (31).

VI. THE PARTONIC INTERPRETATION

At a first sight, the functions $F^{(\nu)}, G^{(\nu)}, F^{5(\nu)}, G^{5(\nu)}$ do not have a simple partonic interpretation. To progress in this direction it is useful to perform the expansions of the propagators,

$$\begin{aligned}
\frac{1}{Q_1^2 + i\epsilon} &\approx \frac{1}{2(\bar{P}\cdot q)}\frac{1}{[z_+ - \xi + \eta z_- + i\epsilon]}, \\
\frac{1}{Q_2^2 + i\epsilon} &\approx -\frac{1}{2(\bar{P}\cdot q)}\frac{1}{[z_+ + \xi + \eta z_- - i\epsilon]},
\end{aligned} \qquad (36)$$

which are valid only in the asymptotic limit. In this limit only the large kinematical invariants and their (finite) ratios are kept. In this expansion the physical scaling variable ξ appears quite naturally and one is led to introduce a new linear combination,

$$t = z_+ + \eta z_-, \qquad (37)$$

to obtain

$$\begin{aligned}
\frac{1}{Q_1^2 + i\epsilon} &\approx \frac{1}{2(\bar{P}\cdot q)}\frac{1}{[t - \xi + i\epsilon]}, \\
\frac{1}{Q_2^2 + i\epsilon} &\approx -\frac{1}{2(\bar{P}\cdot q)}\frac{1}{[t + \xi - i\epsilon]}.
\end{aligned} \qquad (38)$$

Analogously, we rewrite P_z using the variables $\{t, z_-\}$

$$P_z = 2\bar{P}t + \pi z_-, \qquad (39)$$

in terms of a new 4-vector, denoted by π, which is a direct measure of the exchange of transverse momentum with respect to $\bar{P}$:

$$\pi = \Delta + 2\xi\bar{P}. \qquad (40)$$

This quantity is strictly related to $\Delta_\perp$, as given in (4). The dominant (large) components of the process are related to the collinear contributions, and in our calculation the contributions proportional to the vector π will be neglected. This, of course, introduces a violation of the transversality of the process of $O(\Delta_\perp/2\bar{P}\cdot q)$.

Adopting the new variables $\{t, z_-\}$ and the conjugate ones $\{\bar{P}, \pi\}$, the relevant integrals that we need to "reduce" to a single (partonic) variable are contained in the expressions

$$
\begin{aligned}
H_{Q_1}(\xi) &= \int dz_+ dz_- \frac{H(z_+, z_-)}{(Q_1^2 + i\epsilon)} = \frac{1}{2\bar{P}\cdot q}\int Dz \frac{H(t+\xi z_-, z_-)}{(t-\xi+i\epsilon)} \\
H^{\mu}_{Q_1}(\xi) &= \int dz_+ dz_- \frac{H(z_+, z_-)}{(Q_1^2 + i\epsilon)}[2\bar{P}^\mu z_+ + \Delta^\mu z_-] = \frac{1}{2\bar{P}\cdot q}\int Dz \frac{H(t+\xi z_-, z_-)}{(t-\xi+i\epsilon)}[2\bar{P}^\mu t + \pi^\mu z_-] \\
H^{\mu\nu}_{Q_1}(\xi) &= \int dz_+ dz_- \frac{H(z_+, z_-)}{(Q_1^2 + i\epsilon)^2}[P_z^\mu P_z^\nu + q^\mu P_z^\nu + q^\nu P_z^\mu - g^{\mu\nu} q\cdot P_z] \\
&= \frac{1}{(2\bar{P}\cdot q)^2}\int Dz \frac{H(t+\xi z_-, z_-)}{(t-\xi+i\epsilon)^2}[4\bar{P}^\mu \bar{P}^\nu t^2 + (2q^\mu \bar{P}^\nu + 2q^\nu \bar{P}^\mu)t - g^{\mu\nu}(q\cdot P_z) + \pi^\mu\pi^\nu z_-^2 \\
&\quad + (q^\mu \pi^\nu + q^\nu \pi^\mu) z_- + (2\bar{P}^\mu \pi^\nu + 2\bar{P}^\nu \pi^\mu) t z_-].
\end{aligned}
\tag{41}
$$

Here $H(z_+, z_-)$ is a generic symbol for any of the functions. We have similar expressions for the integrals depending on the momenta Q_2.

