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# Dark Matter relic densities in Stückelberg axion models

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## Introduction

The Standard Model of Particle Physics (SM) represents the most advanced particle physics model and its fundamental features have been tested and confirmed in many experiments[1]. If we consider the particle content of the SM, the only missing particle is the Higgs boson, responsible for the breaking of the initial gauge symmetry  $SU(3) \times SU(2) \times U(1)$  down to the  $SU(3) \times U(1)_{em}$  low energy gauge symmetry and for the generation of the masses of fermions and gauge bosons. The existence of such a particle is the most important prediction of the SM; the experiments performed so far cannot exclude its existence even if the allowed mass window is getting smaller and the existence of a SM Higgs boson should be confirmed or ruled out by the end of this year.

If the SM is the best candidate to be the theory describing Nature at energy scales close to the electroweak scale, it can't be considered the definitive theory. For example it has to be extended in order to account for neutrino masses that, according to many neutrino experiments, are tiny but definitely non-zero[2]. Another important problem affecting the SM is the so-called "hierarchy problem". This problem is connected to the radiative corrections on the Higgs mass. The tree level value for the mass parameter appearing in the Higgs potential should be of order  $(-10^2)$  GeV but this value receives huge radiative corrections. For example, if we write the Yukawa coupling of the Higgs field  $h$  with a Dirac fermion  $f$  as

$$\mathcal{L}_{Yuk} = -y_f h \bar{f} f \quad (1.1)$$

then the one-loop corrections on the propagator due to fermion loops would give, together with some expected logarithmic corrections, the following quadratic correction to the squared Higgs mass

$$\delta m_h^2 = -\frac{y_f^2}{8\pi^2} \Lambda^2 \quad (1.2)$$

where  $\Lambda$  is a cutoff introduced to regularize the loop integral. This cutoff has to be interpreted as

the scale up to which new interactions, other than those appearing in the SM, can be neglected. The natural cutoff in the SM would be the Planck scale ( $M_{Pl}$ ), at which quantum gravity effects are expected to become non-negligible. The choice  $\Lambda = M_{Pl}$  would obviously give a huge contribution. The problem manifests itself in many other radiative correction calculations involving the Higgs field and is not an artifact of the chosen regularization technique. In order to cancel this quadratic divergences without making major changes to the model setup we have to introduce a strong fine tuning between the radiative corrections and the bare Higgs mass but this is considered “unnatural”.

The problem is not restricted only to the Higgs mass, since the value of  $m_h^2$  determines the Higgs vev and, consequently, the masses of all massive particles, such as the massive gauge bosons and fermions.

Several proposals have been made to solve these problems. We list here some of them.

- Technicolor: the basic idea behind these kind of theories is that the Higgs boson isn’t actually a fundamental particle but a condensate of fermions subject to a new gauge interaction which is confining at low energies. For a review see[3].
- Extra-dimensions (flat or warped): the common feature of this kind of theory is the fact that they introduce a certain number of extra space-time dimensions in order to obtain an effective Planck scale which is significantly lower than the expected one, thus reducing the hierarchy between the electroweak and the Planck scale. For an introduction see[4].
- Little Higgs: in these theories approximate global symmetries are used to protect the mass of the Higgs from the quadratic divergences; a review can be found in[5].

Another solution proposed to address the hierarchy problem is based on a symmetry defined “supersymmetry” (SUSY). Initially introduced as an early attempt to construct a string theory in the context of strong interactions[6–8], this idea found many applications in statistical physics, quantum mechanics, field theory and in the modern formulation of string theory. Early attempts to build supersymmetric field theories in 4 dimensions date back to the early 70’s[9–12].

The supersymmetry transformations relate bosons to fermions. In a field theory that is invariant under this symmetry, the fermionic and bosonic degrees of freedom are balanced, we have fermions and bosons that share the same mass and the couplings involving a particle and its supersymmetric partner are strongly related. So, in such a theory, the fermion loop correction on the Higgs mass would be accompanied by a scalar loop correction. We would have a perfect cancellation of the two contributions since they would be defined by the same coupling and we would just have a relative minus sign between the two diagrams. In this case we invoke a symmetry principle in order to protect the Higgs mass from dangerous radiative corrections.

We easily understand that supersymmetry cannot be a symmetry of the model describing Nature at the electroweak scale, since, from what we have said, we should have observed a particle

spectrum exhibiting a perfect boson-fermion symmetry. So we have to break supersymmetry at some scale above the electroweak one in such a way that the masses of the unobserved particles are such that they have escaped detection in the experiments performed so far. Furthermore, we have to break it “softly”, in the sense that we have to preserve the equality of dimensionless couplings in the vertices related by supersymmetry transformations and avoid the introduction of interactions that can lead to new quadratic radiative corrections. So the breaking terms will be mass terms, contributing to the masses of the unobserved particles, and terms parameterized by couplings of positive mass dimension. Clearly, choosing a SUSY breaking scale that is much bigger than the electroweak scale would spoil the advantage of introducing SUSY, reintroducing an hierarchy between the energy scales so the SUSY breaking scale is usually set around the TeV scale. This is compatible with the searches for supersymmetric partners of the known particles performed so far.

The minimal supersymmetric extension of the SM is called Minimal Supersymmetric Standard Model (MSSM). This model has been widely studied in the last decades and it has showed some nice features, such as an enhancement in the unification of the gauge couplings at high energies and the possibility to have a radiative breaking of the electroweak symmetry.

The fact that the SM has to be extended is also suggested by cosmological issues. Recent measurements[13] have confirmed that matter accounts for the 30% of the energy density of the Universe and most of this matter is “dark”, that is it doesn’t have electromagnetic interactions. Furthermore, the dark matter constituents have to be “cold” in the sense that, if the solution has to be found in a particle physics model, the particles that constitute it have to be massive in such a way that they were non relativistic during the phase of structure formation. Massless or too light particles would interfere with the process of structure formation contributing to the smoothing of the primordial density perturbations. This is due to the fact that relativistic species are characterized by a non-negligible pressure while non-relativistic species are essentially pressureless.

If we just consider the SM, the only dark matter candidates are relic neutrinos (a population of neutrinos that has decoupled from the cosmological plasma) but these particles are not “cold” in the sense that we have just explained.

On the contrary, the MSSM and other non-minimal supersymmetric extensions of the SM contain a sector where particles that can give raise to a cold dark matter population are found. For example, in the MSSM the neutral higgsinos (fermionic partners of the neutral Higgs fields) the neutral gauginos (fermionic partners of the neutral electroweak gauge bosons) form a sector, defined “neutralino” sector, where massive ( $\approx 100$  GeV) and weakly coupled states are found. The lightest of these states is also stable and it is interesting to note that its stability is the by-product of the introduction of a symmetry, called  $R$ -symmetry. This symmetry is introduced to forbid the presence in the theory of  $B$  or  $L$ -violating terms that would give, for example, a too

fast decay rate for the proton.

It is also important to try to make a connection between the SM and the new theories proposed as viable high energy theories. For example, it is interesting to understand what are the phenomenological implications of intersecting D-brane models[14–17]. In these models the gauge group contains several abelian factors and these would result in the existence of additional neutral gauge bosons respect to the SM ones. Since these gauge bosons have not been observed, we expect that, if they exist, they have a mass related to a scale which is out of reach for the current experiments. Nevertheless, some effects of these extra symmetries could be observed. We could also imagine that these extra gauge symmetries are anomalous and consider anomaly cancellation mechanisms other than the one that makes the SM anomaly free, namely, charge assignment. Alternative anomaly cancellation mechanisms typically involve axion fields associated to the gauge symmetry. Axions have been studied along the years as a realistic attempt to solve the strong CP problem [18, 19][20–24][25], to which they are closely related, but also as a possible candidate to answer more recent puzzles in cosmology such as the origin of dark energy whose presence has found confirmation in the study of Type I supernovae[26, 27]. In this second case it has been pointed out that the axion field can contribute to the vacuum energy, a possibility that remains realistic if their mass  $m_a$  - which in this case should be  $\sim 10^{-33}$  eV and smaller - is of electroweak[28] and not of QCD origin. In this case they differ significantly from the standard (Peccei-Quinn, PQ) invisible axion. According to this scenario, the vacuum misalignment (see[29, 30] for a discussion in the PQ case) induced at the electroweak scale would guarantee that the degree of freedom associated with the axion field rolls down very slowly towards the minimum of the non-perturbative instanton potential, with  $m_a$  much smaller than the current Hubble rate. Given the rather tight experimental constraints which have significantly affected the parameter space (axion mass and gauge couplings) for PQ axions[31–33], the study of these types of fields has also taken into account the possibility to evade the current bounds[34, 35]. These are summarized both into an upper and a lower bound on the size of  $f_a$ , the axion decay constant, which sets the scale of the misalignment angle  $\theta$ , defined as the ratio of the axion field ( $a$ ) over the PQ scale  $v_{PQ}$  ( $v_{PQ} \sim f_a$ ).

Axion-like particles can be reasonably described by pseudoscalar fields characterized by an enlarged parameter space for mass and couplings, with a direct coupling to the gauge fields whose strength remains unrelated to their mass. They have been at the center of several recent and less recent studies (see for instance[36, 37][38–40, 35, 41, 42]). They are supposed to inherit most of the properties of a typical PQ axion while acquiring some others which are not allowed to it.

We recall that the axion mass (which in the PQ case is  $O(\Lambda_{QCD}^2/f_a)$ ) and the axion coupling to the gauge fields are indeed related by the same constant  $f_a$ . In the PQ case  $f_a$  ( $\sim 10^{10} - 10^{12}$  GeV) makes the axion rather light ( $\sim 10^{-3} - 10^{-5}$  eV) and also very weakly coupled. The same scale

plays a significant role in establishing the axion as a possible dark matter candidate, contributing significantly to the relic densities of cold dark matter. A much smaller value of  $f_a$ , for instance, would diminish significantly the axion contribution to cold dark matter due to the suppression of its abundance ( $Y_\chi$ ) which depends quadratically on  $f_a$ .

It is quite immediate to realize that the gauging of the axionic symmetries, realized by introducing a local anomalous  $U(1)$  allows to leave the mass and the coupling of the axion to the gauge fields unrelated[43, 44], offering a natural theoretical justification for the origin of axion-like particles. We just recall that effective low energy models, incorporating gauged PQ interactions, emerge in several string and supergravity constructions, for instance in orientifold vacua of string theory and in gauged supergravities (see for instance [45, 46]).

This work is organized as follows. In Ch. 2 we present a brief description of supersymmetry. This section is intended to show how supersymmetry can be used to build extensions of the SM so we will restrict ourselves to the case of  $N = 1$  local SUSY. In Ch. 3 we present some details on the processes that lead to the formation of dark matter relic densities. In Ch. 4 we present some features of a non-supersymmetric model, called MLSOM[47], and see how a physical state with a Stückelberg axion component can arise in a model with an abelian  $U(1)$  gauge symmetry broken by the Stückelberg mechanism. We will also consider the possibility that such physical state can contribute to the observed cold dark matter density. In Ch. 5 we introduce a non-minimal supersymmetric extension of the SM and discuss some of its features. This extension includes an extra abelian anomalous factor in the gauge group. We will see that also in this case we can identify a physical state with a component given by the Stückelberg axion introduced to break the extra gauge symmetry and give mass to the gauge boson associated to the anomalous symmetry. We also analyze the production of neutralino relic density in this model using some publicly available numerical codes. Finally, in App. A we include some listing of program codes and outputs and some details about the tools we have used in the relic density calculation of the neutralino relic density.

## 1.1 List of publications

The chapters presented in this thesis are based on the following research papers

- Relic Densities of Gauged Axions and Supersymmetry  
 Claudio Corianò (INFN, Lecce & Salento U.), Marco Guzzi (Southern Methodist U.), Antonio Mariano (INFN, Lecce & Salento U.)  
 Published in Nucl.Phys.Proc.Suppl. 217 (2011) 75-77  
 e-Print: arXiv:1012.2420 [hep-ph]
- Relic Densities of Dark Matter in the  $U(1)$ -Extended NMSSM and the Gauged Axion Supermultiplet

Claudio Corianò (Salento U. & INFN, Lecce), Marco Guzzi (Southern Methodist U.), Antonio Mariano (Salento U. & INFN, Lecce)

e-Print: arXiv:1010.2010 [hep-ph]

- Gauged Axions and their QCD Interactions

Claudio Corianò (Salento U. & INFN, Lecce), Marco Guzzi (Southern Methodist U.), Antonio Mariano (Salento U. & INFN, Lecce)

Published in AIP Conf.Proc. 1317 (2011) 177-184

e-Print: arXiv:1009.5450 [hep-ph]

- Cosmological Properties of a Gauged Axion

Claudio Corianò (Salento U. & INFN, Lecce), Marco Guzzi (Southern Methodist U.), George Lazarides (Aristotle U., Thessaloniki), Antonio Mariano (Salento U. & INFN, Lecce)

Published in Phys.Rev. D82 (2010) 065013

e-Print: arXiv:1005.5441 [hep-ph]

- The Effective Actions of Pseudoscalar and Scalar Particles in Theories with Gauge and Conformal Anomalies

Roberta Armillis, Claudio Corianò, Luigi Delle Rose, Marco Guzzi, Antonio Mariano

Published in Fortsch.Phys. 58 (2010) 708-711

e-Print: arXiv:1001.5240 [hep-ph]

- Searching for an Axion-like Particle at the Large Hadron Collider

Claudio Corianò, Marco Guzzi, Antonio Mariano (Salento U. & INFN, Lecce)

Published in Nuovo Cim. C32N3-4 (2009) 265-267

e-Print: arXiv:0905.4416 [hep-ph]

- A Light Supersymmetric Axion in an Anomalous Abelian Extension of the Standard Model

Claudio Coriano, Marco Guzzi, Antonio Mariano, Simone Morelli (Salento U. & INFN, Lecce) Published in Phys.Rev. D80 (2009) 035006

e-Print: arXiv:0811.3675 [hep-ph]

- Axion and Neutralinos from Supersymmetric Extensions of the Standard Model with anomalous  $U(1)$ 's

Claudio Coriano, Marco Guzzi (Salento U. & INFN, Lecce), Nikos Irges (Wuppertal U.), Antonio Mariano (Salento U. & INFN, Lecce)

Published in Phys.Lett. B671 (2009) 87-90

e-Print: arXiv:0811.0117 [hep-ph]

## Supersymmetry

Supersymmetry has a special status with respect to the other symmetries that are widely used in physics since it is a symmetry that relates fermions and bosons. In a theory which is invariant under supersymmetry transformations we have the same number of fermionic and bosonic degrees of freedom. A consequence of supersymmetry is the fact that the fermionic and the corresponding bosonic degrees of freedom share the same mass, as far as supersymmetry is preserved. So, it is clear that supersymmetry should be broken at low energies, since the particle spectrum we observe is evidently not boson-fermion symmetric.

Supersymmetric extensions of the SM were introduced in order to give a solution to the hierarchy problem that affects the Standard Model of elementary particles (SM); this problem is related to the corrections to the Higgs propagator (and, so, to its mass) since this corrections are quadratic in the cut-off introduced to regularize the integrals. Introducing supersymmetry, we have a cancellation of this quadratic divergences considering the contribution of bosonic and fermionic loops appearing in the self-energy diagrams.

The first supersymmetric extension of the SM was the so-called Minimal Supersymmetric Standard Model (MSSM); in this model for each SM bosonic (fermionic) field we find a corresponding fermionic (bosonic) field. Furthermore, a new Higgs doublet is required if we want to introduce the Yukawa couplings and preserve supersymmetry.

Supersymmetric theories can be defined in a simple and compact form in the superfield and superspace formalism.

### 2.1 The Supersymmetry algebra

According to the Coleman-Mandula “no-go” theorem, if we require the following criteria for a quantum field theory

- the theory is invariant under a group  $G$  that has a subgroup that is locally isomorphic to

the Poincaré group  $P$ ;

- for any mass  $M$  we have a finite number of particles with masses less than  $M$ , all of them corresponding to positive-energy representations of the Poincaré group;
- the scattering amplitudes of elastic scattering are analytical functions of the center of mass energy and of the momentum transfer;
- scattering is non-trivial, i.e. we want to have scattering angles other than 0 and 180 degrees,
- there exists a neighborhood of the identity in  $G$  such that every element in this neighborhood belongs to a one-parameter subgroup. If  $x$  and  $y$  are two one-particle states whose wave-functions are test functions, then the derivative

$$-i \frac{d}{dt}(x, g(t)y) = (x, Ay) \quad (2.1)$$

exists at  $t = 0$  and it defines a continuous function of  $x$  and  $y$  which is linear in  $y$  and anti-linear in  $x$ ,

then the group  $G$  is locally isomorphic to the direct product of the Poincaré group  $P$  and a Lorentz-invariant compact group.

The Poincaré group consists of the Lorentz transformations and translations in a 4-dimensional space-time and is defined by the algebra of the generators

$$\begin{aligned} [M_{\mu\nu}, M_{\rho\sigma}] &= i(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\nu\sigma}M_{\mu\rho}) \\ [M_{\mu\nu}, P_\rho] &= i(\eta_{\nu\rho}P_\mu - \eta_{\mu\rho}P_\nu) \\ [P_\mu, P_\nu] &= 0 \end{aligned} \quad (2.2)$$

where  $\mu, \nu = 0, 1, 2, 3$ ,  $M_{\mu\nu} = -M_{\nu\mu}$  and  $\eta_{\mu\nu}$  represents the space-time metric. According to the Coleman-Mandula theorem, this algebra can only be extended by a finite number of Lorentz scalar operators  $B_i$

$$[M_{\mu\nu}, B_i] = 0 = [P_\mu, B_i] \quad (2.3)$$

that realize a Lie algebra

$$[B_i, B_j] = if_{ijk}B_k. \quad (2.4)$$

In 1975 Haag, Lopuszanski and Sohnius (HLS) showed that extensions of the Poincaré algebra are possible if one takes into account “graded Lie algebras” (also called “superalgebras”)[48].

The simplest extension is realized introducing a Majorana spinor charge  $Q_a$  that satisfies the following commutation and anti-commutation relations

$$\begin{aligned} \{Q_a, \bar{Q}_b\} &= 2\gamma_{ab}^\mu P_\mu & \bar{Q}_b &= (Q^\dagger \gamma^0)_b \\ [Q_a, P_\mu] &= 0 \\ [Q_a, M^{\mu\nu}] &= \frac{i}{4} [\gamma^\mu, \gamma^\nu]_{ab} Q_b \end{aligned} \quad (2.5)$$

where the matrices  $\gamma$  represent the Dirac matrices satisfying the relation

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}. \quad (2.6)$$

The same relations can be expressed using a couple of left and right Weyl spinors. We can assume to have more than one supersymmetry generator described by the left and right Weyl spinors  $Q_A^i, \bar{Q}_A^j$  that transform according to some representation of a compact Lie group defined by the Lorentz scalars  $B_i$ . The Weyl spinors  $Q_A^i$  satisfy the following relation

$$[Q_A^i, B_j] = iS_j^{ik} Q_A^k \quad (2.7)$$

where  $S$  represents the hermitian matrices of the representation. The only possible extension to this superalgebra, according to the HLS theorem, is the following

$$\begin{aligned} \{Q_A^i, Q_B^j\} &= \epsilon_{AB} Z^{ij} & Z^{\alpha\beta} &= -Z^{\beta\alpha} \\ [Z^{ij}, B_k] &= 0 \end{aligned} \quad (2.8)$$

where the  $Z^{ij}$  are called ‘‘central charges’’ since they are invariant under the action of the  $B_k$ ’s.

## 2.2 Representations of the SUSY algebra

The Poincaré algebra is a subalgebra of the supersymmetry algebra so any representation of the latter is also a representation of the former, usually a reducible one. So, an irreducible representation of the SUSY algebra corresponds to several particles that share the same mass since the Poincaré Casimir  $P^2$  is also a Casimir for the SUSY algebra. This is obvious if we recall, from Eq. (2.5), that

$$[P, Q] = 0 = [P, \bar{Q}]. \quad (2.9)$$

The particles that belong to an irreducible representation of the SUSY algebra are bosonic

and fermionic since if  $|\Omega\rangle$  is a state of the irreducible representation, then also  $Q|\Omega\rangle$  and  $\bar{Q}|\Omega\rangle$  are states in the same irreducible representation but their spin differs by  $1/2$  with respect to the spin of  $|\Omega\rangle$ . It can be easily shown that the number of fermionic and bosonic degrees of freedom in an irreducible representation is the same.

A direct consequence of the SUSY algebra is that the energy of a supersymmetric theory is non-negative. In fact, from the SUSY algebra we obtain for the Hamiltonian of the theory

$$\langle\psi|H|\psi\rangle = \frac{1}{4} \left[ |Q_1^I|\psi\rangle|^2 + |\bar{Q}_1^I|\psi\rangle|^2 + |Q_2^I|\psi\rangle|^2 + |\bar{Q}_2^I|\psi\rangle|^2 \right] \geq 0 \quad (2.10)$$

If  $|\psi\rangle$  coincides with the vacuum of the theory and supersymmetry is not spontaneously broken, then all of the fermionic charges annihilate the vacuum and the vacuum energy vanishes

$$\langle\psi|H|\psi\rangle = 0. \quad (2.11)$$

If the vacuum energy is different from zero, then at least one charge does not annihilate the vacuum. This signals that supersymmetry is spontaneously broken.

The Casimirs for the Poincaré algebra are given by  $P^2 = P_\mu P^\mu$  and  $W^2 = W_\mu W^\mu$  with  $W^\mu$  the Pauli-Lubanski vector defined as

$$W_\mu = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} P^\nu M^{\rho\sigma}. \quad (2.12)$$

The eigenvalues for  $P^2$  are given by the squared mass of the representation, while the eigenvalues for  $W^2$  are given by  $-m^2 s(s+1)$  with  $s = 0, \frac{1}{2}, 1, \dots$  in the case massive states and  $W_\mu = \lambda P_\mu$  in the case of a massless state of helicity  $\lambda$ .

For  $N = 1$  supersymmetry  $W^2$  is not a Casimir anymore since it doesn't commute with  $Q$  and  $\bar{Q}$ . In this case we redefine the Pauli-Lubanski vector in order to build a new invariant, called  $C^2$ , defined as

$$\begin{aligned} C^2 &= C_{\mu\nu} C^{\mu\nu} & C_{\mu\nu} &= B_\mu P_\nu - B_\nu P_\mu \\ B_\mu &= W_\mu - \frac{1}{4} \bar{Q}_\alpha \bar{\sigma}_\mu^{\dot{\alpha}\beta} Q_\beta. \end{aligned} \quad (2.13)$$

Let's consider the representations of the SUSY algebra. We will only be concerned with the case  $N = 1$  since this is the case of interest for the MSSM and its extensions.

For a massive state of mass  $m$  we can consider a Lorentz transformation that takes us in the rest frame in which we have  $P_\mu = (m, 0, 0, 0)$ . In this case the second Poincaré invariant will be

$$C^2 = 2m^4 J_i J^i \quad \text{with} \quad J_i = S_i - \frac{1}{4m} \bar{Q} \bar{\sigma}_i Q \quad (2.14)$$

where  $S_i$  are the components of the spin operator. Both  $S_i$  and  $\bar{\sigma}_i^{\dot{\alpha}\beta}$  obey the  $SU(2)$  algebra so  $J_i$  will obey the same algebra

$$[J_i, J_j] = i\epsilon_{ijk}J_k \quad (2.15)$$

and its eigenvalues will be  $j(j+1)$  with  $j$  integer or half-integer.

$J_i$  commutes with  $Q$  and  $\bar{Q}$  and this is trivial since we are in the rest frame ( $\vec{P} = 0$ ). Once  $m$  and  $j$  are fixed to define a state  $|m, j\rangle$  we can obtain a new state by the action of the supersymmetry generators

$$|\Omega\rangle = Q_1 Q_2 |m, j\rangle. \quad (2.16)$$

Since  $Q_1 |\Omega\rangle = Q_2 |\Omega\rangle = 0$ ,  $\Omega$  is a ‘‘Clifford vacuum’’ with respect to the fermionic operators  $Q_1$ ,  $Q_2$  and it has a  $2j+1$  degeneracy.

As a consequence  $J_i$  acts on  $|\Omega\rangle$  as the spin operator  $S_i$ , so  $|\Omega\rangle$  is actually an eigenstate of spin and any irreducible representation of the SUSY algebra can be classified in terms of mass and spin.

If we define the creation and annihilation operators in terms of the supersymmetry charges

$$a_{1,2} = \frac{1}{\sqrt{2m}} Q_{1,2} \quad a_{1,2}^\dagger = \frac{1}{\sqrt{2m}} \bar{Q}_{1,2} \quad (2.17)$$

then, for a given Clifford vacuum, the massive SUSY irreducible representation will be given by

$$|\Omega\rangle \quad a_1^\dagger |\Omega\rangle \quad a_2^\dagger |\Omega\rangle \quad \frac{1}{\sqrt{2}} a_1^\dagger a_2^\dagger |\Omega\rangle = \frac{1}{\sqrt{2}} a_2^\dagger a_1^\dagger |\Omega\rangle. \quad (2.18)$$

We have  $4(2j+1)$  states. If we consider the spin of these states for  $|\Omega\rangle = |m, j, j_3\rangle$ , we get  $s_3 = j_3, j_3 - \frac{1}{2}, j_3 + \frac{1}{2}, j_3$ .

In the simplest case,  $j = 0$ , we get a scalar, a pseudo-scalar (since parity swaps  $a_1^\dagger$  and  $a_2^\dagger$ ) and a Weyl fermion; all of these states will share the same mass  $m$ .

In the case of massless states we consider a vector in the light-like reference frame  $P_\mu = (E, 0, 0, E)$ . In this case

$$C^2 = -2E^2(B_0 - B_3)^2 = -\frac{1}{2}E^2 \bar{Q}_2 Q_2 \bar{Q}_2 Q_2 = 0. \quad (2.19)$$

From the superalgebra we also get:

$$\begin{aligned} \{Q_i, \bar{Q}_i\} &= 4E \\ \{Q_{\dot{2}}, \bar{Q}_{\dot{2}}\} &= 0. \end{aligned} \quad (2.20)$$

We can define a vacuum state  $|\Omega\rangle$  as we did in the massive case. In this case the states created by the creation operator  $\bar{Q}_2$  have zero norm

$$\langle\Omega|Q_2\bar{Q}_2|\Omega\rangle = 0 \quad . \quad (2.21)$$

so we have only one pair of creation and annihilation operators given by

$$a = \frac{1}{2\sqrt{E}}Q_1 \quad a^\dagger = \frac{1}{2\sqrt{E}}\bar{Q}_1. \quad (2.22)$$

$|\Omega\rangle$  is non-degenerate and has helicity  $\lambda$ ; since the creation operator  $a^\dagger$  transforms in the representation  $(0, \frac{1}{2})$  under the Lorentz group, it increases helicity by  $1/2$ , so massless  $N = 1$  SUSY irreducible representations contain two states of helicity  $\lambda$  and  $\lambda + 1/2$ . The representation we've obtained is not a CPT eigenstate in general. In order to restore CPT invariance, we require two massless SUSY irreducible representations with helicity such that we obtain four states with helicities  $\lambda, \lambda + \frac{1}{2}, -\lambda - \frac{1}{2}, -\lambda$  by the action of the creation operator.

## 2.3 Superspace and superfields formalism

The  $N = 1$  SUSY superalgebra can be rewritten in terms of commutators introducing the left and right Weyl spinors  $\theta$  and  $\bar{\theta}$ :

$$\{\theta^\alpha, \theta^\beta\} = \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = \{\theta^\alpha, \bar{\theta}_{\dot{\beta}}\} = 0. \quad (2.23)$$

and rewriting

$$[\theta Q, \bar{\theta}\bar{Q}] = 2\theta\sigma^\mu\bar{\theta}P_\mu \quad [\theta Q, \theta Q] = 0 \quad [\bar{\theta}\bar{Q}, \bar{\theta}\bar{Q}] = 0 \quad (2.24)$$

A generic element of the group whose generator satisfy this Lie algebra can be expressed in exponential form

$$G(x, \theta, \bar{\theta}, \omega) = e^{i[-x^\mu P_\mu + \theta Q + \bar{\theta}\bar{Q}]} e^{-\frac{i}{2}\omega^{\mu\nu} M_{\mu\nu}}. \quad (2.25)$$

From this form it is evident that the variables  $(x^\mu, \theta, \bar{\theta})$  parametrize an eight dimensional coset space known as  $N = 1$  rigid superspace where ‘‘rigid’’ refers to the fact that we are considering global supersymmetry transformations.

We define the following properties of the derivatives with respect to the Grassmannian super-

space coordinates

$$\begin{aligned}
\partial_\alpha &= \frac{\partial}{\partial \theta^\alpha} & \partial^\alpha &= \frac{\partial}{\partial \theta_\alpha} = -\epsilon^{\alpha\beta} \partial_\beta & \bar{\partial}^{\dot{\alpha}} &= \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} & \bar{\partial}_{\dot{\alpha}} &= \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} = -\epsilon_{\dot{\alpha}\dot{\beta}} \bar{\partial}^{\dot{\beta}} \\
\partial_\alpha \theta^\beta &= \delta_\alpha^\beta & \bar{\partial}^{\dot{\alpha}} \bar{\theta}_{\dot{\beta}} &= \delta_{\dot{\beta}}^{\dot{\alpha}} & \partial^\alpha \theta^\beta &= -\epsilon^{\alpha\beta} & \partial_\alpha \theta_\beta &= -\epsilon_{\alpha\beta} \\
\bar{\partial}^{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} &= -\epsilon^{\dot{\alpha}\dot{\beta}} & \bar{\partial}_{\dot{\alpha}} \bar{\theta}_{\dot{\beta}} &= -\epsilon_{\dot{\alpha}\dot{\beta}} \\
\partial_\alpha \theta^\beta \theta^\gamma &= \delta_\alpha^\beta \theta^\gamma - \delta_\alpha^\gamma \theta^\beta & \partial_\alpha \theta \theta &= 2\theta_\alpha & \bar{\partial}_{\dot{\alpha}} \bar{\theta} \bar{\theta} &= -2\bar{\theta}_{\dot{\alpha}} & \partial^2 \theta \theta &= 4 & \bar{\partial}^2 \bar{\theta} \bar{\theta} &= 4. \quad (2.26)
\end{aligned}$$

We can also define the integration with respect to the Grassmann variables, the Berezin integral. For a single Grassmann number  $\theta$  it is defined as

$$\int d\theta \theta = 1 \quad \int d\theta = 0 \quad \int d\theta f(\theta) = f_1 \quad (2.27)$$

using the Taylor series expansion  $f(\theta) = f_0 + \theta f_1$ . This expansion is finite since, being  $\theta$  a Grassmann number,  $\theta\theta = 0$ .

From these definitions we can easily conclude that the Berezin integration is translationally invariant, is equivalent to differentiation and a Grassmann delta function can be defined as  $\delta(\theta) = \theta$ .

We can easily generalize this definitions of the Berezin integral to the case of the superspace coordinates using the conventions

$$d^2\theta = -\frac{1}{4} d\theta^\alpha \theta^\beta \epsilon_{\alpha\beta} \quad d^2\bar{\theta} = -\frac{1}{4} d\bar{\theta}_{\dot{\alpha}} \bar{\theta}_{\dot{\beta}} \epsilon^{\dot{\alpha}\dot{\beta}} \quad d^4\theta = d^2\theta d^2\bar{\theta} \quad (2.28)$$

chosen in such a way that

$$\int d^2\theta (\theta\theta) = 1 \quad \int d^2\bar{\theta} (\bar{\theta}\bar{\theta}) = 1. \quad (2.29)$$

We can define the SUSY covariant derivatives for  $N = 1$  rigid superspace as

$$D_\alpha = \partial_\alpha + i\sigma_{\alpha\dot{\beta}}^\mu \bar{\theta}^{\dot{\beta}} \partial_\mu \quad \bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i\theta^\beta \sigma_{\beta\dot{\alpha}}^\mu \partial_\mu$$

where the components of  $\sigma^\mu$  are the  $2 \times 2$  identity matrix,  $\sigma^0$ , and the Pauli matrices  $\sigma^i$ ,  $i = 1, 2, 3$ .

A scalar superfield  $\Phi(x, \theta, \bar{\theta})$  is a scalar function defined on the  $N = 1$  rigid superspace. Given the properties of Grassmann variables we can expand it in a finite Taylor series

$$\begin{aligned}
\Phi(x, \theta, \bar{\theta}) &= f(x) + \theta\phi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta} n(x) + \theta\sigma^\mu\bar{\theta} v_\mu(x) + \theta\theta\bar{\theta}\bar{\lambda}(x) + \\
&\quad \bar{\theta}\bar{\theta}\theta\psi(x) + \theta\theta\bar{\theta}\bar{\theta} d(x). \quad (2.30)
\end{aligned}$$

The fields appearing in this expansion are called “component fields” and are, in general, complex functions of the space-time coordinates.

If we consider an infinitesimal supersymmetry transformation of the superfield we obtain the infinitesimal transformations for the component fields. The result is that the infinitesimal transformation for a given component field can be expressed in terms of the other component fields so the general scalar superfield forms a basis for a linear representation of  $N = 1$  SUSY. This representation, though, is reducible since we can impose some constraints among the component fields obtaining that they still satisfy the same transformation rules.

Given a generic scalar superfield, we can obtain other superfields acting with the SUSY covariant or space-time derivatives since these derivatives commute with the supersymmetry generators.

## 2.4 Constrained superfields

An  $N = 1$  chiral superfield is obtained imposing the following covariant constraint on the generic superfield  $\Phi$

$$\bar{D}_{\dot{\alpha}}\Phi = 0. \quad (2.31)$$

The most general solution to this constraint is given by

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) = & A(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) + i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}A(x) + \frac{i}{\sqrt{2}}\theta\theta\partial_{\mu}\psi(x)\sigma^{\mu}\bar{\theta} - \\ & \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\square A(x). \end{aligned} \quad (2.32)$$

This shows that chiral superfields only contain left-handed Weyl spinors.

An infinitesimal  $N = 1$  SUSY transformation on the chiral superfield gives

$$\delta A = \sqrt{2}\xi\psi \quad \delta\psi = \sqrt{2}\xi F + \sqrt{2}i\sigma^{\mu}\bar{\xi}\partial_{\mu}A \quad \delta F = -\sqrt{2}i\partial_{\mu}\psi\sigma^{\mu}\bar{\xi}. \quad (2.33)$$

Right-handed chiral superfields are defined by the constraint

$$D_{\alpha}\Phi^{\dagger} = 0 \quad (2.34)$$

and their fermionic component field is a right handed Weyl fermion.

Vector superfields are defined imposing a covariant reality constraint on the general scalar superfield

$$V(x, \theta, \bar{\theta}) = V(x, \theta, \bar{\theta})^{\dagger} \quad (2.35)$$

In components we have 4 real scalars, 2 complex Weyl spinors and 1 real vector and in terms of components of the generic superfield we have the constraints

$$f = f^* \quad \bar{\chi} = \phi^* \quad m = n^* \quad v_\mu = v_\mu^* \quad \bar{\lambda} = \psi^* \quad d = d^* \quad (2.36)$$

A vector superfield can be built starting from a chiral superfield  $\Phi(x, \theta, \bar{\theta})$  in the following ways

$$\Phi(x, \theta, \bar{\theta}) + \Phi(x, \theta, \bar{\theta})^\dagger \quad \text{or} \quad \Phi(x, \theta, \bar{\theta})^\dagger \Phi(x, \theta, \bar{\theta}). \quad (2.37)$$

If we take the first combination in component fields we have

$$\begin{aligned} \Phi + \Phi^\dagger = & (A + A^*) + \sqrt{2}\theta\psi + \sqrt{2}\bar{\theta}\bar{\psi} + \theta\theta F + \bar{\theta}\bar{\theta}F^* + i\theta\sigma^\mu\bar{\theta}\partial_\mu(A - A^*) + \\ & \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi + \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^\mu\partial_\mu\bar{\psi} - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\square(A + A^*). \end{aligned} \quad (2.38)$$

so we can define the gauge transformation for a vector field as

$$V \rightarrow V + \Phi + \Phi^\dagger. \quad (2.39)$$

This gives the expected transformation rules for the vector component

$$v_\mu \rightarrow v_\mu + \partial_\mu\Lambda \quad \Lambda = i(A - A^*). \quad (2.40)$$

Any action built in terms of superfields and invariant under the gauge transformation just defined will be independent from some of the components of the vector superfields. This means that we can decompose a vector superfield in the following way

$$V(x, \theta, \bar{\theta}) = V_{WZ}(x, \theta, \bar{\theta}) + \Phi(x, \theta, \bar{\theta}) + \Phi(x, \theta, \bar{\theta})^\dagger \quad (2.41)$$

in which the superfield  $V_{WZ}$  contains only four of the original nine components

$$V_{WZ}(x, \theta, \bar{\theta}) = \theta\sigma^\mu\bar{\theta}v_\mu + \theta\theta\bar{\theta}\bar{\lambda} + \bar{\theta}\bar{\theta}\theta\lambda + \theta\theta\bar{\theta}\bar{\theta}d. \quad (2.42)$$

The superfield  $V_{WZ}$  is called Wess-Zumino gauge fixed vector superfield. This gauge-fixed form leaves untouched the usual abelian gauge freedom for the vector component.

It is clear that supersymmetry “breaks” the Wess-Zumino decomposition but, starting from vector superfields, we can construct superfields containing only the component fields appearing

in  $V_{WZ}$ . We do this by defining the left and right handed spinor superfields

$$\begin{aligned} W_\alpha &= -\frac{1}{4}\bar{D}\bar{D}D_\alpha V(x, \theta, \bar{\theta}), \\ \bar{W}_{\dot{\alpha}} &= -\frac{1}{4}DD\bar{D}_{\dot{\alpha}}V(x, \theta, \bar{\theta}). \end{aligned} \quad (2.43)$$

or, equivalently

$$\begin{aligned} W_\alpha &= -\frac{1}{8}\bar{D}\bar{D}e^{-2V}D_\alpha e^{2V}, \\ \bar{W}_{\dot{\alpha}} &= \frac{1}{8}DDe^{2V}\bar{D}_{\dot{\alpha}}e^{-2V}. \end{aligned} \quad (2.44)$$

The importance of the second definition will be more clear when we will define non-abelian gauge fields.

The superfields we have just defined are chiral and anti-chiral superfields since they satisfy the chiral/anti-chiral superfield constraints. They also satisfy a new constraint

$$\bar{D}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}} = D^\alpha W_\alpha \quad (2.45)$$

and they are invariant under the supergauge transformation defined in Eq. (2.39). This means that, without loss of generality, we can use the Wess-Zumino gauge-fixed vector superfields in their definition. We get

$$\begin{aligned} W_\alpha &= \lambda_\alpha(y) + 2\theta_\alpha d(y) + \frac{i}{2}(\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha (\partial_\mu v_\nu - \partial_\nu v_\mu)(y) - i(\theta\theta)\sigma_{\alpha\dot{\beta}}^\mu \partial_\mu \bar{\lambda}^{\dot{\beta}}(y), \\ \bar{W}_{\dot{\alpha}} &= \bar{\lambda}_{\dot{\alpha}}(y^\dagger) + 2\bar{\theta}_{\dot{\alpha}} d(y^\dagger) - \frac{i}{2}(\bar{\sigma}^\mu \sigma^\nu \bar{\theta})_{\dot{\alpha}} (\partial_\mu v_\nu - \partial_\nu v_\mu)(y^\dagger) - (\bar{\theta}\bar{\theta})\bar{\sigma}_{\dot{\alpha}}^{\mu\beta} \partial_\mu \lambda_\beta(y^\dagger) \end{aligned} \quad (2.46)$$

where  $y$  is defined by the change of coordinates

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}. \quad (2.47)$$

These superfields represent an irreducible SUSY multiplet called “field strength multiplet” since it contains the field strength of the vector component of the superfield.