The integration over the z_- variable in the integrals shown above is performed by introducing a suitable spectral representation of the function $H(t, +\xi z_-, z_-)$. As shown in [10], we can classify these representations by the $n = 0, 1, \ldots,$ powers of the variable z_-,

$$
\hat{h}_n(t/\tau, \xi) = \int dz_- z_-^n \hat{h}\left(\frac{t}{\tau} + \xi z_-, z_-\right). \tag{42}
$$

With the help of this relation, one obtains

$$
\begin{aligned}
\hat{H}_n(t, \xi) &= \frac{1}{t^n}\int dz_- z_-^n H(t+\xi z_-, z_-) \\
&= \frac{1}{t^n}\int_0^1 \frac{d\tau}{\tau}\tau^n \hat{h}_n(t/\tau, \xi) \\
&= \int_t^{\mathrm{sign}(t)} \frac{d\lambda}{\lambda}\lambda^{-n}\hat{h}_n(\lambda, \xi).
\end{aligned}
\tag{43}
$$

The result of this manipulation is to generate single-valued distribution amplitudes from double-valued ones. In the single-valued distributions $\hat{h}_n(t, \xi)$, the new scaling variables t and ξ have a partonic interpretation. ξ measures the asymmetry between the momenta of the initial and final states, while it can be checked that the support of the variable t is the interval $[-1, 1]$. The twist-2 part of the Compton amplitude is related only to the $n = 0$ moment of z_-. Before performing the z_- integration in each integral of Eq. (41) using Eq. (43)—a typical example is $H^{\mu\nu}_{Q_1}(\xi)$—we reduce such integrals to the sum of two terms using the identity

$$
\begin{aligned}
\int_{-1}^1 dt \frac{t^m}{(t\pm\xi\mp i\epsilon)^2}\hat{H}_n(t, \xi) = \int_{-1}^1 dt \frac{t^{m-1}}{(t\pm\xi\mp i\epsilon)} &\times\Big[\hat{H}_n(t, \xi) \\
&- \frac{1}{t^n}\hat{h}_n(t, \xi)\Big].
\end{aligned}
\tag{44}
$$

As shown in [9], after the z_- integration, the integrals in (41) can be rewritten in the form

$$
\begin{aligned}
H_{Q_1}(\xi) &= \frac{1}{2\bar{P}\cdot q}\int_{-1}^1 dt \frac{\hat{H}_0(t, \xi)}{(t-\xi+i\epsilon)}, \qquad H^\mu_{Q_1}(\xi) = \frac{2\bar{P}^\mu}{2\bar{P}\cdot q}\int_{-1}^1 dt \frac{t\hat{H}_0(t, \xi)}{(t-\xi+i\epsilon)} + O(\pi^\mu), \\
H^{\mu\nu}_{Q_1}(\xi) &= \frac{1}{(2\bar{P}\cdot q)^2}\int_{-1}^1 dt \frac{[2\hat{H}_0(t, \xi) - \hat{h}_0(t, \xi)]}{(t-\xi+i\epsilon)} 4\bar{P}^\mu \bar{P}^\nu t + \frac{1}{(2\bar{P}\cdot q)^2}\int_{-1}^1 dt \frac{[\hat{H}_0(t, \xi) - \hat{h}_0(t, \xi)]}{(t-\xi+i\epsilon)} \\
&\quad \times \{(2q^\mu \bar{P}^\nu + 2q^\nu \bar{P}^\mu - g^{\mu\nu} 2q\cdot\bar{P})\} + O(\pi^\mu\pi^\nu),
\end{aligned}
\tag{45}
$$

where, again, we are neglecting contributions from the terms proportional to π^μ, subleading in the deeply virtual limit. The quantities that actually have a strict partonic interpretation are the $\hat{h}_0^a(t, \xi)$ functions, as argued in Ref. [13]. The identification of the leading twist contributions is performed exactly as in [10]. We use a suitable form of the polarization vectors (for the gauge bosons) to generate the helicity components of the amplitudes and perform the asymptotic (DVCS) limit in order to identify the leading terms. Terms of $O(1/\sqrt{2\bar{P}\cdot q})$ are suppressed and are not kept into account. Below we will show only the tensor structures which survive after this limit.