We can obtain a finite supergauge transformation by exponentiation

$$e^V \rightarrow e^{\Phi^\dagger} e^V e^\Phi. \quad (2.48)$$

The generalization to the abelian case can be obtained defining

$$V = T^a V_a \quad \Phi = T^a \Phi_a \quad (2.49)$$

where the  $T^a$ 's represent the hermitian generators of the non-abelian gauge group satisfying the Lie algebra and normalization condition

$$[T^a, T^b] = if^{abc} T^c \quad \text{tr}(T^a T^b) = \delta^{ab}. \quad (2.50)$$

It can be shown that once again we can obtain a Wess-Zumino gauge fixed form for the vector superfield that leaves intact the gauge freedom. Field strength superfields can be defined as showed in Eq. (2.44). These superfields transform covariantly under a gauge transformation

$$W_\alpha \rightarrow e^{-2\Phi} W_\alpha e^{2\Phi} \quad \bar{W}_{\dot{\alpha}} \rightarrow e^{2\Phi^\dagger} \bar{W}_{\dot{\alpha}} e^{-2\Phi^\dagger}. \quad (2.51)$$

In component fields the field strength superfield  $W_\alpha$  is given by

$$W_\alpha = \lambda_\alpha(y) + 2\theta_\alpha d(y) + \sigma_\alpha^{\mu\nu\beta} \theta_\beta F_{\mu\nu}(y) - i\theta\theta \sigma_{\alpha\dot{\beta}}^\mu \nabla_\mu \bar{\lambda}^{\dot{\beta}}(y), \quad (2.52)$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu v_\nu - \partial_\nu v_\mu + i[v_\mu, v_\nu] \\ \nabla_\mu \bar{\lambda}^{\dot{\beta}} &= \partial_\mu \bar{\lambda}^{\dot{\beta}} + i[v_\mu, \bar{\lambda}^{\dot{\beta}}] \end{aligned} \quad (2.53)$$

and we recover the usual Yang-Mills field strength and gauge covariant derivative.

## Dark Matter relics

Many observational evidences suggest that the energy density of the Universe is very close to the critical density, i.e. the energy density that gives a flat Universe. Furthermore, the main contribution to this energy density should come from a form of energy, dubbed Dark Energy, that should have negative pressure and whose origin is still unknown; it should account for the  $\approx 70\%$  of the observed energy density. The remaining 30% should be given by matter but, according to the limits coming from Big Bang Nucleosynthesis, most of this,  $\approx 25\%$  of the total energy density, should be non-baryonic. It should also be “dark” in the sense that it shouldn’t interact with photons since this would make it easy to detect. So, the most important effects for this type of matter should be the gravitational ones. Indeed, gravitational effects, seen in the rotation curves of galaxies and in gravitational lensing observations, seem to confirm the existence of Dark Matter. An appropriate Cold Dark Matter density is also needed to explain the formation of structures in the Early Universe since Dark Matter does not couple to photons and so can easily make initially over-dense regions more and more dense by gravitational attraction. In order for this to happen it has also to be non-relativistic and this is what the adjective “cold” refers to.

In what follows we will review the possible processes that lead to the formation of a Dark Matter population. We start with some basic facts about the description of an expanding isotropic and homogeneous Universe.

On cosmological scales the content of the Universe can be described as a perfect fluid whose properties are defined by the energy-momentum tensor

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu} , \quad (3.1)$$

where  $U^\mu$  is the four-velocity of the fluid and  $\rho$  and  $p$  are, respectively, the energy density and the pressure of the fluid in its rest frame. This energy-momentum tensor describes a homogeneous and isotropic Universe. We can also define an entropy density in the rest frame,  $s$ , in terms of the

pressure, the energy density and the temperature of the fluid

$$s = \frac{p + \rho}{T}. \quad (3.2)$$

The energy-momentum tensor satisfies the covariant conservation law

$$\nabla_{\mu} T^{\mu\nu} = 0 \quad (3.3)$$

and in the case of a perfect fluid it gives

$$\dot{\rho} + 3H(\rho + p) = 0; \quad (3.4)$$

it can be seen as the analogous of the first law of thermodynamics

$$dU = T dS - p dV \quad (3.5)$$

in the case of an adiabatic transformation ( $dS = 0$ ).

The metric associated to such a space-time is the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a(t) (dx^2 + dy^2 + dz^2) \quad (3.6)$$

where  $a(t)$  is a function of time describing the expansion of the Universe and is called “scale factor”.

### 3.1 Thermal relics

When we consider the behavior of the cosmological plasma during the expansion of the Universe, it is important to define if a component of the fluid is in equilibrium with the rest of the plasma or not. We say that a component of the cosmological plasma is in thermal equilibrium with the rest of the plasma if the interaction rate for the particles that constitute that component is bigger than the expansion rate, that is

$$\Gamma \gg H \quad (3.7)$$

where  $\Gamma$  represents the interaction rate and  $H$  is the Hubble constant. The Hubble constant is defined in terms of the factor  $a(t)$  as  $H = \dot{a}/a$ . The condition in (3.7) is satisfied or not, depending on the strength of the interactions for the given component, on the number density of the species taking part in that process and their masses and on the temperature of the cosmological plasma. When the equilibrium condition does not apply for a given species, we say that that species has

“decoupled” from the rest of the plasma or “frozen out”.

In the radiation dominated phase the expansion is typically slow with respect to the interaction rate so all of the species in the plasma are in equilibrium but this regime has to break eventually. In fact the interaction rate  $\Gamma$  is defined by

$$\Gamma = n\langle\sigma v\rangle \quad (3.8)$$

where  $n$  is the number density of a certain species and  $\langle\sigma v\rangle$  represents a thermally averaged cross section. So, since the universe is expanding, the interaction rate will become smaller and smaller until all of the species will be out of equilibrium respect to a plasma made of the remaining massless particles, namely, the photons of the Cosmic Microwave Background.

As far as a species is in equilibrium, we can describe it using a distribution function that gives the number density of particles with a momentum around a certain value. If, for a given species, we consider the distribution function  $f = f(\vec{p})$  then we can define the number density, the energy density and the pressure as

$$\begin{aligned} n &= \frac{g}{(2\pi)^3} \int f(\vec{p})d^3p \\ \rho &= \frac{g}{(2\pi)^3} \int E(\vec{p})f(\vec{p})d^3p \\ p &= g(2\pi)^3 \int \frac{|\vec{p}|^2}{3E(\vec{p})}f(\vec{p})d^3p. \end{aligned} \quad (3.9)$$

where  $g$  represents the number of spin states of the corresponding particles. If we are in thermal equilibrium at a temperature  $T$  we have only two possible forms for the distribution function, the Fermi-Dirac or the Bose-Einstein distribution

$$f(\vec{p}) = \frac{1}{e^{E(\vec{p})/T} \pm 1} \quad (3.10)$$

where  $E(\vec{p}) = m^2 + |\vec{p}|^2$ .

It is interesting to see the results that we get for the number density, energy density and pressure for bosons and fermions in the ultra-relativistic and non-relativistic limits. These are obtained in the limits  $T \gg m$  and  $T \ll m$  respectively, with  $m$  the mass of the particles.

In the relativistic case we get

$$n = \begin{cases} \frac{\zeta(3)}{\pi^2} g T^3 & \text{bosons} \\ \left(\frac{3}{4}\right) \frac{\zeta(3)}{\pi^2} g T^3 & \text{fermions} \end{cases} \quad \rho = \begin{cases} \frac{\pi^2}{30} g T^4 & \text{bosons} \\ \left(\frac{7}{8}\right) \frac{\pi^2}{30} g T^4 & \text{fermions} \end{cases} \quad p = \frac{1}{3}\rho. \quad (3.11)$$

In the non-relativistic case the situation is quite different, we have no difference between bosons and fermions and we get

$$n = g \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T} \quad \rho = nm \quad p = nT. \quad (3.12)$$

We see that the number density for a given species is exponentially suppressed when the species becomes non-relativistic, namely when the temperature becomes less than the mass of the particle. This can be interpreted as the result of the fact that in a plasma that is cooling down it's more and more difficult to produce a particle with a mass higher than the temperature of the plasma. Moreover, we see that for a non-relativistic species, pressure is negligible while it's not so for a relativistic species.

We also note that the energy density for the relativistic species is proportional to  $T^4$  while for a non-relativistic species it is proportional to  $T^3$  so we expect to have a “radiation dominated” era at high temperatures and a “matter dominated” era at low temperatures.

If a massive species stays in equilibrium with the cosmological plasma it will be completely depleted while the temperature goes down and it will contribute its energy and entropy densities to the cosmological plasma. This is what happens for species that interact efficiently with the plasma. We can roughly identify such particles with electrically charged and strongly interacting particles.

If a species has only non-efficient interactions, like the weak interactions are, then it will decouple from the rest of the plasma and we will be left with a relic population of particles of this species. This decoupling can happen when the species is relativistic or when it is non-relativistic, depending on its mass, and the produced dark matter is called “hot” or “cold” respectively. The adjective “dark” is referred to the fact that the particle that constitute this kind of matter don't interact with photons.

### 3.1.1 Hot dark matter

An example of relativistic decoupling is the decoupling of neutrinos. It takes place around a temperature of 1 MeV and gives a Hot Dark Matter population. After the decoupling the neutrino temperature will scale as  $1/a$ , where  $a$  is the scale factor of the FRW metric, and will remain equal to the photon background temperature, unless the photon background, being still coupled to other species, undergoes some kind of process that modifies its temperature. Actually, this is what happens since, shortly after the neutrino decoupling, the temperature drops under the mass of the electrons so the positrons and the electrons annihilate transferring their energy to the photon population. So, if neutrinos were massless, we would expect a neutrino background colder than the photon background. This picture is made more complex considering the fact that the CMB temperature today is  $3K \simeq 10^{-4}$  eV and this temperature is well below the expected neutrino

masses so relic neutrinos should be non-relativistic today. The estimated contribution of relic neutrinos to the density parameter can account only for a negligible fraction of the observed dark matter abundance. This is expected also considering the fact that the neutrinos are relativistic for most part of the expansion of the universe and larger values of the neutrino number density would contribute to excessively smooth out the cosmological perturbations that are fundamental in the structure formation process.

### 3.1.2 Cold dark matter

In this section we will follow [49, 50].

The case of a non-relativistic decoupling is more subtle. In fact, from Eq. (3.12) we see that for a non-relativistic species the number density changes rapidly with the temperature so it's important to determine precisely the decoupling temperature. This is usually done integrating numerically the Boltzmann equations for the species that eventually decouples and the species that interact with it. If we consider a process like  $P_1, P_2 \leftrightarrow P_3, P_4$  where  $P_i$  represents a generic species, then the evolution of the number density of one of the particles involved in the process, say  $n_1$ , is defined by the Boltzmann equation

$$\frac{1}{a^3} \frac{d(n_1 a^3)}{dt} = \prod_{i=1}^4 \int \frac{d^3 p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) |\mathcal{M}|^2 e^{-\frac{E_1 + E_2}{T}} \left( \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right) \quad (3.13)$$

where  $a$  represents the cosmological scale factor,  $p_i$  represent the momentum of the particle  $P_i$ ,  $E_i$  is its energy,  $\mathcal{M}$  is the amplitude of the process (assumed to be time reversal invariant) and  $n_i^{(0)}$  represents the equilibrium density for the  $i$  species (see Eqs. (3.11), (3.12)). Defining the thermally averaged cross section as

$$\langle \sigma v \rangle = \frac{1}{n_1^0 n_2^0} \prod_{i=1}^4 \int \frac{d^3 p_i}{(2\pi)^3 2E_i} e^{-\frac{E_1 + E_2}{T}} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) |\mathcal{M}|^2 \quad (3.14)$$

we can rewrite the Boltzmann equation as

$$\frac{1}{a^3} \frac{d(n_1 a^3)}{dt} = n_1^0 n_2^0 \langle \sigma v \rangle \left( \frac{n_3 n_4}{n_3^0 n_4^0} - \frac{n_1 n_2}{n_1^0 n_2^0} \right). \quad (3.15)$$

If the interaction rate  $n_2^{(0)} \langle \sigma v \rangle$  is such that the comoving number density  $n_1 a^3$  is conserved, then we get the following equilibrium condition on the number densities

$$\frac{n_3 n_4}{n_3^0 n_4^0} = \frac{n_1 n_2}{n_1^0 n_2^0}. \quad (3.16)$$

If we consider the case in which  $P_1 = P_2$  and  $P_1$  is non relativistic while  $P_3$  and  $P_4$  are relativistic we have  $n_1 = n_2$  and  $n_{3,4} = n_{3,4}^0$  and the Boltzmann equation gets the form

$$\frac{1}{a^3} \frac{dn_1 a^3}{dt} = \langle \sigma v \rangle [(n_1^0)^2 - (n_1)^2]. \quad (3.17)$$

We define the variables

$$Y = \frac{n_1}{s} \quad x = \frac{m_1}{T} \quad (3.18)$$

where  $m_1$  is the mass of the particle  $P_1$  and  $s$  is the entropy density, given by

$$s = \sqrt{\frac{2}{45}} \pi g_* T^3 \quad (3.19)$$

with  $g_*$  the number of the effectively massless degrees of freedom. We can express the equilibrium values  $Y_0$  and  $n_1^0$  in terms of the new variables as

$$n_1^0 = e^{-x} \frac{m_1^3}{\sqrt{(2\pi x)^3}} \quad Y_0 = \frac{45}{2\pi^2 g_*} e^{-x} \sqrt{\left(\frac{x}{2\pi}\right)^3}. \quad (3.20)$$

We can also rewrite the Boltzmann equation in terms of the new variables

$$\frac{dY}{dx} = -\frac{1}{x^2} \frac{s(m_1)}{H(m_1)} \langle \sigma v \rangle (Y^2 - Y_0^2) \quad (3.21)$$

where  $s(m_1) = x^3 s(T)$ ,  $H(m_1) = x^2 H(T)$  and

$$H(T) = \frac{\pi}{3} \sqrt{\frac{g_*}{10}} \frac{T^2}{M_{Pl}}. \quad (3.22)$$

We notice that the ratio  $s(m_1)/H(m_1)$  does not depend on the temperature (or, equivalently, on  $x$ ) since

$$\frac{s(m_1)}{H(m_1)} = \frac{6\sqrt{10g_*}}{45} m_1 M_{Pl}. \quad (3.23)$$

We also expect that the thermally averaged cross section does not depend on the temperature when  $T < m_1$  so we define the new variable

$$y = \frac{s(m_1)}{H(m_1)} \langle \sigma v \rangle Y \quad (3.24)$$

in terms of which the Boltzmann equation becomes

$$\frac{dy}{dx} = -\frac{1}{x^2} (y^2 - y_0^2). \quad (3.25)$$

The solution of this equation is usually determined numerically. Nevertheless, we can find an analytic approximate solution defining  $x_f$  as  $x_f = m_1/T_{f.o.}$  where  $T_{f.o.}$  represents the freeze out temperature. Then we assume that  $Y \simeq Y_0$  for  $x < x_f$  and  $Y \gg Y_0$  for  $x > x_f$  since we know that  $Y_0$  decreases exponentially for increasing  $x$ . So we can easily integrate the differential equation and we get

$$\frac{1}{y(\infty)} - \frac{1}{y(x_f)} \simeq \frac{1}{y(\infty)} = \frac{1}{x_f} \Rightarrow y(\infty) = x_f. \quad (3.26)$$

We can obtain an estimate for  $x_f$  (and for the freeze out temperature) defining it as the value of  $x$  for which  $y_0(x)$  equals  $x_f$ . Using the definition of  $y_0$  and typical values for the mass of the particle, for the thermally averaged cross section and for the number of effectively massless degrees of freedom we get

$$x_f \approx 24 + \ln\left(\frac{m_1}{100 \text{ GeV}}\right) + \ln\left(\frac{\langle\sigma v\rangle}{10^{-9} \text{ GeV}^{-2}}\right) - \frac{1}{2} \ln\left(\frac{g_*}{100}\right). \quad (3.27)$$

The final expression for the cold dark matter relic density can be expressed as

$$\rho_1 = m_1 n_1(t_0) = m_1 Y(\infty) s = \frac{m_1 s x_f H(m_1)}{\langle\sigma v\rangle s(m_1)} \quad (3.28)$$

where we have used Eqs. (3.18), (3.24) and (3.26).

In the SM we don't have any particle that can decouple when it is non relativistic but in many extensions of the SM we have such particles, which are usually defined WIMPs (Weakly Interacting Massive Particles). If a WIMP is also stable, it would give rise to a Cold Dark Matter (CDM) population since it would be non-relativistic (thus cold) since the time it decouples from the cosmological plasma. For example, this is the case of the Lightest Supersymmetric Particle (LSP) in a supersymmetric extension of the SM with conserved R-parity.

## 3.2 Non-thermal relics: the case of vacuum misalignment

We can obtain a dark matter population from a mechanism that is essentially different from the thermal production considered so far. Suppose we have a scalar field  $\phi$  subject to a potential

$V(\phi)$ ; the corresponding action will be given by

$$S = \int d^4x \sqrt{g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) \quad (3.29)$$

where  $g$  represents the determinant of the metric tensor  $g_{\mu\nu}$ . If this field is spatially homogeneous we will have, in a FRW background, the equation of motion

$$\partial_{tt} \phi + 3H \partial_t \phi + \frac{\partial V(\phi)}{\partial \phi} = 0 \quad (3.30)$$

where  $H = \dot{a}/a$  is the Hubble constant. The requirement of homogeneity is easily satisfied assuming that we are at the end of an inflationary phase such that the field is homogeneous on scales smaller than the horizon.

Following[51], if the potential consists only of a mass term, that is

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \quad (3.31)$$

then the equation describes an harmonic oscillator with a damping term parameterized by the Hubble constant. In the case  $3H > m$  the oscillator is overdamped so the field will assume a constant value. In this case we will have an energy density given by  $m^2 \phi_0^2/2$  where  $\phi_0$  is the value of the field. When the overdamping condition breaks, because the Hubble constant becomes smaller and/or the mass of the field becomes bigger, the field will start to oscillate around the minimum of the potential. An approximate solution for this oscillation is given by

$$\langle \phi \rangle = A(t) \cos(mt) \quad \frac{d}{dt}(mA^2) = -3H(mA^2). \quad (3.32)$$

Integrating the second equation, assuming that the mass of the field is constant or slowly varying respect to the period of the oscillations, we get

$$\left( \frac{A(t)}{A(t_i)} \right)^2 = \left( \frac{a(t)}{a(t_i)} \right)^{-3}, \quad (3.33)$$

where  $a(t_i)$  represents the FRW scale factor at the beginning of the oscillatory motion. The mean of the field value on an oscillation will be proportional to the amplitude of the oscillation (assuming that this amplitude does not vary much in an oscillation period), so the energy density will be proportional to the square of the amplitude of the oscillation. This means that the energy density associated to this oscillating field scales as  $a(t)^{-3}$ , that is like a non-relativistic fluid (see Sec. 3.1). Even if the mass of the field is such that the corresponding particle should be relativistic at the temperature at which the oscillations begin, the production mechanism is such that we have a zero momentum Bose condensate that behaves like non-relativistic matter and is

“colder” than the cosmological plasma. A necessary condition for this to happen is that the field is homogeneous, in such a way that we can have coherent oscillations of the field around the minimum of the potential. As we have already said, this condition can be ensured by invoking an inflationary phase. Such a phase is introduced to solve several issues as, for example, the isotropy of the Cosmic Microwave Background (CMB) and the generation of the primordial perturbations that lead to the formation of structures and to the anisotropies in the CMB.

The mechanism of generation of dark-matter like energy density by coherent oscillations of an homogeneous field around a minimum of the potential is referred to as “vacuum misalignment” or “vacuum realignment” since it doesn’t apply if the field is initially frozen on the vacuum. It has been introduced for the first time in the case of the Peccei-Quinn (PQ) axion[52, 51, 53]. In that case the axion field is massless and remains so until non perturbative QCD effect, which turn on around the QCD phase transition scale  $\Lambda_{QCD} \approx 200$  MeV, raise the mass of the axion to a very small value that allows the field to oscillate around the minimum. The mass for a PQ axion is roughly given by  $\Lambda_{QCD}^2/f_a$  with  $f_a$  the axion decay constant usually taken as  $10^{10} \lesssim f_a \lesssim 10^{12}$  GeV.