VII. ORGANIZING THE COMPTON AMPLITUDES

In order to give a more compact expression for the amplitudes of our processes, we define

$$
\begin{aligned}
g^{T\mu\nu} &= -g^{\mu\nu} + \frac{q^\mu \bar{P}^\nu}{(q\cdot\bar{P})} + \frac{q^\nu \bar{P}^\mu}{(q\cdot\bar{P})}, \\
\alpha(t) &= \frac{g_u}{(t-\xi+i\epsilon)} - \frac{g_d}{(t+\xi-i\epsilon)}, \\
\beta(t) &= \frac{g_u}{(t-\xi+i\epsilon)} + \frac{g_d}{(t+\xi-i\epsilon)}.
\end{aligned}
\tag{46}
$$

Calculating all the integrals in Eqs. (30), (32), (34), and (35), we rewrite the expressions of the amplitudes as follows:

$$\begin{aligned} T^{W^+}_{\mu\nu} &= iU_{ud}\overline{U}(P_2)[i(\tilde{S}^u_{\mu\nu} + S^{5u}_{\mu\nu}) + \varepsilon^u_{\mu\nu} + \tilde{\varepsilon}^{5u}_{\mu\nu} - i(\tilde{S}^d_{\mu\nu} + S^{5d}_{\mu\nu}) - \varepsilon^d_{\mu\nu} - \tilde{\varepsilon}^{5d}_{\mu\nu}]U(P_1),\\ T^{W^-}_{\mu\nu} &= -iU_{du}\overline{U}(P_2)[i(\tilde{S}^u_{\mu\nu} + S^{5u}_{\mu\nu}) + \varepsilon^u_{\mu\nu} + \tilde{\varepsilon}^{5u}_{\mu\nu} - i(\tilde{S}^d_{\mu\nu} + S^{5d}_{\mu\nu}) - \varepsilon^d_{\mu\nu} - \tilde{\varepsilon}^{5d}_{\mu\nu}]_{g_u\to g_d}U(P_1),\\ T^{Z_0}_{\mu\nu} &= i\overline{U}(P_2)[g_u g_{uV}(S^u_{\mu\nu} - i\varepsilon^{5u}_{\mu\nu}) - g_u g_{uA}(\tilde{S}^{5u}_{\mu\nu} - i\tilde{\varepsilon}^u_{\mu\nu}) - g_d g_{dV}(S^d_{\mu\nu} - i\varepsilon^{5d}_{\mu\nu}) + g_d g_{dA}(\tilde{S}^{5d}_{\mu\nu} - i\tilde{\varepsilon}^d_{\mu\nu})]U(P_1), \end{aligned} \tag{47}$$

where, suppressing all the subleading terms in the tensor structures, we get

$$\begin{aligned} \overline{U}(P_2)\tilde{S}^{a\mu\nu}U(P_1) = &\int_{-1}^1 dt\alpha(t)\frac{g^{T\mu\nu}}{2\bar{P}\cdot q}\Big[\overline{U}(P_2)\not{q}U(P_1)\hat{f}^a_0(t,\xi) + \overline{U}(P_2)\Big(i\frac{\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M}\Big)U(P_1)\hat{g}^a_0(t,\xi)\Big]\\ &+\int_{-1}^1 dt\beta(t)\frac{\bar{P}^\mu\bar{P}^\nu}{(\bar{P}\cdot q)^2}\Big[\overline{U}(P_2)\not{q}U(P_1)t\hat{f}^a_0(t,\xi) + \overline{U}(P_2)\Big(i\frac{\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M}\Big)U(P_1)t\hat{g}^a_0(t,\xi)\Big], \end{aligned} \tag{48}$$

while for the $\varepsilon^{a\mu\nu}$ expression we obtain

$$\overline{U}(P_2)\varepsilon^{a\mu\nu}U(P_1) = \epsilon^{\mu\alpha\nu\beta}\frac{2q_\alpha\bar{P}_\beta}{(2\bar{P}\cdot q)^2}\int_{-1}^1 dt\beta(t)\Big[\overline{U}(P_2)\not{q}U(P_1)\hat{f}^a_0(t,\xi) + \overline{U}(P_2)\Big(i\frac{\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M}\Big)U(P_1)\hat{g}^a_0(t,\xi)\Big]. \tag{49}$$