## Dark matter in the MLSOM

The Minimal Low Scale Orientifold Model (MLSOM) [47] is an extension of the Standard Model. It is a two Higgs doublet model where the gauge group contains an extra U(1) gauge factor and the associated gauge symmetry is supposed to be anomalous. The restoration of the gauge invariance is achieved through Wess-Zumino interaction terms involving a Stückelberg field (also called “gauged axion”). The Stückelberg field takes also part in the generation of a mass term for the extra gauge boson associated with the anomalous symmetry. This gauge boson, above the electroweak symmetry breaking scale, receives a mass through the Stückelberg mechanism and the gauged axion in this case is simply the Goldstone boson “eaten” by the gauge boson in order to become massive.

At the electroweak symmetry breaking scale the situation is modified since the mass of the  $Z'$  receives a correction from the Higgs mechanism and the Goldstone boson becomes a linear combination of the the gauged axion and some components of the Higgs fields of the model. As we will see, this leaves space for a physical state that has an axion component.

This chapter is based on [54].

### 4.1 Definitions and conventions

The MLSOM is defined by an effective action whose structure is given by

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_{an} + \mathcal{S}_{WZ} + \mathcal{S}_{CS} \quad (4.1)$$

where  $\mathcal{S}_0$  describes the usual gauge degrees of freedom of the Standard Model with the following modifications

- we have 2 Higgs doublets,  $H_u$  and  $H_d$  that couple through Yukawa couplings to the up-type and down-type fermions<sup>1</sup> respectively,

<sup>1</sup>With down-type fermions we refer to the down quarks and to the massive leptons

- we have an extra abelian gauge symmetry, broken through the Stückelberg mechanism at the scale  $M$ , and the corresponding massive gauge boson. The Stückelberg lagrangian is given by

$$\mathcal{L} = \frac{1}{2} \left( \partial_\mu b - M B_\mu \right)^2 \quad (4.2)$$

where  $M$  is a mass parameter that we will refer to as to the Stückelberg mass. In what follows we will refer to the field  $b$  as to the axion; we will see, in fact, that it shifts under a gauge transformation and this behavior recalls that of axionic fields, such as the Peccei-Quinn (PQ) axion[18].

The charge assignment under the extra abelian gauge group is generic, so this symmetry can be anomalous.

A complete expression for the lagrangian  $\mathcal{L}_0$  is given in[44]. Here we briefly describe the structure of the anomalous contributions and of the induced counterterms for the restoration of gauge invariance in the 1-loop effective action.

In Eq. (4.1) the anomalous contributions coming from the 1-loop triangle diagrams involving abelian and non-abelian gauge interactions are represented by the term  $\mathcal{S}_{an}$  and are summarized by the expression

$$\mathcal{S}_{an} = \frac{1}{2} \langle T_{BWW} BWW \rangle + \frac{1}{2} \langle T_{BGG} BGG \rangle + \frac{1}{3!} \langle T_{BBB} BBB \rangle + \frac{1}{2} \langle T_{BYY} BYY \rangle + \frac{1}{2} \langle T_{YBB} YBB \rangle, \quad (4.3)$$

where the symbols  $\langle \rangle$  denote integration and the letters  $G, W, Y, B$  refer to the  $SU(3)$ ,  $SU(2)$ ,  $U(1)_Y$  and  $U(1)_B$  gauge bosons respectively. For instance, the contributions in configuration space for the first term are given explicitly by

$$\langle T_{BWW} BWW \rangle \equiv \int dx dy dz T_{BWW}^{\lambda\mu\nu,ij}(z, x, y) B^\lambda(z) W_i^\mu(x) W_j^\nu(y) \quad (4.4)$$

where  $T_{BWW}$  denotes the anomalous triangle diagram with one  $B$  and two  $W$  external gauge lines. The Wess-Zumino (WZ) counterterms are given by

$$\begin{aligned} \mathcal{S}_{WZ} = & \frac{C_{BB}}{M} \langle b F_B \wedge F_B \rangle + \frac{C_{YY}}{M} \langle b F_Y \wedge F_Y \rangle + \frac{C_{YB}}{M} \langle b F_Y \wedge F_B \rangle + \\ & \frac{F}{M} \langle b \text{Tr}[F_W \wedge F_W] \rangle + \frac{D}{M} \langle b \text{Tr}[F_G \wedge F_G] \rangle, \end{aligned} \quad (4.5)$$

while the gauge dependent Chern-Simons (CS) abelian and non abelian counterterms [55] take the form

$$\mathcal{S}_{CS} = d_1 \langle BY \wedge F_Y \rangle + d_2 \langle YB \wedge F_B \rangle$$

$$+ c_1 \langle \epsilon^{\mu\nu\rho\sigma} B_\mu C_{\nu\rho\sigma}^{SU(2)} \rangle + c_2 \langle \epsilon^{\mu\nu\rho\sigma} B_\mu C_{\nu\rho\sigma}^{SU(3)} \rangle. \quad (4.6)$$

The non-abelian CS forms are given by

$$C_{\mu\nu\rho}^{SU(2)} = \frac{1}{6} \left[ W_\mu^i \left( F_{i,\nu\rho}^W + \frac{1}{3} g_2 \epsilon^{ijk} W_\nu^j W_\rho^k \right) + c.p. \right], \quad (4.7)$$

$$C_{\mu\nu\rho}^{SU(3)} = \frac{1}{6} \left[ G_\mu^a \left( F_{a,\nu\rho}^G + \frac{1}{3} g_3 f^{abc} G_\nu^b G_\rho^c \right) + c.p. \right], \quad (4.8)$$

where  $c.p.$  denotes cyclic permutations.

#### 4.1.1 Gauge bosons - fermions interactions

The gauge covariant derivatives are defined as

$$D_\mu = \partial_\mu + i g_s T^a G_\mu^a + i g_2 \tau^i W_\mu^i + \frac{i}{2} g_Y q^Y Y_\mu + \frac{i}{2} g_B q^B B_\mu, \quad (4.9)$$

with  $T^a$  and  $\tau^i$  given by

$$T^a = \frac{\lambda^a}{2} \quad \tau^i = \frac{\sigma^i}{2}, \quad (4.10)$$

where  $\lambda^a$  and  $\sigma^i$  are the Gell-Mann and Pauli matrices with  $a = 1, 2, \dots, 8$  and  $i = 1, 2, 3$ . This choice of the covariant derivative defines the gauge variations of the fields; in particular, under the extra abelian group transformations we have

$$B'_\mu = B_\mu + \partial_\mu \theta \quad b' = b + M\theta \quad \phi' = e^{-i \frac{1}{2} g_B q_\phi^B \theta} \phi. \quad (4.11)$$

Note that the gauge transformation for the axion  $b$  and for the gauge field  $B_\mu$  are defined in such a way that the Stückelberg lagrangian, defined in Eq. (4.2), is left invariant.

We define the lepton doublet as

$$L_i = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix} \quad (4.12)$$

where  $i = 1, 2, 3$  denotes the family index. The interaction Lagrangian for the leptons is given by

$$\begin{aligned} \mathcal{L}_{int}^{lep} = & \left( \bar{\nu}_{Li} \quad \bar{e}_{Li} \right) \gamma^\mu \left[ -g_2 \tau^a W_\mu^a + \frac{1}{4} g_Y Y_\mu - \frac{1}{2} g_B q_L^B B_\mu \right] \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix} + \\ & \bar{e}_{Ri} \gamma^\mu \left[ \frac{1}{2} g_Y Y_\mu - \frac{1}{2} g_B q_{e_R}^B B_\mu \right] e_{Ri} \end{aligned} \quad (4.13)$$

where  $e_{Ri}$  represents the right-handed lepton components.

Writing the quark doublet as

$$Q_i = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix} \quad (4.14)$$

where  $i = 1, 2, 3$  is a generation index, we obtain the interaction Lagrangian for the quarks

$$\begin{aligned} \mathcal{L}_{int}^{quarks} = & (\bar{u}_{Li} \quad \bar{d}_{Li}) \gamma^\mu \left[ -g_s T^a G_\mu^a - g_2 \tau^a W_\mu^a - \frac{1}{12} g_Y Y_\mu - \frac{1}{2} g_B q_Q^B B_\mu \right] \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix} + \\ & + \bar{u}_{Ri} \gamma^\mu \left[ -g_s T^a G_\mu^a - g_2 \tau^a W_\mu^a - \frac{1}{3} g_Y Y_\mu - \frac{1}{2} g_B q_{u_R}^B B_\mu \right] u_{Ri} \\ & + \bar{d}_{Ri} \gamma^\mu \left[ -g_s T^a G_\mu^a - g_2 \tau^a W_\mu^a + \frac{1}{6} g_Y Y_\mu - \frac{1}{2} g_B q_{d_R}^B B_\mu \right] d_{Ri} \end{aligned} \quad (4.15)$$

where the color indices are understood and  $u_{Ri}$  and  $d_{Ri}$  are the right-handed up and down quarks fields.

#### 4.1.2 The Yukawa couplings and the axi-higgs

We work with a 2-Higgs doublet model, and therefore we parametrize the Higgs fields in terms of 8 real degrees of freedom as

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \quad H_d = \begin{pmatrix} H_d^+ \\ H_d^0 \end{pmatrix} \quad (4.16)$$

where  $H_u^+$ ,  $H_d^+$  and  $H_u^0$ ,  $H_d^0$  are complex fields. Specifically

$$H_u^+ = \frac{\text{Re}H_u^+ + i\text{Im}H_u^+}{\sqrt{2}} \quad H_d^- = \frac{\text{Re}H_d^- + i\text{Im}H_d^-}{\sqrt{2}} \quad H_u^- = H_u^{+*} \quad H_d^+ = H_d^{-*}. \quad (4.17)$$

Expanding around the vacuum we get for the uncharged components

$$H_u^0 = v_u + \frac{\text{Re}H_u^0 + i\text{Im}H_u^0}{\sqrt{2}} \quad H_d^0 = v_d + \frac{\text{Re}H_d^0 + i\text{Im}H_d^0}{\sqrt{2}}. \quad (4.18)$$

We define  $\cos \beta = v_d/v$ ,  $\sin \beta = v_u/v$  with  $v^2 = v_d^2 + v_u^2$  constrained to give the correct masses for the  $W$  and  $Z$  gauge bosons. As a consequence we will have  $\tan \beta = v_u/v_d$ .

The couplings of the two Higgses to the fermion sector are described by the Yukawa Lagrangian

$$\mathcal{L}_{Yuk} = -\Gamma^d \bar{Q} H_d d_R - \Gamma^d \bar{d}_R H_d^\dagger Q - \Gamma^u \bar{Q}_L (i\sigma_2 H_u^*) u_R - \Gamma^u \bar{u}_R (i\sigma_2 H_u^*)^\dagger Q_L -$$

$$\Gamma^e \bar{L} H_d e_R - \Gamma^e \bar{e}_R H_d^\dagger L, \quad (4.19)$$

where the Yukawa coupling constants  $\Gamma^d, \Gamma^u$  and  $\Gamma^e$  run over the three generations, i.e.  $u = \{u, c, t\}$ ,  $d = \{d, s, b\}$  and  $e = \{e, \mu, \tau\}$ . After the electroweak symmetry breaking the fermions appearing in the Yukawa lagrangian acquire a mass given by

$$m_u = v_u \Gamma^u \quad m_\nu = v_u \Gamma^\nu \quad m_d = v_d \Gamma^d \quad m_e = v_d \Gamma^e \quad (4.20)$$

where we have suppressed the generation index.

Rotating the CP-odd and CP-even neutral Higgs sectors onto the mass eigenstates we obtain

$$\begin{aligned} H_u^0 &= v_u + \frac{\text{Re}H_u^0 + i \text{Im}H_u^0}{\sqrt{2}} \\ &= v_u + \frac{(h^0 \sin \alpha - H^0 \cos \alpha) + i (O_{11}^\chi G_0^1 + O_{21}^\chi G_0^2 + O_{31}^\chi \chi)}{\sqrt{2}} \end{aligned} \quad (4.21)$$

$$\begin{aligned} H_d^0 &= v_d + \frac{\text{Re}H_d^0 + i \text{Im}H_d^0}{\sqrt{2}} \\ &= v_d + \frac{(h^0 \cos \alpha + H^0 \sin \alpha) + i (O_{12}^\chi G_0^1 + O_{22}^\chi G_0^1 + O_{32}^\chi \chi)}{\sqrt{2}} \end{aligned} \quad (4.22)$$

where  $\alpha$  represents the rotation angle that diagonalizes the  $2 \times 2$  CP-even mass matrix, whose eigenstates are  $h^0$  and  $H^0$ , and  $O^\chi$  represents the rotation matrix that diagonalizes the  $3 \times 3$  CP-odd Higgs sector which consists of  $\text{Im}H_d^0$ ,  $\text{Im}H_u^0$  and  $b$ ; the components of this matrix will be defined in Sec. 4.3. We will see that this sector contains the two neutral Goldstones of the theory (corresponding to the massive neutral gauge bosons  $Z$  and  $Z'$ ) and a physical state that we will call  $\chi$ . This state will inherit a Yukawa interaction with the fermions due to its Higgs components as can be easily seen from Eq. (4.22).

The couplings of the  $h^0$  boson to fermions are given by

$$\mathcal{L}_{Yuk}(h^0) = -\Gamma^d \bar{d}_L d_R \left( \frac{\cos \alpha}{\sqrt{2}} h^0 \right) - \Gamma^u \bar{u}_L u_R \left( \frac{\sin \alpha}{\sqrt{2}} h^0 \right) - \Gamma^e \bar{e}_L e_R \left( \frac{\cos \alpha}{\sqrt{2}} h^0 \right) + h.c. \quad (4.23)$$

The couplings of the  $H^0$  boson to the fermions are

$$\mathcal{L}_{Yuk}(H^0) = -\Gamma^d \bar{d}_L d_R \left( \frac{\sin \alpha}{\sqrt{2}} H^0 \right) - \Gamma^u \bar{u}_L u_R \left( -\frac{\cos \alpha}{\sqrt{2}} H^0 \right) - \Gamma^e \bar{e}_L e_R \left( \frac{\sin \alpha}{\sqrt{2}} H^0 \right) + h.c. \quad (4.24)$$

### 4.1.3 The neutral gauge bosons sector

The mass matrix for the neutral gauge boson sector is a  $3 \times 3$  matrix that receives contributions from the covariant derivatives acting on the Higgs fields that get a nonzero expectation value

(that is, from the usual Higgs mechanism) and from the Stückelberg lagrangian that contains a mass term for the gauge boson  $B$ .

The mass eigenstates can be expressed in terms of the interaction eigenstates introducing a rotation matrix  $O^A$  such that

$$\begin{pmatrix} A_\gamma \\ Z \\ Z' \end{pmatrix} = O^A \begin{pmatrix} W_3 \\ A^Y \\ B \end{pmatrix}. \quad (4.25)$$

At the first order in  $\epsilon = x_B/M^2$  with  $x_B = (q_u^B v_u^2 + q_d^B v_d^2)$  the rotation matrix is given by

$$O^A \simeq \begin{pmatrix} \frac{g_Y}{g} & \frac{g_2}{g} & 0 \\ \frac{g_2}{g} + O(\epsilon_1^2) & -\frac{g_Y}{g} + O(\epsilon_1^2) & \frac{g}{2}\epsilon_1 \\ -\frac{g_2}{2}\epsilon_1 & \frac{g_Y}{2}\epsilon_1 & 1 + O(\epsilon_1^2) \end{pmatrix} \quad (4.26)$$

More details can be found in[44]. Here we just point out that the photon eigenstate  $A_\gamma$  is expressed as the same linear combination obtained in the Standard Model and that a possible mixing effects only the  $Z$  and the  $Z'$ . Furthermore this mixing essentially depends on the ratio  $v/M$  and vanishes if the Stückelberg scale is much bigger that the electroweak scale.

## 4.2 General features of models with gauged axions: the Stückelberg field

In this section we briefly review the main features of the class of models that we address, discussing specifically the Stückelberg field  $b$  which accompanies their anomalous  $U(1)_B$  symmetry.

Intersecting brane models are one of the constructions where these types of generalized axions appear[16, 56, 57]. In the case in which several stacks of branes are introduced, each stack being the domain in which fields charged under the gauge symmetry  $U(N)$  live, several intersecting stacks generate, at their intersections, fields with the quantum numbers of all the unitary gauge groups of the construction, such as  $U(N_1) \times U(N_2) \times \dots \times U(N_k) = SU(N_1) \times U(1) \times SU(N_2) \times U(1) \times \dots \times SU(N_k) \times U(1)$ . In realistic models, the phases of the extra  $U(1)$ 's are rearranged in terms of an anomaly-free generator, with an (anomaly free) hypercharge  $U(1)$  (or  $U(1)_Y$ ) times extra  $U(1)$ 's which are anomalous, carrying both their own anomalies and the mixed anomalies with all the fields of the Standard Model.

For instance, a simple realization of the Standard Model is obtained by taking 3 stacks of branes: a first stack of 3 branes, with a symmetry  $U(3)$ , a second stack of 2 branes, with a symmetry  $U(2)$  and an extra single brane  $U(1)$ , giving a gauge structure of the form  $SU(3) \times SU(2) \times U(1) \times U(1) \times U(1)$ . Linear combinations of the generators of the three  $U(1)$ 's allow to rewrite the entire

abelian symmetry in the form  $U(1)_Y \times U(1)' \times U(1)''$ . The original basis for the  $U(1)$ 's is also called “the brane basis”, while the reorganization of the generators in the form of “hypercharge plus reminder” goes under the name of “the hypercharge basis”. Explicit assignments can be found in the recent literature[16, 56, 58].

We will be using the notation  $U(1)_B \times U(1)_C$  to refer to the factor  $U(1)' \times U(1)''$  of the gauge structure in the hypercharge basis.

The two extra  $U(1)$ 's are in a “broken” phase. For instance, if we denote with  $B$  and  $C$  the gauge bosons corresponding to the two abelian groups, the kinetic terms of these fields are given by

$$\mathcal{L}_{St} = \frac{1}{2} (\partial_\mu b - M_1 B_\mu)^2 + \frac{1}{2} (\partial_\mu c - M_2 C_\mu)^2 \quad (4.27)$$

which is the well-known Stückelberg form[59, 60].  $M_1$  and  $M_2$  are also called Stückelberg masses while  $b$  and  $c$  are two pseudoscalars known as Stückelberg fields (or Stückelberg axions). The Stückelberg symmetry of the Lagrangian in Eq. (4.27) is revealed by acting with gauge transformations of the gauge fields  $B$  and  $C$ , under which their corresponding axions  $b$  and  $c$  vary by a local shift

$$\begin{aligned} \delta_B B_\mu &= \partial_\mu \theta_B & \delta_B b &= M_1 \theta_B \\ \delta_C C_\mu &= \partial_\mu \theta_C & \delta_C c &= M_2 \theta_C, \end{aligned}$$

parameterized by the local gauge parameters  $\theta_B$  and  $\theta_C$ . In the literature, the Stückelberg symmetry is presented as a way to give a mass to an abelian gauge field but still preserving the gauge symmetry of the theory. However, a more careful look at this symmetry shows that its realization is the same one obtained, for instance, in an abelian Higgs model parameterizing the Higgs field using a module field and a phase field and setting the module field to a vev (vacuum expectation value) corresponding to the Stückelberg mass.

The Stückelberg lagrangian can also be obtained following a dualization procedure (see, for instance, [61]) in which the massive anomalous gauge bosons acquires a mass through the presence of “ $A \wedge F$ ” couplings in the effective string theory description. In this case the starting Lagrangian of the effective theory involves an antisymmetric rank-2 tensor  $A_{\mu\nu}$ , that is a 2-form, coupled to the field strength  $F_{\mu\nu}$  of an anomalous gauge boson (here denoted by  $B$ )

$$\mathcal{L} = -\frac{1}{12} H^{\mu\nu\rho} H_{\mu\nu\rho} - \frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} + \frac{M}{4} \epsilon^{\mu\nu\rho\sigma} A_{\mu\nu} F_{\rho\sigma}, \quad (4.28)$$

where

$$H_{\mu\nu\rho} = \partial_\mu A_{\nu\rho} + \partial_\rho A_{\mu\nu} + \partial_\nu A_{\rho\mu} \quad F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (4.29)$$

are the kinetic terms for the 2-form  $A$  and the 1-form  $B$  and  $g$  is an arbitrary constant. The third contribution in Eq. (4.28) is an interaction term that we call  $A \wedge F$ .

The Lagrangian is dualized by using a “first order” formalism, that is  $H$  is treated as an independent field respect to the antisymmetric field  $A_{\mu\nu}$ . This requires the introduction of a Lagrangian multiplier defined in terms of a new field,  $b(x)$ , in order to recover the definition of  $H = dA$ , that is its dependence on  $A$ , as the equations of motion of  $b$ . Using this formalism the lagrangian becomes

$$\mathcal{L}_0 = -\frac{1}{12} H^{\mu\nu\rho} H_{\mu\nu\rho} - \frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} - \frac{M}{6} \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho} B_\sigma + \frac{1}{6} b(x) \epsilon^{\mu\nu\rho\sigma} \partial_\mu H_{\nu\rho\sigma}. \quad (4.30)$$

The appearance of a scale  $M$  in this Lagrangian is of paramount importance both in the analysis of the relic densities of axions generated by the dualization of this action and in determining the mass of the extra anomalous U(1) gauge boson, which has been analyzed in detail in previous works[62]. It defines the energy region where the Green-Schwarz mechanism comes into play to cancel the anomaly in orientifold vacua of string theory[47]. Clearly, it is part of a far more involved field theory Lagrangian which, in general, is not included in the field theory analysis of this mechanism, since an expansion up to operators of dimension 5 is considered.

The last term in Eq. (4.30) is necessary in order to reobtain Eq. (4.28) from Eq. (4.30). If, instead, we integrate by parts the last term of the Lagrangian given in Eq. (4.30) and solve for  $H$  we find

$$H^{\mu\nu\rho} = -\epsilon^{\mu\nu\rho\sigma} (MB_\sigma - \partial_\sigma b). \quad (4.31)$$

Inserting this back into Eq. (4.30) we obtain the expression

$$\mathcal{L}_A = -\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} (MB_\sigma - \partial_\sigma b)^2 \quad (4.32)$$

which is the Stückelberg form for the mass term of  $B$ .

This rearrangement of the degrees of freedom, valid in a classical sense[63], and the mapping of the possible physical phases of these two model theories are an example of the connection between Lagrangians of antisymmetric tensor fields and their dual formulations, that in this specific case is an abelian massive gauge theory in a Stückelberg form (see for instance the discussion in[64]).

The axion field generated by the dualization mechanism appears to be a Nambu-Goldstone mode, which could be absorbed by a unitary gauge choice in the Stückelberg phase of the model. However, as discussed in[47], we will allow a mixing between this mode and the Higgs sector at the electroweak phase transition, by introducing an extra potential which respects the gauge symmetry and whose origin has been left, so far, unspecified.

$f$	$Q$	$u_R$	$d_R$	$L$	$e_R$
$q^B$	$q_Q^B$	$q_{u_R}^B$	$q_{d_R}^B$	$q_L^B$	$q_{e_R}^B$

$f$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_B$
$Q$	3	2	1/6	$q_Q^B$
$u_R$	3	1	2/3	$q_Q^B + q_u^B$
$d_R$	3	1	-1/3	$q_Q^B - q_d^B$
$L$	1	2	-1/2	$q_L^B$
$e_R$	1	1	-1	$q_L^B - q_d^B$
$H_u$	1	2	1/2	$q_u^B$
$H_d$	1	2	1/2	$q_d^B$

**Table 4.1:** Charges of the fermion and of the scalar fields

For this reason, at low energy, the counting of the physical degrees of freedom in the pseudoscalar sector of the model is performed in the combined Higgs-Stückelberg phase and a massive physical axion emerges from the combination of the phases of the Higgses and of the Stückelberg field. In models with several  $U(1)$ 's this construction is slightly more involved but, also in this case, we get a physical state with an axion component [47] whose mass is controlled by the size of the extra potential.

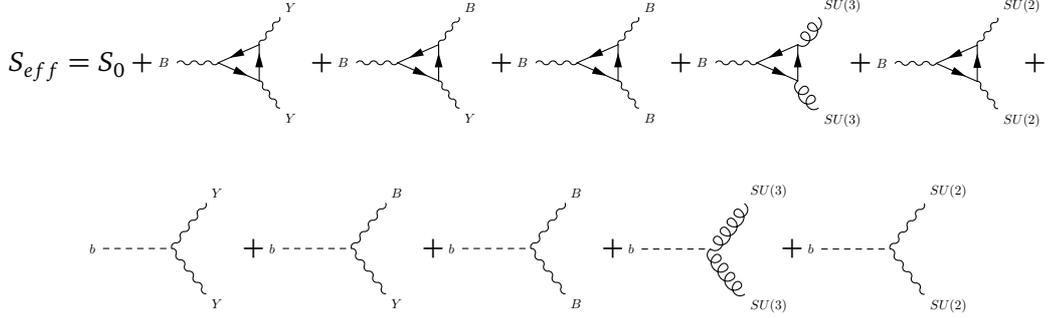
The Stückelberg Lagrangian that we have reviewed is part of the classical action  $S_0$  that also includes the remaining gauge kinetic terms of the theory at classical level, for a gauge symmetry  $SU(3) \times SU(2) \times U(1)_Y$ .

#### 4.2.1 Charge assignments and counterterms

We briefly comment on the list of the charge assignments of the single extra  $U(1)$  model, which is given in Tab. 4.1. Specifically,  $q_L^B, q_Q^B$  denote the charges of the left-handed lepton doublet ( $L$ ) and of the quark doublet ( $Q$ ), while  $q_{u_r}^B, q_{d_r}^B, q_{e_r}^B$  are the charges of the right-handed  $SU(2)$  singlets (quarks and leptons). We denote with  $\Delta q^B = q_u^B - q_d^B$  the difference between the two charges of the up and down Higgses ( $q_u^B, q_d^B$ ) respectively. The trilinear anomalous gauge interactions induced by the anomalous  $U(1)$  and the relative counterterms, which are all parts of the 1-loop effective action, are illustrated in Fig. 4.1. The numerical values of the counterterms appearing on the second line of Fig. 4.1 (see Eq. (4.5) for the definition) are fixed by the conditions of gauge invariance of the lagrangian and are summarized by the following relations

$$C_{YY} = -\frac{1}{6}q_Q^B + \frac{4}{3}q_{u_R}^B + \frac{1}{3}q_{d_R}^B - \frac{1}{2}q_L^B + q_{e_R}^B,$$

$$C_{YB} = -\left(q_Q^B\right)^2 + 2\left(q_{u_r}^B\right)^2 - \left(q_{d_r}^B\right)^2 + \left(q_L^B\right)^2 - \left(q_{e_r}^B\right)^2,$$



**Figure 4.1:** Anomalous contributions to the Lagrangian and WZ counterterms

$$\begin{aligned}
C_{BB} &= -6 (q_Q^B)^3 + 3 (q_{u_R}^B)^3 + 3 (q_{d_R}^B)^3 - 2 (q_L^B)^3 + (q_{e_R}^B)^3, \\
D &= \frac{1}{2} (-2q_Q^B + q_{d_R}^B + q_{u_R}^B), \\
F &= \frac{1}{2} (-q_L^B - 3q_Q^B).
\end{aligned} \tag{4.33}$$

They are, respectively, the counterterms for the cancellation of the mixed anomaly  $U(1)_B U(1)_Y^2$  and  $U(1)_Y U(1)_B^2$ , the counterterm for the  $BBB$  anomaly vertex or  $U(1)_B^3$  anomaly and those of the  $U(1)_B SU(3)^2$  and  $U(1)_B SU(2)^2$  anomalies. From the Yukawa couplings we get the following constraints on the  $U(1)_B$  charges

$$q_Q^B - q_d^B - q_{d_R}^B = 0 \quad q_Q^B + q_u^B - q_{u_R}^B = 0 \quad q_L^B - q_d^B - q_{e_R}^B = 0. \tag{4.34}$$

In Tab. 4.1 we show the expressions of the free  $U(1)_B$  charges appearing on each generation, having taken into account these constraints. Using these constraints on the charges we can also eliminate some of them in the expression of the counterterms, obtaining

$$\begin{aligned}
C_{YY} &= \frac{1}{6} (3q_L^B + 9q_Q^B + 8\Delta q^B) \\
C_{YB} &= 2 \left[ q_d^B (q_L^B + 3q_Q^B) + 2\Delta q^B (q_d^B + q_Q^B) + (\Delta q^B)^2 \right] \\
C_{BB} &= (q_L^B - q_d^B)^3 + 3 (q_d^B + q_Q^B + \Delta q^B)^3 + 3 (q_Q^B - q_d^B)^3 - 2 (q_L^B)^3 - 6 (q_Q^B)^3 \\
D &= \frac{\Delta q^B}{2} \\
F &= \frac{1}{2} (-q_L^B - 3q_Q^B).
\end{aligned} \tag{4.35}$$

The solutions given above are generic, in the sense that they parameterize, in principle, an infinite class of models whose charge assignments under  $U(1)_B$  are arbitrary, with the charges

on the last column of Tab. 4.1 taken as their free parameters. One can immediately observe that the Stückelberg axion has interactions with all of the gauge sectors of the model. Furthermore, in the case  $\Delta q^B = 0$ , the  $U(1)_B SU(3)^2$  anomaly disappears but, as we will see in Sec. 4.3, in this case the higgs-axion mixing disappears too.

### 4.3 The scalar potential

The scalar sector is characterized by a rather standard electroweak potential involving two Higgs doublets,  $V_{PQ}(H_u, H_d)$ , plus one extra contribution, denoted as  $V_{\not{P}Q}(H_u, H_d, b)$ , which mixes the Higgs sector with the Stückelberg axion  $b$ . The total scalar potential will be a sum of the two contributions

$$V = V_{PQ}(H_u, H_d) + V_{\not{P}Q}(H_u, H_d, b). \quad (4.36)$$

The expressions of the two contributions to the scalar potential are

$$\begin{aligned} V_{PQ} &= \mu_u^2 H_u^\dagger H_u + \mu_d^2 H_d^\dagger H_d + \lambda_{uu}(H_u^\dagger H_u)^2 + \lambda_{dd}(H_d^\dagger H_d)^2 - 2\lambda_{ud}(H_u^\dagger H_u)(H_d^\dagger H_d) + 2\lambda'_{ud}|H_u^T \tau_2 H_d|^2 \\ V_{\not{P}Q} &= \lambda_0(H_u^\dagger H_d e^{-ig_B(q_u - q_d)\frac{b}{2M}}) + \lambda_1(H_u^\dagger H_d e^{-ig_B(q_u - q_d)\frac{b}{2M}})^2 + \lambda_2(H_u^\dagger H_u)(H_u^\dagger H_d e^{-ig_B(q_u - q_d)\frac{b}{2M}}) + \\ &\quad \lambda_3(H_d^\dagger H_d)(H_u^\dagger H_d e^{-ig_B(q_u - q_d)\frac{b}{2M}}) + \text{h.c.} \end{aligned} \quad (4.37)$$

The terms in  $V_{\not{P}Q}$  are allowed by the gauge symmetry of the model and are parameterized by one dimensionful ( $\lambda_0$ ) and three dimensionless constants ( $\lambda_1, \lambda_2, \lambda_3$ ). In what follows we will rescale  $\lambda_0$  by the electroweak scale  $\nu = \sqrt{\nu_u^2 + \nu_d^2}$  ( $\lambda_0 \equiv \bar{\lambda}_0 \nu^2$ ) in order to obtain a homogeneous expression of the mass of the axi-higgs as a function of the relevant scales of the model which are the electroweak vev  $\nu$  and the Stückelberg mass  $M$ .

We focus our attention just on the CP-odd sector of the total potential, which is the only one that is relevant for our discussion. The expansion of this potential around the electroweak vacuum is given by the parameterization

$$H_u = \begin{pmatrix} H_u^+ \\ \nu_u + H_u^0 \end{pmatrix} \quad H_d = \begin{pmatrix} H_d^+ \\ \nu_d + H_d^0 \end{pmatrix} \quad (4.38)$$

where we have used, as usual, the gauge symmetries of the model to place the vev on the neutral components of the Higgs doublets.

The potential, in the CP-odd sector, is characterized by two null eigenvalues corresponding to two neutral Goldstone modes ( $G_0^1, G_0^2$ ) and an eigenvalue corresponding to a massive state with an axion component that we will call  $\chi$ . In the  $(\text{Im}H_d^0, \text{Im}H_u^0, b)$  basis we get the following

normalized eigenstates

$$\begin{aligned}
G_0^1 &= \frac{1}{\sqrt{v_u^2 + v_d^2}}(v_d, v_u, 0) \\
G_0^2 &= \frac{1}{\sqrt{g_B^2(q_d - q_u)^2 v_d^2 v_u^2 + 2M^2(v_d^2 + v_u^2)}} \left( -\frac{g_B(q_d - q_u)v_d v_u^2}{\sqrt{v_u^2 + v_d^2}}, \frac{g_B(q_d - q_u)v_d^2 v_u}{\sqrt{v_d^2 + v_u^2}}, \sqrt{2M} \sqrt{v_u^2 + v_d^2} \right) \\
\chi &= \frac{1}{\sqrt{g_B^2(q_d - q_u)^2 v_u^2 v_d^2 + 2M^2(v_d^2 + v_u^2)}} \left( \sqrt{2M} v_u, -\sqrt{2M} v_d, g_B(q_d - q_u)v_d v_u \right) \quad (4.39)
\end{aligned}$$

and we indicate with  $O^\chi$  the orthogonal matrix which connects the interaction and the mass basis

$$\begin{pmatrix} G_0^1 \\ G_0^2 \\ \chi \end{pmatrix} = O^\chi \begin{pmatrix} \text{Im}H_d^0 \\ \text{Im}H_u^0 \\ b \end{pmatrix}. \quad (4.40)$$

The matrix  $O^\chi$  is easily obtained from the normalized eigenvectors and is given by

$$O^\chi = \begin{pmatrix} \frac{v_d}{v} & \frac{v_u}{v} & 0 \\ -\frac{g_B(q_d - q_u)v_d v_u^2}{v \sqrt{g_B^2(q_d - q_u)^2 v_d^2 v_u^2 + 2M^2 v^2}} & \frac{g_B(q_d - q_u)v_d^2 v_u}{v \sqrt{g_B^2(q_d - q_u)^2 v_d^2 v_u^2 + 2M^2 v^2}} & \frac{\sqrt{2M}v}{\sqrt{g_B^2(q_d - q_u)^2 v_d^2 v_u^2 + 2M^2 v^2}} \\ \frac{\sqrt{2M}v_u}{\sqrt{g_B^2(q_d - q_u)^2 v_u^2 v_d^2 + 2M^2 v^2}} & -\frac{\sqrt{2M}v_d}{\sqrt{g_B^2(q_d - q_u)^2 v_u^2 v_d^2 + 2M^2 v^2}} & \frac{g_B(q_d - q_u)v_d v_u}{\sqrt{g_B^2(q_d - q_u)^2 v_u^2 v_d^2 + 2M^2 v^2}} \end{pmatrix}. \quad (4.41)$$

The massive state  $\chi$  inherits the WZ interaction from its axion component. It can be expressed in terms of the interaction basis states as

$$\chi = O_{31}^\chi \text{Im}H_d + O_{32}^\chi \text{Im}H_u + O_{33}^\chi b. \quad (4.42)$$

Notice that the rotation of  $b$  into the physical axion  $\chi$  involves a factor  $O_{33}^\chi$  which is of order  $v/M$ . As a consequence the  $\chi$  inherited interaction is suppressed by a scale  $M^2/v$ . This scale is the product of two contributions: a  $1/M$  suppression coming from the original Wess-Zumino counterterm of the Lagrangian ( $b/MF\tilde{F}$ ) and a factor  $v/M$  obtained by the projection of  $b$  onto  $\chi$ .

The resulting coupling appears as a coefficient in the interaction of the physical axion with two photons

$$g_{\gamma\gamma}^\chi \chi F_\gamma \tilde{F}_\gamma \quad (4.43)$$

and is given by

$$g_{\gamma\gamma}^\chi = \left( F O_{W_{3\gamma}}^A O_{W_{3\gamma}}^A + C_{YY} O_{Y\gamma}^A O_{Y\gamma}^A \right) O_{33}^\chi. \quad (4.44)$$

It is defined by a combination of matrix elements of the rotation matrices  $O^A$  and  $O^\chi$ , together with some of the counterterm parameters,  $F$  and  $C_{YY}$ , defined in Eq. (4.35).  $O^A$  is the matrix that rotates the neutral gauge bosons from the interaction to the mass eigenstates defined in Eq. (4.25).

Defining  $g^2 = g_2^2 + g_Y^2$ , the coefficient  $g_{\gamma\gamma}^\chi$  can be rewritten as

$$g_{\gamma\gamma}^\chi = \frac{g_B g_Y^2 g_2^2}{32\pi^2 M g^2} O_{33}^\chi \sum_f \left( -q_{fL}^B + q_{fR}^B \left( q_{fR}^Y \right)^2 - q_{fL}^B \left( q_{fL}^Y \right)^2 \right). \quad (4.45)$$

Notice that this expression is cubic in the gauge coupling constants, since factors such as  $g_2/g$  and  $g_Y/g$  are mixing angles, while the factor  $1/\pi^2$  originates from the anomaly. Therefore one obtains a general size for  $g_{\gamma\gamma}^\chi$  of order  $O(g^3 v/M^2)$ , with charges which are, in general, of order unity.

Apart from the WZ interaction with the photons,  $\chi$  also inherits a Yukawa interaction with the massive fermions of the theory because of its Higgs components. Considering the definition of the Yukawa couplings in Eq. (4.19) the fermion interactions for  $\chi$  are given by

$$\mathcal{L}_{Yuk}(\chi) = -\frac{i}{\sqrt{2}} \chi \left( O_{31}^\chi \Gamma^d \bar{d}_L d_R + O_{31}^\chi \Gamma^e \bar{e}_L e_R + O_{32}^\chi \Gamma^u \bar{u}_L u_R \right) + h.c. \quad (4.46)$$

In Sec. 4.4 we will refer to the coefficients coming from this lagrangian as to  $c_{\chi f}$  with  $f$  representing a specific quark or lepton.

### 4.3.1 Periodicity of the $V'$ potential

The CP-odd sector potential has a well-defined periodicity if we parametrize the Higgs fields in a polar form. To identify the corresponding phase in the Higgs-neutral CP-odd sector, we introduce the following parameterization of the neutral components in the broken electroweak phase

$$H_u^0 = \frac{1}{\sqrt{2}} \left( \sqrt{2}v_u + \rho_u^0(x) \right) e^{i \frac{F_u^0(x)}{\sqrt{2}v_u}} \quad H_d^0 = \frac{1}{\sqrt{2}} \left( \sqrt{2}v_d + \rho_d^0(x) \right) e^{i \frac{F_d^0(x)}{\sqrt{2}v_d}}, \quad (4.47)$$

where we have introduced two phase fields  $F_u$  and  $F_d$  and two module fields  $\rho_u^0$  and  $\rho_d^0$ . The potential is periodic with respect to the linear combination of fields given by

$$\theta(x) \equiv \frac{g_B(q_d - q_u)}{2M} b(x) - \frac{1}{\sqrt{2}v_u} F_u^0(x) + \frac{1}{\sqrt{2}v_d} F_d^0(x). \quad (4.48)$$

Using the matrix  $O^\chi$  to rotate on the physical basis, the phase describing the periodicity of the potential turns out to be proportional to the physical axion, modulo a dimensionful constant that we will call  $\sigma_\chi$

$$\theta(x) \equiv \frac{\chi(x)}{\sigma_\chi}, \quad (4.49)$$

with

$$\sigma_\chi \equiv \frac{2v_u v_d M}{\sqrt{g_B^2 (q_d - q_u)^2 v_d^2 v_u^2 + 2M^2 (v_d^2 + v_u^2)}}. \quad (4.50)$$

In the limit  $M \gg v$  we have  $\sigma_\chi \simeq v \sin 2\beta / \sqrt{2}$ .