Passing to the $S^{a\mu\nu}$ and $\tilde{\varepsilon}^{a\mu\nu}$ tensors, which appear in the Z_0 neutral current exchange, we get the following formulas:

$$\begin{aligned} \overline{U}(P_2)S^{a\mu\nu}U(P_1) = &\int_{-1}^1 dt\Big(\frac{1}{t-\xi+i\epsilon} + \frac{1}{t+\xi-i\epsilon}\Big)\Big\{\frac{g^{T\mu\nu}}{2\bar{P}\cdot q}\Big[\overline{U}(P_2)\not{q}U(P_1)\hat{f}^a_0(t,\xi) + \overline{U}(P_2)\\ &\times\Big(i\frac{\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M}\Big)U(P_1)\hat{g}^a_0(t,\xi)\Big]\Big\} + \int_{-1}^1 dt\Big(\frac{1}{t-\xi+i\epsilon} - \frac{1}{t+\xi-i\epsilon}\Big)\cdot\frac{\bar{P}^\mu\bar{P}^\nu}{(\bar{P}\cdot q)^2}\\ &\times\Big[\overline{U}(P_2)\not{q}U(P_1)t\hat{f}^a_0(t,\xi) + \overline{U}(P_2)\Big(i\frac{\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M}\Big)U(P_1)t\hat{g}^a_0(t,\xi)\Big], \end{aligned} \tag{50}$$

$$\begin{aligned} \overline{U}(P_2)\tilde{\varepsilon}^{a\mu\nu}U(P_1) = &\int_{-1}^1 dt\Big(\frac{1}{t-\xi+i\epsilon} - \frac{1}{t+\xi-i\epsilon}\Big)\epsilon^{\mu\alpha\nu\beta}\frac{2q_\alpha\bar{P}_\beta}{(2\bar{P}\cdot q)^2}\Big\{\Big[\overline{U}(P_2)\not{q}U(P_1)\hat{f}^a_0(t,\xi) + \overline{U}(P_2)\\ &\times\Big(i\frac{\sigma^{\alpha\beta}q_\alpha\Delta_\beta}{M}\Big)U(P_1)\hat{g}^a_0(t,\xi)\Big]\Big\}. \end{aligned} \tag{51}$$

Obviously the $\tilde{S}^{a5\mu\nu}$, $S^{a5\mu\nu}$, $\tilde{\varepsilon}^{a5\mu\nu}$, and $\varepsilon^{a5\mu\nu}$ expressions are obtained by the substitution (31).

At this stage, to square the amplitude, we need to calculate the following quantity, separately for the two charged processes:

$$T^2 = |T_{\mathrm{DVNS}}|^2 + T_{\mathrm{DVNS}}T^*_{\mathrm{BH}} + T_{\mathrm{BH}}T^*_{\mathrm{DVNS}} + |T_{\mathrm{BH}}|^2, \tag{52}$$

which simplifies in the neutral case, since it reduces $|T_{\mathrm{DVNS}}|^2$ [2]. Equations (47)–(51) and their axial counterparts are our final result and provide a description of the deeply virtual amplitude in the electroweak sector for charged and neutral currents. The result can be expressed in terms of a small set of parton distribution functions which can be easily related to generalized parton distributions, as in standard DVCS.

VIII. CONCLUSIONS

We have presented an application/extension of a method, which has been formulated in the past for the identification of the leading twist contributions to the parton amplitude in the generalized Bjorken region, to the electroweak case. We have considered the special case of a deeply virtual kinematics. We have focused our attention on processes initiated by neutrinos. From the theoretical and experimental viewpoints, the study of these processes is of interest, since very little is known of the neutrino interaction at intermediate energy in these more complex kinematical domains.

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