In order to extract the mass term for  $\chi$ , we can rewrite the part of the potential that depends only on  $\chi$  after electroweak symmetry breaking as

$$V_{\text{PQ}}^\chi = 4v_u v_d (\lambda_2 v_d^2 + \lambda_3 v_u^2 + \lambda_0) \cos\left(\frac{\chi}{\sigma_\chi}\right) + 2\lambda_1 v_u^2 v_d^2 \cos\left(2\frac{\chi}{\sigma_\chi}\right). \quad (4.51)$$

From the expansion of the arguments of the cosines we obtain the mass for the physical axion  $\chi$

$$m_\chi^2 = \frac{2v_u v_d}{\sigma_\chi^2} (\bar{\lambda}_0 v^2 + \lambda_2 v_d^2 + \lambda_3 v_u^2 + 4\lambda_1 v_u v_d) \simeq \frac{2v^2}{\sin 2\beta} (\bar{\lambda}_0 + \lambda_2 \cos^2 \beta + \lambda_3 \sin^2 \beta + 2\lambda_1 \sin 2\beta) \quad (4.52)$$

where we have used the expression for  $\sigma_\chi$  in the limit  $M \gg v$ .

One point that needs to be stressed is the fact that, after the electroweak symmetry breaking, the potential for  $\chi$  is parameterized by  $\chi/\sigma_\chi$  while the interaction of  $\chi$  with the gauge fields is suppressed by  $M^2/v$ . This makes this situation essentially different from the PQ case since in that case both the mass and the interaction with the gauge fields are suppressed by the axion decay constant,  $f_a$ .

## 4.4 Decays of axion-like particles

The physical state in the CP-odd sector inherits the Yukawa interactions from its Higgs components; these interactions are proportional to the components of the rotation matrix  $O^\chi$  and to the mass of the fermions that appear in the Yukawa couplings.

The presence of these interactions increases the number of the decay modes and, in particular, induces new channels in its decay rate into gauge bosons, mediated by fermion loops. In this section we perform a complete study of the decay rate under the assumption that the mass of  $\chi$

is in the meV region and below. In particular, in the case of a very light axi-Higgs, the decays into massless vector bosons are all dominated by the Wess-Zumino contributions, which are far larger than those coming from the fermion loops. These results will be used in the study of the relic densities of this particle which will be presented in the next section. Here we compare the results of the decay rates for  $\chi$  with those of the Peccei-Quinn (PQ) axion that we are going to compute from scratch.

The coupling of the PQ axion to the fermions is given by

$$\mathcal{L}_f = i g_f \frac{m_f}{f_a} a \bar{\psi}_f \gamma^5 \psi_f, \quad (4.53)$$

where  $a$  represents the PQ axion field,  $m_f$  is the mass of the fermion of flavor  $f$ , and the coupling  $g_f = Q_{f_R}^{PQ} - Q_{f_L}^{PQ}$  is given in terms of the chiral PQ charges of each fermion. We denote with  $f_a$  the axion decay constant; this parameter is bounded from astrophysical and cosmological constraints to be in the range  $10^8 \text{ GeV} \leq f_a \leq 10^{12} \text{ GeV}$ .

The interaction of the PQ axion with photons is given by

$$\mathcal{L}_{a\gamma\gamma} = \frac{G_{a\gamma\gamma}}{4} a F^{\mu\nu} \tilde{F}_{\mu\nu} = -G_{a\gamma\gamma} a \vec{E} \cdot \vec{B}. \quad (4.54)$$

where  $\vec{E}$  and  $\vec{B}$  are, respectively, the electric and magnetic fields and the coupling  $G_{a\gamma\gamma}$  is the sum of a model dependent term and of a second term which depends only on the ratio of the quark masses

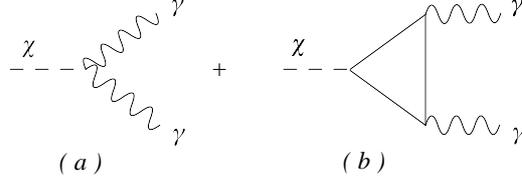
$$G_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a} \left( \sum_f Q_f^{PQ} (Q_f^{em})^2 - \frac{2}{3} \frac{4+z}{1+z} \right), \quad (4.55)$$

where the quark-mass ratio is  $z = m_u/m_d$ , while the  $Q_f^{em}$ 's are the electric charges of the quarks. Since the coefficient  $G_{a\gamma\gamma}$  is model dependent we have several possibilities[65]. We define  $G_{a\gamma\gamma}^0$  the coupling  $G_{a\gamma\gamma}$  in the case

$$\sum_f Q_f^{PQ} (Q_f^{em})^2 = 0. \quad (4.56)$$

This definition is useful if we want to consider the model dependent and model independent contributions separately. We also make the usual choice,  $z = 0.56$ . This choice gives the following decay rate into two photons as function of the axion mass  $m_a$

$$\Gamma_{a\gamma\gamma} = \frac{(G_{a\gamma\gamma}^0)^2}{64\pi} m_a^3 = 1.1 \times 10^{-24} s^{-1} \left( \frac{m_a}{\text{eV}} \right)^5. \quad (4.57)$$



**Figure 4.2:** Contributions to the  $\chi \rightarrow \gamma\gamma$  decay.

More generally, we want to write the decay rate separating the contribution from the Wess-Zumino interactions from those which are obtained from the fermion loop corrections. We obtain

$$d\Gamma_{PQ}(a \rightarrow \gamma\gamma) = \frac{1}{2m_a} \sum_{pol} |\mathcal{M}_{PQ}|^2 \frac{d\vec{k}_1}{(2\pi)^3 k_1^0} \frac{d\vec{k}_2}{(2\pi)^3 k_2^0} (2\pi)^4 \delta^{(4)}(k - k_1 - k_2), \quad (4.58)$$

where the squared amplitude is given by

$$\begin{aligned} \sum_{pol} |\mathcal{M}_{PQ}|^2 &= \sum_{pol} |\mathcal{M}_{WZ} + \mathcal{M}_{loop}|^2 \\ &= m_a^4 \left[ \frac{1}{2} (G_{a\gamma\gamma}^0)^2 + \frac{1}{2} \left| \sum_f N_c(f) \frac{\tau_f f(\tau_f)}{4\pi^2 m_f} e^2 Q_f^2 g_f \frac{m_f}{f_a} \right|^2 + G_{a\gamma\gamma}^0 \sum_f N_c(f) \frac{\tau_f f(\tau_f)}{4\pi^2 m_f} e^2 Q_f^2 g_f \frac{m_f}{f_a} \right], \end{aligned} \quad (4.59)$$

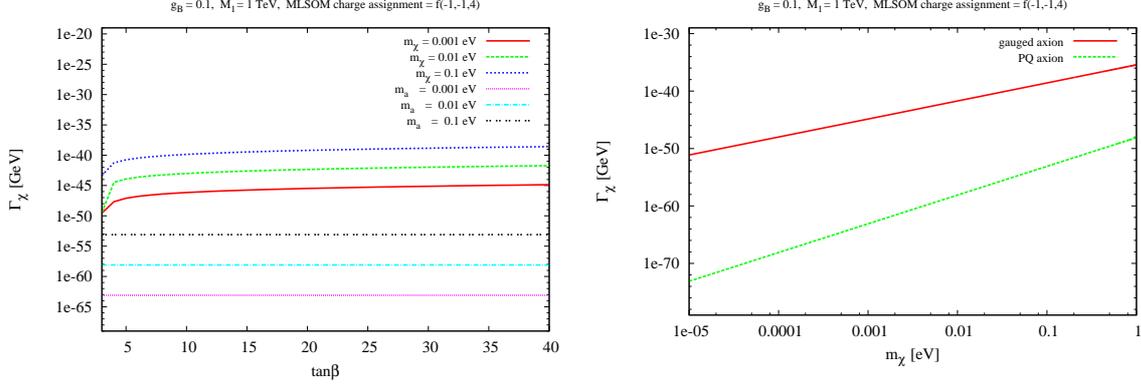
where  $N_c(f)$  is the color factor for the fermion in the loop and the functions  $\tau_f$  and  $f(\tau_f)$  depend on the mass of the fermion circulating in the loop. They are defined as

$$\text{Re}[f(\tau_f)] = \begin{cases} \arcsin^2(\tau_f^{-1/2}) & \tau_f \geq 1 \\ -\frac{1}{4} \left[ \log^2\left(\frac{1+\sqrt{1-\tau_f}}{1-\sqrt{1-\tau_f}}\right) - \pi^2 \right] & \tau_f < 1 \end{cases} \quad (4.60)$$

while its imaginary part is

$$\text{Im}[f(\tau_f)] = \begin{cases} 0 & \tau_f \geq 1 \\ \frac{\pi}{2} \left[ \log\left(\frac{1+\sqrt{1-\tau_f}}{1-\sqrt{1-\tau_f}}\right) \right] & \tau_f < 1 \end{cases} \quad (4.61)$$

where  $\tau_f = 4m_f^2/m_a^2$ . In the case we are considering  $\tau_f \gg 1$ , that is the axion mass is much smaller than the mass of the fermions circulating in the loop, and in this limit  $\tau_f f(\tau_f) \approx 1$ .



**Figure 4.3:** Total decay rate of the axi-Higgs for several mass values. Here, for the PQ axion, we have chosen  $f_a = 10^{10}$  GeV.

As we move to compute the decay of  $\chi$  and assume a free varying mass for this particle, the WZ interaction is given by

$$\mathcal{M}_{WZ}^{\mu\nu}(\chi \rightarrow \gamma\gamma) = 4 g_{\gamma\gamma}^\chi \epsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}. \quad (4.62)$$

with the coupling  $g_{\gamma\gamma}^\chi$  defined in Eq. (4.45). In Fig. 4.2a we have isolated the massless contribution to the decay rate coming from the WZ counterterm  $\chi F_\gamma \tilde{F}_\gamma$  whose expression is

$$\Gamma_{WZ}(\chi \rightarrow \gamma\gamma) = \frac{m_\chi^3}{4\pi} \left( g_{\gamma\gamma}^\chi \right)^2. \quad (4.63)$$

Combining also in this case the tree level decay with the 1-loop amplitude, we obtain for  $\chi \rightarrow \gamma\gamma$  the amplitude

$$\mathcal{M}^{\mu\nu}(\chi \rightarrow \gamma\gamma) = \mathcal{M}_{WZ}^{\mu\nu} + \mathcal{M}_{loop}^{\mu\nu}. \quad (4.64)$$

In this case the rates are derived from the expression

$$\Gamma_\chi \equiv \Gamma(\chi \rightarrow \gamma\gamma) = \frac{m_\chi^3}{32\pi} \left[ 8(g_{\gamma\gamma}^\chi)^2 + \frac{1}{2} \left| \sum_f N_c(f) i \frac{\tau_f f(\tau_f)}{4\pi^2 m_f} e^2 Q_f^2 c_{\chi f} \right|^2 + 4g_{\gamma\gamma}^\chi \sum_f N_c(f) i \frac{\tau_f f(\tau_f)}{4\pi^2 m_f} e^2 Q_f^2 c_{\chi f} \right]. \quad (4.65)$$

In the equation above both the direct ( $\sim (g_{\gamma\gamma}^\chi)^2$ ) and the interference ( $\sim g_{\gamma\gamma}^\chi$ ) contributions are suppressed as inverse powers of the Stückelberg mass. We also remind that the coefficient

$c_{\chi f}$  represents the couplings of the axi-higgs to the fermions defined in Eq. (4.46) and that these couplings are not suppressed as they are in the PQ case. We show the results of the comparative study in Fig. 4.3, where in the left panel we present results for the decay rates of  $\chi \rightarrow \gamma\gamma$  for several values of the axion mass as a function of  $\tan\beta = v_u/v_d$ . The plots indicate a very mild dependence of the rates on this parameter, even for rather large variations. In the same plot the rates for the PQ case are shown as constant lines, just for comparison. Notice that we have chosen a rather low Stückelberg mass, with  $M = 1$  TeV. The charge assignment of the anomalous model have been denoted as  $f(-1, 1, 4)$ , where we have used the convention

$$f\left(q_{Q_L}^B, q_L^B, \Delta q^B\right) \equiv \left(q_{Q_L}^B, q_{u_R}^B; q_{d_R}^B, q_L^B, q_{e_R}^B, q_u^B, q_d^B\right). \quad (4.66)$$

The charges depend only upon the three free parameters  $q_{Q_L}^B, q_L^B, \Delta q^B$ . The parametric solution of the anomaly equations of the model  $f(q_{Q_L}^B, q_L^B, \Delta q^B)$ , for the particular choice  $q_{Q_L}^B = -1, q_L^B = -1$ , reproduces the entire charge assignment of a special class of intersecting brane models (see [56, 61] and the discussion in [66])

$$f(-1, -1, 4) = (-1, 0, 0, -1, 0, +2, -2). \quad (4.67)$$

In Fig. 4.3 (right panel) we show the decay rates as a function of the axion mass in both cases, having chosen a nominal mass range for this particle varying between  $10^{-5} - 1$  eV. One can immediately observe that the rates for the PQ case are smaller than those for the Stückelberg by a factor of  $10^{20} - 10^{12}$ , nevertheless the axi-Higgs  $\chi$  has a lifetime which is much bigger than the current age of the universe.

Concerning the possibility to detect the axion through its two-photon decay channel, its tiny mass and the smaller value of its lifetime unfortunately do not allow to set significant constraints on its possible parameter space. The situation, in this case, is rather different from that of other dark matter candidates, such as, for instance, the gravitinos, which have been widely investigated recently[67–69]. In fact, the allowed parameter space where the constraints derived from those previous studies apply, concerns a region in the plane  $(\tau_{DM}, m_{DM})$  - with  $\tau_{DM}$  being the lifetime of a generic dark matter particle and  $m_{DM}$  its mass - which is bounded by the intervals  $10^{26} \text{ s} < \tau_{DM} < 10^{35} \text{ s}$  and  $10^{-5} \text{ GeV} < m_{DM} < 10^2 \text{ GeV}$ .

While the value of  $\tau_\chi$  for the axion can reasonably reach the lower edge of the scanned region in  $\tau_{DM}$ , by an adjustment of its coupling  $g_B$  and charge assignments of the anomalous  $U(1)$ , its mass is definitely too small to be excluded by these types of analysis. These studies are, obviously, very interesting for candidates of heavier mass, such as gravitinos. Similar considerations apply in the case of LHC studies, given the small production rates for a very light axion. For much heavier axions, instead, these types of studies have been performed quite recently[66], but the behavior of this particle, in this case, is akin a light Higgs rather than a long-lived light pseudoscalar.

## 4.5 Axion dark matter

As we have discussed previously, at the electroweak scale, a mixing between the various phases of the Higgs potential allows to identify the physical state with an axion component,  $\chi$ . In general, this is misaligned with respect to the minimum of the potential generated at this transition, with a misalignment that, as we have pointed out, is parameterized by the value of  $\theta = \chi/\sigma_\chi$ .

The analysis of the relic density is then performed rather straightforwardly, following a standard approach borrowed from the PQ case. For this goal, we define the abundance of  $\chi$  at the oscillation temperature  $T_i$  as

$$Y_\chi(T_i) \equiv \left. \frac{n_\chi}{s} \right|_{T_i}. \quad (4.68)$$

We know that the vacuum misalignment mechanism generates a condensate that behaves like cold dark matter. Since the energy density associated to cold dark matter scales as  $a^{-3}$ , where  $a$  represents the Friedmann-Robertson-Walker scale factor, and the entropy density also scales as  $a^{-3}$ , then  $Y_\chi$  is a conserved variable. The condition for the  $\chi$  field to start oscillating and to appear as dark matter is (see Sec. 3.2)

$$m_\chi(T_i) = 3H(T_i), \quad (4.69)$$

where  $m_\chi(T_i)$  is the mass of  $\chi$  and  $H(T_i)$  the Hubble constant at the oscillation temperature  $T_i$ . This temperature will be equal or smaller than the electroweak symmetry breaking scale since the mass of  $\chi$  is generated at this scale. We can re-express the Hubble constant in Eq. (4.69) in terms of the number of effectively massless degrees of freedom at the oscillation temperature  $g_{*,T_i}$ , that is

$$m_\chi(T_i) = \sqrt{\frac{4}{5} \pi^3 g_{*,T_i}} \frac{T_i^2}{M_P}. \quad (4.70)$$

If we consider the expression of the mass of  $\chi$  obtained in Eq. (4.52) then we obtain an estimate of the minimum value of the couplings appearing in the potential in order for the oscillations to start at the temperature  $T_i$

$$\frac{2v^2}{\sin 2\beta} \left( \bar{\lambda}_0 + \lambda_2 \cos^2 \beta + \lambda_3 \sin^2 \beta + 2\lambda_1 \sin 2\beta \right) \simeq \frac{4}{5} \pi^3 g_{*,T_i} \frac{T_i^4}{M_P^2}. \quad (4.71)$$

Assuming  $\bar{\lambda}_0 = \lambda_2 = \lambda_3 = \lambda_1 = \lambda$  we get

$$\lambda = \left( \frac{\sin 2\beta}{1 + \sin 2\beta} \right) \left( \frac{T_i^4}{v^2 M_P^2} \right) \frac{\pi^3 g_{*,T_i}}{5}. \quad (4.72)$$

The  $\beta$  dependent factor varies from 0.28 to 0.04 for  $\tan \beta$  going from 5 to 50. The number of effectively massless degrees of freedom at the electroweak phase transition is  $\approx 100$  and we have  $v \approx 174$  GeV,  $M_p = 2.43 \times 10^{18}$  GeV. Using these values and requiring  $T_i = v$  we get  $\lambda \simeq 10^{-31}$ . Clearly, smaller values of the coupling  $\lambda$  would give a oscillation temperature smaller than the electroweak scale.

We can express the entropy density in terms of the oscillation temperature  $T_i$  as

$$s = \frac{2\pi^2 g_{*,T_i} T_i^3}{45}. \quad (4.73)$$

The energy density of the  $\chi$  field at the oscillation temperature depends on the amplitude of the oscillation and, as we have said, it corresponds to the energy density of non-relativistic dark matter. This means that we can obtain an estimate of the number density  $n_\chi$  as follows

$$\rho = \frac{1}{2} m_\chi (T_i)^2 \chi_i^2 = n_\chi m_\chi (T_i) \Rightarrow n_\chi = \frac{1}{2} m_\chi (T_i) \chi_i^2 \quad (4.74)$$

where  $\chi_i$  represents the initial amplitude of the oscillation. In terms of the initial angle of misalignment  $\theta_i$  and using the mass coming from the oscillation condition, Eq. (4.68) becomes

$$Y_\chi(T_i) = \frac{45 \sigma_\chi^2 \theta_i^2}{2 \sqrt{5\pi g_{*,T_i}} T_i M_p}. \quad (4.75)$$

Using the conservation of the abundance  $Y_{\chi 0} = Y_\chi(T_i)$  (where the index 0 refers to the present time), the expression of the contribution to the energy density today as a fraction of the total energy density is given by

$$\Omega_\chi^{mis} = Y_\chi(T_i) m_\chi \frac{s_0}{\rho_c} = \frac{n_\chi}{s} \Big|_{T_i} m_\chi \frac{s_0}{\rho_c} = \frac{45 \theta_i^2}{2 \sqrt{5\pi g_{*,T_i}} M_p} \frac{\sigma_\chi^2 m_\chi s_0}{T_i \rho_c}. \quad (4.76)$$

The values of the critical energy density ( $\rho_c$ ) and the entropy density today are estimated as [70, 71]

$$\rho_c = 5.3 \cdot 10^{-6} \text{ GeV/cm}^3 \quad s_0 = 2970 \text{ cm}^{-3}, \quad (4.77)$$

while for the initial misalignment angle we have  $\theta_i \simeq 1$ . If we assume  $\bar{\lambda}_0 = \lambda_2 = \lambda_3 = \lambda_1 = \lambda$  and take the expressions for the mass of  $\chi$  and  $\sigma_\chi$  in the limit  $M \gg v$  and  $T_i = v$  we get

$$\Omega_\chi^{mis} = \sqrt{\lambda} \theta_i^2 \left( \frac{45}{2 \sqrt{5\pi g_{*,T_i}} M_p} \frac{s_0}{\rho_c} v^2 \right) \left( \sin^2 2\beta \sqrt{\frac{1 + \sin 2\beta}{\sin 2\beta}} \right). \quad (4.78)$$

The  $\beta$  dependent factor varies from 0.28 to 0.008 for  $\tan\beta$  in the range 5 – 50, the factor depending on the Planck and electroweak scales is  $\approx 4 \times 10^{-6}$  while  $\theta_i \approx 1$ . Putting everything together we can write

$$\Omega_\chi^{mis} = \sqrt{\lambda} \times \begin{cases} 1.13 \times 10^{-6} & \tan\beta = 5 \\ 3.29 \times 10^{-8} & \tan\beta = 50 \end{cases} . \quad (4.79)$$

This result has to be compared with the result for the lifetime of  $\chi$ . In fact, in order to have dark matter production from the misalignment mechanism we have to require that the particles composing the generated condensate are long lived. Since the electroweak phase transition takes place at  $t \approx 10^{-12}$ s, then the lifetime required for  $\chi$  to be observed as dark matter today is essentially equal to the age of the Universe,  $t_0 = 13.75 \text{ Gyr} = 4.3 \times 10^{17} \text{ s}$ [13]. So we have to require that

$$\Gamma_\chi \leq \frac{\hbar}{t_0} \simeq 10^{-43} \text{ GeV}. \quad (4.80)$$

According to the results obtained in Sec. 4.4, the mass of the axion has to be  $\lesssim 10^{-2}$  eV and this would require  $\lambda \lesssim 10^{-28}$ . We can imagine that we can increase the axion mass and at the same time suppress the decay rate choosing different parameters for the decay rate calculation. For example, we could choose bigger values of the Stückelberg mass in order to suppress the WZ interaction. Nevertheless we can easily see that the couplings appearing in the fermion triangle diagram are not suppressed, so the obtained upper bound is quite general.

In the case of a Peccei-Quinn axion the situation is quite different and we are going to show why. The misalignment angle in the PQ case is defined as

$$\theta(x) = \frac{a(x)}{f_a} \quad (4.81)$$

where  $a(x)$  represents the axion field and  $f_a$  is called axion decay constant. The axion receives a mass contribution due to QCD instanton effects that switch on at the QCD phase transition scale  $\Lambda_{QCD}$ , namely when the strong interactions become non-perturbative and explicitly break the  $U(1)$  PQ symmetry down to a discrete symmetry  $Z_N$ . These effects become important at the scale  $\Lambda_{QCD}$  since their strength is typically suppressed by the factor  $e^{-2\pi/\alpha_s}$  so they are enhanced for bigger values of  $\alpha_s$ .

The mass of the axion is temperature dependent and its zero temperature value is given by

$$m_a = \frac{\Lambda_{QCD}^2}{f_a}. \quad (4.82)$$

It is important to note that the axion decay constant  $f_a$  enters as a suppression scale in all the

interactions involving the PQ axion. Making  $f_a$  big enough we can make the PQ axion invisible and at the same time we are lowering its mass.

We have seen that the dark matter density coming from the misalignment mechanism depends on the product  $\sigma_\chi^2 m_\chi / T_i$  which is of order  $v^2$  if we take  $T_i = v$ . In the PQ case the analogous product becomes  $f_a^2 (\Lambda_{QCD}^2 / f_a) / \Lambda_{QCD} = f_a \Lambda_{QCD}$  so, since  $f_a \gg v$  we have an enhancement in the dark matter density. Actually we have to ensure that the value of  $f_a$  is such that it doesn't give a dark matter density that exceeds the experimental bounds.

So we can put a lower bound on  $f_a$  requiring that the axion interactions are suppressed, since they would enhance the energy loss in stars and sensibly alter the stellar evolution[34]; on the other hand we can put an upper bound on  $f_a$  requiring that the obtained dark matter density is such that  $\Omega \lesssim 1$ . This bounds easily translate into an upper and lower bound on the axion mass and the window for the mass of the PQ axion that we are left with is given by  $10^{-5} \text{eV} \lesssim m_a \lesssim 10^{-2} \text{eV}$ [72, 73].

## 4.6 Conclusions

We have discussed the most salient cosmological features of models containing gauged axions, obtained from the gauging of an anomalous symmetry. The gauging allows to define a consistent theory for axion-like particles, which generalize many of the properties of PQ axions. They have appeared for the first time in the study of intersecting branes, but their features are quite generic. They are constructed as effective theories containing minimal gauge interactions which restore gauge invariance of the effective action in the presence of an anomalous  $U(1)$  symmetry, and no further requirements. Differently from the PQ case, here there is no concept of an original PQ symmetry, broken at a very large scale, with the axion taking the role of a Goldstone mode that acquires a mass at the QCD phase transition. Rather, the physical axion emerges directly at the electroweak phase transition, when Higgs-axion mixing occurs and the physical states with axion component is identified.

Our analysis represents, more generally, a description of the fate of the Stückelberg field in cosmology, from the defining Stückelberg phase of the theory at a large scale (defined by the value of the Stückelberg mass) down to the electroweak symmetry breaking scale, when this field appears as a component of a physical state.

We have pointed out that the dark matter production in this setup cannot account for a sizable fraction of the observed dark matter density. This is related to the fact that the relic density is defined by the electroweak scale while in the PQ case the same role is played by the axion decay constant, which is several orders of magnitude bigger than the electroweak scale. We have also shown that the axion decay cannot be suppressed by increasing the Stückelberg mass since the Yukawa interactions that the axi-higgs inherits from its Higgs components always

give an unsuppressed decay channel. So the only possible suppression of the decay rate comes from the mass of the axi-higgs.

## The USSM-A

The USSM-A is a non-minimal supersymmetric extension of the Standard Model. Its main feature is the presence of an extra anomalous  $U(1)$  gauge group factor. The charges of the fields under this extra symmetry are not chosen in order to cancel the gauge anomalies by charge assignment but a Wess-Zumino mechanism for the cancellation of the anomalies is introduced. This mechanism requires the presence of a Stückelberg axion, a field that shifts under a gauge transformation and takes an important role in the mechanism that generates the mass of the extra neutral gauge boson associated to the extra  $U(1)$ .

The model shares some features with another non-minimal extension that includes an extra  $U(1)$  symmetry that is non-anomalous by charge assignment, the USSM[74–76]. Another non-minimal extension with an extra anomalous  $U(1)$  can be found in[77–79]. The supersymmetric Stückelberg lagrangian has been introduced in[80, 81].

In the sections that follow we will analyze the model focusing on the calculation of the relic density of dark matter. We will see that we can investigate two dark matter sources, namely, the thermal production of a population of relic neutralinos and the non-thermal production of a population of “cold” scalar particles. This chapter is based on[82, 83].

### 5.1 Definitions and conventions

The gauge structure of the model is given by  $SU(3) \times SU(2) \times U(1)_Y \times U(1)_B$ , where  $U(1)_B$  represents the gauge group associated to the anomalous symmetry while the remaining group factor is the SM gauge group. In all the Lagrangians below we implicitly sum over the three lepton and quarks generations. A list of the fundamental superfields and charge assignments can be found in Tab. 5.1. The components of the superfields appearing in the model are listed in Tab. 5.2. See Sec. 2.4 for the definition of chiral and vector superfields and some details on the construction of supersymmetric Lagrangians.

Superfields	SU(3)	SU(2)	$U(1)_Y$	$U(1)_B$
$\hat{\mathbf{b}}(x, \theta, \bar{\theta})$	<b>1</b>	<b>1</b>	0	$s$
$\hat{S}(x, \theta, \bar{\theta})$	<b>1</b>	<b>1</b>	0	$B_S$
$\hat{L}(x, \theta, \bar{\theta})$	<b>1</b>	<b>2</b>	-1/2	$B_L$
$\hat{R}(x, \theta, \bar{\theta})$	<b>1</b>	<b>1</b>	1	$B_R$
$\hat{Q}(x, \theta, \bar{\theta})$	<b>3</b>	<b>2</b>	1/6	$B_Q$
$\hat{U}_R(x, \theta, \bar{\theta})$	$\bar{\mathbf{3}}$	<b>1</b>	-2/3	$B_{U_R}$
$\hat{D}_R(x, \theta, \bar{\theta})$	$\bar{\mathbf{3}}$	<b>1</b>	+1/3	$B_{D_R}$
$\hat{H}_1(x, \theta, \bar{\theta})$	<b>1</b>	<b>2</b>	-1/2	$B_{H_1}$
$\hat{H}_2(x, \theta, \bar{\theta})$	<b>1</b>	<b>2</b>	1/2	$B_{H_2}$

**Table 5.1:** Charge assignment of the model; the symbol “s” in place of the  $U(1)_B$  charge of the axion superfield represents the fact that the field shifts under a  $U(1)_B$  gauge transformation.

The Lagrangian can be expressed as

$$\mathcal{L}_{USSM-A} = \mathcal{L}_{USSM} + \mathcal{L}_b \quad (5.1)$$

where the contribution of the terms depending on the Stückelberg axion superfield are included in  $\mathcal{L}_b$ . The first term is given by

$$\mathcal{L}_{USSM} = \mathcal{L}_{lep} + \mathcal{L}_{quark} + \mathcal{L}_{Higgs} + \mathcal{L}_{gauge} + \mathcal{L}_{SMT} + \mathcal{L}_{GMT} \quad (5.2)$$

with contributions from the usual supersymmetric lagrangian terms for leptons, quarks, Higgs, gauge fields and the SUSY breaking terms. These contributions are defined in terms of appropriate integrals over the Grassmann coordinates of the superspace  $(\theta, \bar{\theta})$  and polynomials in the superfields. The Lagrangian terms containing kinetic and gauge interaction terms for leptons and quarks are defined as

$$\mathcal{L}_{lep} = \int d^4\theta \left[ \hat{L}^\dagger e^{2g_s \hat{W} + g_Y \hat{Y} + g_B \hat{B}} \hat{L} + \hat{R}^\dagger e^{g_Y \hat{Y} + g_B \hat{B}} \hat{R} \right] \quad (5.3)$$

$$\mathcal{L}_{quark} = \int d^4\theta \left[ \hat{Q}^\dagger e^{2g_s \hat{G} + 2g_2 \hat{W} + g_Y \hat{Y} + g_B \hat{B}} \hat{Q} + \hat{U}_R^\dagger e^{2g_s \hat{G} + g_Y \hat{Y} + g_B \hat{B}} \hat{U}_R + \hat{D}_R^\dagger e^{2g_s \hat{G} + g_Y \hat{Y} + g_B \hat{B}} \hat{D}_R \right] \quad (5.4)$$

where  $g_s$ ,  $g_2$ ,  $g_Y$  and  $g_B$  represent the gauge couplings of the model. The vector superfields appearing in the exponentials are defined in terms of the anti-commuting coordinates  $(\theta, \bar{\theta})$ . So, given the anti-commutation properties of the Grassmann variables, the exponentials appearing in the lagrangians just defined can be expanded in a finite Taylor series. This means that the Berezin integrals are actually performed on polynomials in the superfields.

The Higgs sector consists of two  $SU(2)$  doublets, denoted as  $\hat{H}_1$  and  $\hat{H}_2$ , and a superfield  $\hat{S}$

Superfield	Bosonic	Fermionic	Auxiliary
$\hat{\mathbf{b}}(x, \theta, \bar{\theta})$	$b(x)$	$\psi_b(x)$	$F_b(x)$
$\hat{S}(x, \theta, \bar{\theta})$	$S(x)$	$\tilde{S}(x)$	$F_S(x)$
$\hat{L}(x, \theta, \bar{\theta})$	$\tilde{L}(x)$	$L(x)$	$F_L(x)$
$\hat{R}(x, \theta, \bar{\theta})$	$\tilde{R}(x)$	$\bar{R}(x)$	$F_R(x)$
$\hat{Q}(x, \theta, \bar{\theta})$	$\tilde{Q}(x)$	$Q(x)$	$F_Q(x)$
$\hat{U}_R(x, \theta, \bar{\theta})$	$\tilde{U}_R(x)$	$\bar{U}_R(x)$	$F_{U_R}(x)$
$\hat{D}_R(x, \theta, \bar{\theta})$	$\tilde{D}_R(x)$	$\bar{D}_R(x)$	$F_{D_R}(x)$
$\hat{H}_1(x, \theta, \bar{\theta})$	$H_1(x)$	$\tilde{H}_1(x)$	$F_{H_1}(x)$
$\hat{H}_2(x, \theta, \bar{\theta})$	$H_2(x)$	$\tilde{H}_2(x)$	$F_{H_2}(x)$
$\hat{B}(x, \theta, \bar{\theta})$	$B_\mu(x)$	$\lambda_B(x)$	$D_B(x)$
$\hat{Y}(x, \theta, \bar{\theta})$	$A_\mu^Y(x)$	$\lambda_Y(x)$	$D_Y(x)$
$\hat{W}^i(x, \theta, \bar{\theta})$	$W_\mu^i(x)$	$\lambda_{W^i}(x)$	$D_{W^i}(x)$
$\hat{G}^a(x, \theta, \bar{\theta})$	$G_\mu^a(x)$	$\lambda_{G^a}(x)$	$D_{G^a}(x)$

**Table 5.2:** Superfields and their components. The last four superfields are vector superfields while the remaining ones a left chiral superfields.

which is a singlet under the SM gauge group. The doublets are analogous to those found in the MSSM while the singlet, which is charged under the extra  $U(1)$ , takes a role in the generation of the  $\mu$ -term found in the MSSM and in the Higgs mechanism that breaks the extra  $U(1)$  symmetry. The same symmetry is also broken by the Stückelberg mechanism and we will comment on the consequences of this in Sec. 5.1.3.

From the list of the superfield charges in Tab. 5.1 we notice that the two Higgs doublets have opposite hypercharge. The presence of two Higgs doublets is necessary since the superpotential, in order to be invariant under supersymmetry transformations, has to be an holomorphic function of the superfields, i.e., it cannot be defined using both a superfield and its complex conjugate. So, if we want to give mass to the “up” and “down” fermions through the Yukawa couplings, we need two different superfields with opposite hypercharges.

The introduction of the singlet superfield  $\hat{S}$  is common to several extensions of the MSSM. This field is introduced in order to give a solution to the so-called  $\mu$ -problem: the MSSM superpotential is characterized by the mass parameter  $\mu$ ; this has to be in the range  $10^2 - 10^3$  GeV, that is very close to the SUSY breaking scale, in order to realize the expected electroweak symmetry breaking in that model. But this poses a problem, since  $\mu$  is not a SUSY breaking parameter and its not clear why its value should be close to the SUSY breaking scale.

Furthermore, apart from being part of a possible solution of the  $\mu$ -problem, the singlet superfield  $\hat{S}$  is introduced in the USSM[74], a supersymmetric extensions of the SM that include an extra  $U(1)$  gauge symmetry, in order to be able to break the extra gauge symmetry through the Higgs mechanism and obtain an extra neutral gauge boson. In this case the singlet field is a

singlet under the SM gauge group but is charged under the extra  $U(1)$ .

The kinetic and interaction terms for the Higgs fields are given by

$$\mathcal{L}_{Higgs} = \int d^4\theta \left[ \hat{H}_1^\dagger e^{2g_2\hat{W}+g_Y\hat{Y}+g_B\hat{B}} \hat{H}_1 + \hat{H}_2^\dagger e^{2g_2\hat{W}+g_Y\hat{Y}+g_B\hat{B}} \hat{H}_2 + \hat{S}^\dagger e^{g_B\hat{B}} \hat{S} + \mathcal{W} \delta^2(\bar{\theta}) + \bar{\mathcal{W}} \delta^2(\theta) \right] \quad (5.5)$$

where  $\mathcal{W}$  denotes the superpotential which is given by

$$\mathcal{W} = \lambda \hat{S} \hat{H}_1 \cdot \hat{H}_2 + y_e \hat{H}_1 \cdot \hat{L} \hat{R} + y_d \hat{H}_1 \cdot \hat{Q} \hat{D}_R + y_u \hat{H}_2 \cdot \hat{Q} \hat{U}_R. \quad (5.6)$$

This superpotential is very close to the MSSM superpotential, with the  $\mu$ -term  $\mu \hat{H}_1 \cdot \hat{H}_2$  replaced by  $\lambda \hat{S} \hat{H}_1 \cdot \hat{H}_2$  with  $\lambda$  a dimensionless coupling.

Some comments on the MSSM superpotential are required. If we only require gauge invariance and renormalizability we could include in the superpotential more terms but all of this terms violate baryon (B) or total lepton number (L) conservation by one unit. For example we could consider the terms

$$\mathcal{W}_L = \lambda^{ijk} \hat{L}_i \hat{L}_j \hat{R}_k \quad \mathcal{W}_B = \lambda'^{ijk} \hat{D}_{Ri} \hat{D}_{Rj} \hat{U}_{Rk} \quad (5.7)$$

where  $i, j, k$  represent family indices and  $\lambda, \lambda'$  are dimensionless couplings. These terms are experimentally forbidden since they would give, for example, a too fast decay rate for the proton. Analogous terms don't exist in the SM and would be represented by non-renormalizable terms. In order to solve this problem, a global symmetry, called  $R$ -parity or matter-parity, has been introduced in the MSSM. The quantum number associated to matter-parity is defined, for any superfield, as

$$P_M = (-1)^{3(B-L)}. \quad (5.8)$$

where  $B$  and  $L$  represent the baryon and lepton numbers of the superfield. It can be easily seen that lepton and quark superfields have  $P_M = -1$ , while Higgs and gauge superfields have  $P_M = +1$ . It can also be easily seen that the terms included in the superpotential have  $P_M = +1$  while the baryon and lepton number violating terms have  $P_M = -1$  thus, in order to avoid such terms, it is required that only positive matter-parity terms appear in the superpotential.

In terms of the particles that appear in the theory, matter-parity can be re-expressed in the form of a so-called  $R$ -parity, defined as

$$P_R = (-1)^{3(B-L)+2s} \quad (5.9)$$

where  $s$  represents the spin of the particle. Given this definition, it is easily understood that

the different components fields in a superfield have different  $R$ -parity charge, since their spins differ by  $1/2$  while the lepton and baryon number are the same for all of the fields in a supermultiplet. The interesting thing is that the SM particles and the Higgs bosons have  $P_R = +1$  while sfermions, gauginos and higgsinos, which are also called “sparticles” or “supersymmetric particles”, have  $P_R = -1$ . If we require  $R$ -parity conservation we can’t have any mixing between particles and sparticles and, furthermore, in the vertices of the theory the sparticles appear in even number. This feature easily results in the stability of the Lightest Supersymmetric Particle (LSP). In fact, according to what we have just said, a supersymmetric particle should decay producing an odd number of sparticles but, if the decaying particle is the lightest sparticle, this process is kinematically forbidden.

These considerations, referred to the MSSM superpotential, entirely translate to the superpotential that we are taking into account. So, even in this case, the LSP is stable and its relic abundance is a possible source of cold dark matter. We will consider the calculation of the relic densities of the LSP in Sec. 5.6.

The gauge kinetic terms are given by

$$\mathcal{L}_{gauge} = \frac{1}{4} \int d^4\theta \left[ \frac{1}{4g_s^2} \mathcal{G}^\alpha \mathcal{G}_\alpha + \frac{1}{4g_2^2} W^\alpha W_\alpha + W^{Y\alpha} W_\alpha^Y + W^{B\alpha} W_\alpha^B \right] \delta^2(\bar{\theta}) + h.c. \quad (5.10)$$

where the supersymmetric field strengths appear. These are defined in terms of the vector superfields and the supersymmetric covariant derivatives, for example

$$W_\alpha = -\frac{1}{4} \bar{D} \bar{D} e^{2g_2 \hat{V}} D_\alpha e^{-2g_2 \hat{V}} \quad (5.11)$$

with  $\hat{V} = T^a \hat{V}^a$ , where  $\hat{V}^a$  are vector superfields in the adjoint representation of  $SU(2)$ , and

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} - i \sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu \quad \bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i \theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu. \quad (5.12)$$

For the abelian field strength the definition is analogous, for example

$$W_\alpha^Y = -\frac{1}{4} \bar{D} \bar{D} D_\alpha V^Y \quad (5.13)$$

where  $V^Y$  represents the vector superfield associated to the hypercharge gauge group.

Since supersymmetry is not present at the electroweak scale, it has to be broken at a certain scale above the electroweak one; this scale is typically chosen to lie around the TeV scale. In order to realize this breaking in the lagrangian we include, as in the MSSM, mass terms for the sparticles and trilinear couplings that break supersymmetry explicitly. These terms are contained

in the Lagrangian terms

$$\begin{aligned} \mathcal{L}_{SMT} = & - \int d^4\theta \delta^4(\theta, \bar{\theta}) [M_L^2 \hat{L}^\dagger \hat{L} + m_R^2 \hat{R}^\dagger \hat{R} + M_Q^2 \hat{Q}^\dagger \hat{Q} + m_U^2 \hat{U}_R^\dagger \hat{U}_R + m_D^2 \hat{D}_R^\dagger \hat{D}_R + \\ & m_1^2 \hat{H}_1^\dagger \hat{H}_1 + m_2^2 \hat{H}_2^\dagger \hat{H}_2 + m_S^2 \hat{S}^\dagger \hat{S} + (a_\lambda \hat{S} \hat{H}_1 \cdot \hat{H}_2 + h.c.) + (a_e \hat{H}_1 \cdot \hat{L} \hat{R} + h.c.) + \\ & (a_d \hat{H}_1 \cdot \hat{Q} \hat{D}_R + h.c.) + (a_u \hat{H}_2 \cdot \hat{Q} \hat{U}_R + h.c.)] \end{aligned} \quad (5.14)$$

$$\begin{aligned} \mathcal{L}_{GMT} = & - \int d^4\theta \left[ \frac{1}{2} (M_G G^\alpha G_\alpha + M_W W^\alpha W_\alpha + M_Y W^{Y\alpha} W_\alpha^Y + M_B W^{B\alpha} W_\alpha^B + \right. \\ & \left. M_{YB} W^{Y\alpha} W_\alpha^B) + h.c. \right] \delta^4(\theta, \bar{\theta}) \end{aligned} \quad (5.15)$$

The parameters  $M_L, M_Q, m_R, m_{U_R}, m_{D_R}, m_1, m_2, m_S$  are the mass parameters of the explicit supersymmetry breaking, while  $a_e, a_\lambda, a_u, a_d$  are couplings with mass dimension one. The gauge mass terms include the MSSM gaugino mass terms parameterized by  $M_G, M_W, M_Y$ , a mass term for the gaugino associated to the extra  $U(1)$  symmetry introduced and a mixed gaugino mass term between the hypercharge and extra  $U(1)$  gauginos.

The interactions and dynamics of the axion superfield are defined in  $\mathcal{L}_b$ . This Lagrangian contains the Stückelberg term  $\mathcal{L}_{St}$ , the Wess-Zumino term  $\mathcal{L}_{WZ}$  and a SUSY breaking term  $\mathcal{L}_{bMT}$ . The first one contains the kinetic terms for the components of the axion superfield and the Stückelberg mass term for the extra gauge boson. The second term describes the Wess-Zumino interactions that are required for the anomaly cancellation while the third and last term contains the SUSY breaking terms for the axion superfield components. These terms are defined by the following integrals over the Grassmann coordinates

$$\begin{aligned} \mathcal{L}_{St} &= \frac{1}{2} \int d^4\theta (\hat{\mathbf{b}} + \hat{\mathbf{b}}^\dagger + \text{sqr}t{2} M_{St} \hat{B})^2 \\ \mathcal{L}_{WZ} &= -\frac{1}{2} \int d^4\theta \left\{ \left[ \frac{c_G}{M_{St}} \text{Tr}(GG) \hat{\mathbf{b}} + \frac{c_W}{M_{St}} \text{Tr}(WW) \hat{\mathbf{b}} \right. \right. \\ & \quad \left. \left. + \frac{c_Y}{M_{St}} \hat{\mathbf{b}} W_\alpha^Y W^{Y,\alpha} + \frac{c_B}{M_{St}} \hat{\mathbf{b}} W_\alpha^B W^{B,\alpha} + \frac{c_{YB}}{M_{St}} \hat{\mathbf{b}} W_\alpha^Y W^{B,\alpha} \right] \delta(\bar{\theta}^2) + h.c. \right\}. \\ \mathcal{L}_{bMT} &= -\frac{1}{2} m_{\text{Re}b}^2 \text{Re}b^2 - \frac{1}{2} M_{\psi_b} \bar{\psi}_b \psi_b \end{aligned} \quad (5.16)$$

We define the scalar component of the axion superfield  $\hat{b}$  in terms of a real and an imaginary part and we will refer to the corresponding fields as to the ‘‘saxion’’ and the ‘‘axion’’ respectively. These fields are defined as

$$b = \frac{1}{\sqrt{2}} (\text{Re}b + i \text{Im}b) \quad (5.17)$$

then we can express  $\mathcal{L}_{St}$  in terms of component fields as

$$\begin{aligned} \mathcal{L}_{St} = & \frac{1}{2} \left( \partial_\mu \text{Im } b + M_{St} B_\mu \right)^2 + \frac{1}{2} \partial_\mu \text{Re } b \partial^\mu \text{Re } b + \frac{i}{2} \psi_b \sigma^\mu \partial_\mu \psi_b + \frac{i}{2} \bar{\psi}_b \bar{\sigma}^\mu \partial_\mu \bar{\psi}_b + F_b^\dagger F_b + \\ & M_{St} \text{Re } b D_B + M_{St} (i \psi_b \lambda_B + h.c.) \end{aligned} \quad (5.18)$$

while for the WZ terms we get

$$\begin{aligned} \mathcal{L}_{WZ} = & \frac{c_G}{M_{St}} \left( \frac{1}{16} F_b \lambda_G^a \lambda_G^a + \frac{i}{8\sqrt{2}} D_G^a \lambda_G^a \psi_b - \frac{1}{16\sqrt{2}} G_{\mu\nu}^a \psi_b \sigma^\mu \bar{\sigma}^\nu \lambda_G^a + \frac{1}{64\sqrt{2}} \text{Im } b \tilde{G}^{a\mu\nu} G_{\mu\nu}^a + \right. \\ & \left. \frac{1}{8\sqrt{2}} \text{Im } b \lambda_G^a \sigma_\mu D^\mu \bar{\lambda}_G^a + \frac{1}{16\sqrt{2}} \text{Re } b D_G^a D_G^a - \frac{1}{16\sqrt{2}} \text{Re } b G^{a\mu\nu} G_{\mu\nu}^a - \frac{i}{8\sqrt{2}} \text{Re } b \lambda_G^a \sigma_\mu D^\mu \bar{\lambda}_G^a + h.c. \right) + \\ & \frac{c_W}{M_{St}} \left( \frac{1}{16} F_b \lambda_W^i \lambda_W^i + \frac{i}{8\sqrt{2}} D_W^i \lambda_W^i \psi_b - \frac{1}{16\sqrt{2}} W_{\mu\nu}^i \psi_b \sigma^\mu \bar{\sigma}^\nu \lambda_W^i + \frac{1}{64\sqrt{2}} \text{Im } b \tilde{W}^{i\mu\nu} W_{\mu\nu}^i + \right. \\ & \left. \frac{1}{8\sqrt{2}} \text{Im } b \lambda_W^i \sigma_\mu D^\mu \bar{\lambda}_W^i + \frac{1}{16\sqrt{2}} \text{Re } b D_W^i D_W^i - \frac{1}{16\sqrt{2}} \text{Re } b W^{i\mu\nu} W_{\mu\nu}^i - \frac{i}{8\sqrt{2}} \text{Re } b \lambda_W^i \sigma_\mu D^\mu \bar{\lambda}_W^i + h.c. \right) + \\ & \frac{c_Y}{M_{St}} \left( \frac{1}{2} F_b \lambda_Y \lambda_Y + \frac{i}{\sqrt{2}} D_Y \lambda_Y \psi_b - \frac{1}{2\sqrt{2}} F_{\mu\nu}^Y \psi_b \sigma^\mu \bar{\sigma}^\nu \lambda_Y + \frac{1}{8\sqrt{2}} \text{Im } b \tilde{F}^{Y\mu\nu} F_{\mu\nu}^Y - \frac{1}{\sqrt{2}} \text{Im } b \lambda_Y \sigma_\mu \partial^\mu \bar{\lambda}_Y + \right. \\ & \left. \frac{1}{2\sqrt{2}} \text{Re } b D_Y D_Y - \frac{1}{2\sqrt{2}} \text{Re } b F^{Y\mu\nu} F_{\mu\nu}^Y - \frac{i}{\sqrt{2}} \text{Re } b \lambda_Y \sigma_\mu \partial^\mu \bar{\lambda}_Y + h.c. \right) + \\ & \frac{c_B}{M_{St}} \left( \frac{1}{2} F_b \lambda_B \lambda_B + \frac{i}{\sqrt{2}} D_B \lambda_B \psi_b - \frac{1}{2\sqrt{2}} F_{\mu\nu}^B \psi_b \sigma^\mu \bar{\sigma}^\nu \lambda_B + \frac{1}{8\sqrt{2}} \text{Im } b \tilde{F}^{B\mu\nu} F_{\mu\nu}^B - \frac{1}{\sqrt{2}} \text{Im } b \lambda_B \sigma_\mu \partial^\mu \bar{\lambda}_B + \right. \\ & \left. \frac{1}{2\sqrt{2}} \text{Re } b D_B D_B - \frac{1}{2\sqrt{2}} \text{Re } b F^{B\mu\nu} F_{\mu\nu}^B - \frac{i}{\sqrt{2}} \text{Re } b \lambda_B \sigma_\mu \partial^\mu \bar{\lambda}_B + h.c. \right) + \\ & \frac{c_{YB}}{M_{St}} \left( -\frac{1}{2} F_b \lambda_B \lambda_Y - \frac{i}{2\sqrt{2}} D_Y \lambda_B \psi_b - \frac{i}{2\sqrt{2}} D_B \lambda_Y \psi_b + \frac{1}{4\sqrt{2}} F_{\mu\nu}^B \psi_b \sigma^\mu \bar{\sigma}^\nu \lambda_Y + \frac{1}{4\sqrt{2}} F_{\mu\nu}^Y \psi_b \sigma^\mu \bar{\sigma}^\nu \lambda_B - \right. \\ & \left. \frac{1}{8\sqrt{2}} \text{Im } b \tilde{F}^{Y\mu\nu} F_{\mu\nu}^B + \frac{1}{2\sqrt{2}} \text{Im } b \lambda_Y \sigma_\mu \partial^\mu \bar{\lambda}_B + \frac{1}{2\sqrt{2}} \text{Im } b \lambda_B \sigma_\mu \partial^\mu \bar{\lambda}_Y - \right. \\ & \left. \frac{1}{2\sqrt{2}} \text{Re } b D_Y D_B + \frac{1}{2\sqrt{2}} \text{Re } b F^{Y\mu\nu} F_{\mu\nu}^B - \frac{i}{2\sqrt{2}} \text{Re } b \lambda_Y \sigma_\mu \partial^\mu \bar{\lambda}_B - \frac{i}{2\sqrt{2}} \text{Re } b \lambda_B \sigma_\mu \partial^\mu \bar{\lambda}_Y + h.c. \right). \end{aligned} \quad (5.19)$$

The Lagrangian  $\mathcal{L}_{St}$  is invariant under the  $U(1)_B$  gauge transformations defined by

$$\begin{aligned} \delta_B \hat{B} &= \hat{\Lambda} + \hat{\Lambda}^\dagger \\ \delta_B \hat{b} &= -2M_{St} \hat{\Lambda} \end{aligned} \quad (5.20)$$

where  $\hat{\Lambda}$  is an arbitrary chiral superfield. So the scalar component of  $\hat{b}$ , that consists of the saxion and the axion fields, shifts under a  $U(1)_B$  gauge transformation.

The coefficients  $c_I \equiv (c_G, c_W, c_Y, c_B, c_{YB})$  appearing in the WZ Lagrangian are fixed by the conditions of gauge invariance and are functions of the charges of the fields appearing in the model.

Extracting the group factors we have

$$\begin{aligned} c_B &= -\frac{\mathcal{A}_{BBB}}{384\pi^2} & c_Y &= -\frac{\mathcal{A}_{BYY}}{128\pi^2} & c_{YB} &= -\frac{\mathcal{A}_{BBY}}{128\pi^2} \\ c_W &= -\frac{\mathcal{A}_{BWW}}{64\pi^2} & c_G &= -\frac{\mathcal{A}_{BGG}}{64\pi^2}. \end{aligned} \quad (5.21)$$

The coefficients  $\mathcal{A}$  are defined by the conditions of gauge invariance of the effective action and are related, respectively, to the anomalies

$$\{U(1)_B^3\} \quad \{U(1)_B, U(1)_Y^2\} \quad \{U(1)_B^2, U(1)_Y\} \quad \{U(1)_B, SU(2)^2\} \quad \{U(1)_B, SU(3)^2\}. \quad (5.22)$$

These relations appear in the anomalous variation ( $\delta_B$ ) of the supersymmetric 1-loop effective action of the model, which forces the introduction of supersymmetric PQ-like interactions, i.e. the WZ terms, for its overall vanishing. Formally we have the relation

$$\delta_B(B_i)\mathcal{S}_{1loop} + \delta_B(c_I(B_i))\mathcal{S}_{WZ} = 0, \quad (5.23)$$

where the anomalous variation can be parameterized by the 4 charges  $B_i$  together with the coefficients  $c_I(B_i)$  in front of the WZ counter-terms. In these notations, the index  $I$  runs over all the 5 mixed-anomaly conditions  $B^3, BY^2, B^2Y, BW^2, BG^2$ .

These coefficients assume the form

$$\begin{aligned} \mathcal{A}_{BBB} &= -3B_{H_1}^3 - 3B_{H_1}^2(3B_L + 18B_Q - 7B_S) - 3B_{H_1}[3B_L^2 + B_S(18B_Q - 7B_S)] \\ &\quad + 3B_L^3 + B_S(27B_Q^2 - 27B_S B_Q + 8B_S^2) \\ \mathcal{A}_{BYY} &= \frac{1}{2}(-3B_L - 9B_Q + 7B_S) \\ \mathcal{A}_{BBY} &= 2B_{H_1}(3B_L + 9B_Q - 5B_S) + B_S(12B_Q - 5B_S) \\ \mathcal{A}_{BWW} &= \frac{1}{2}(3B_L + 9B_Q - B_S) \\ \mathcal{A}_{BGG} &= \frac{3}{2}B_S. \end{aligned} \quad (5.24)$$

We have expressed all the anomaly equations in terms of only 4 charges,  $B_i \equiv (B_{H_1}, B_S, B_Q, B_L)$ , which are defined in Tab. 5.1. This has been done using the relations on the charges obtained requiring the gauge invariance of the superpotential. These charges can be taken as fundamental parameters of the model. Their independent variation allows to scan the entire spectra of these models with no reference to any specific construction.

Before coming to the definition of the charge assignments we pause for a remark. As we are going to show in the next sections, the scalar potential takes a non-local form unless all the

anomaly coefficients in Eq. (5.24) are zero. Such potential can however be expanded, in a low energy description, in powers of  $\text{Re}b/M_{St}$  assuming that the Stückelberg scale is higher than the SUSY breaking scale.

We define the function which allows to identify all the charges in terms of the free ones as  $f$ . This is formally given by

$$f(B_Q, B_L, B_{H_1}, B_S) = (B_Q, B_{U_R}, B_{D_R}, B_L, B_R, B_{H_1}, B_{H_2}, B_S). \quad (5.25)$$

In the numerical analysis, the charges of Eq. (5.25) will be assigned as

$$f(2, 1, -1, 3) = (2, 0, -1, 1, 0, -1, -2, 3) \quad (5.26)$$

and we will comment on some possible variation on these values.

### 5.1.1 Equations of motion for the $D$ and $F$ fields

The lagrangian defined so far contains terms that involve  $D$  and  $F$  fields. As usual in the formulation of supersymmetric lagrangians there are no kinetic terms for these fields if we consider the expansion in component fields. So we can express these fields in terms of the other fields of the model using their equations of motion and eliminate them from the lagrangian. This procedure is a common procedure in the MSSM and its extensions. In our case the procedure is straightforward for the  $F$  fields but for the  $D$  fields it is more involved due to the presence of the Wess-Zumino supersymmetric lagrangian. As we can see in the component expansion of the WZ lagrangian in Eq. (5.19), we have terms that involve two  $D$  fields and the real part of the scalar component of the axion superfield,  $\text{Re}b$ , which we have called saxion. If we consider the variation of the action with respect to the D-fields and solve the obtained equations for these, we get non-polynomial expressions

$$\begin{aligned} D_{W^i, OS} &= \frac{1}{16 + 4\frac{c_W}{M_{St}}\text{Re}b} \left[ i\sqrt{2}\frac{c_W}{M_{St}} (\bar{\psi}_b \bar{\lambda}_W^i - \lambda_W^i \psi_b) + \right. \\ &\quad \left. 16g_2 (H_u^\dagger \tau^i H_u + H_d^\dagger \tau^i H_d + \tilde{Q}_L^\dagger \tau^i \tilde{Q}_L + \tilde{L}^\dagger \tau^i \tilde{L}) \right] \\ D_{G^a, OS} &= \frac{1}{16 + 4\frac{c_G}{M_{St}}\text{Re}b} \left[ i\sqrt{2}\frac{c_G}{M_{St}} (\bar{\psi}_b \bar{\lambda}_G^a - \lambda_G^a \psi_b) + \right. \\ &\quad \left. 16g_s (\tilde{Q}_L^\dagger T^a \tilde{Q}_L + \tilde{U}_R^\dagger T^a \tilde{U}_R + \tilde{D}_R^\dagger T^a \tilde{D}_R) \right], \end{aligned} \quad (5.27)$$

where the sum over lepton generations and quark families is understood.

The EOM are more involved when we consider the abelian D-fields since in this case in the

WZ lagrangian we have a term that mixes  $D_Y$  and  $D_B$ . The EOM are given by

$$\begin{aligned}
D_{B,OS} &= \frac{1}{12 + 12\sqrt{2}\text{Re}b(c_B + c_Y)/M_{St} - 6\text{Re}b^2(c_{YB}^2 - 4c_Y c_B)/M_{St}^2} \\
&\left\{ \left[ 2 \frac{c_B}{M_{St}} \left( \sqrt{2} + 2 \frac{c_Y}{M_{St}} \text{Re}b \right) - \frac{c_{YB}^2}{M_{St}^2} \text{Re}b \right] (-3i\lambda_B \psi_{\mathbf{b}} + h.c.) - 12M_{St} \text{Re}b - 12\sqrt{2}c_Y \text{Re}b^2 + \right. \\
&12g_B \left( 1 + \sqrt{2} \frac{c_Y}{M_{St}} \text{Re}b \right) (B_S S^\dagger S + B_{H_1} H_1^\dagger H_1 + B_{H_2} H_2^\dagger H_2 + B_{D_R} \tilde{D}_R^\dagger \tilde{D}_R + B_{U_R} \tilde{U}_R^\dagger \tilde{U}_R + \\
&B_Q \tilde{Q}^\dagger \tilde{Q} + B_R \tilde{R}^\dagger \tilde{R} + B_L \tilde{L}^\dagger \tilde{L}) + 3\sqrt{2} \frac{c_{YB}}{M_{St}} (i\lambda_Y \psi_{\mathbf{b}} + h.c.) + \\
&\left. \sqrt{2} \frac{c_{YB}}{M_{St}} g_Y \text{Re}b (-3H_1^\dagger H_1 + 3B_{H_2} H_2^\dagger H_2 + 2\tilde{D}_R^\dagger \tilde{D}_R - 4\tilde{U}_R^\dagger \tilde{U}_R + \tilde{Q}^\dagger \tilde{Q} + 6\tilde{R}^\dagger \tilde{R} - 3\tilde{L}^\dagger \tilde{L}) \right\} \\
D_{Y,OS} &= \frac{1}{12 + 12\sqrt{2}\text{Re}b(c_B + c_Y)/M_{St} - 6\text{Re}b^2(c_{YB}^2 - 4c_Y c_B)/M_{St}^2} \\
&\left\{ 3 \frac{c_{YB}^2}{M_{St}^2} \text{Re}b (i\lambda_Y \psi_{\mathbf{b}} + h.c.) + 3\sqrt{2} \frac{c_{YB}}{M_{St}} (i\lambda_Y \psi_{\mathbf{b}} + h.c.) - 6\sqrt{2} c_{YB} \text{Re}b^2 + \right. \\
&6\sqrt{2} \frac{c_{YB}}{M_{St}} \text{Re}b g_B (B_S S^\dagger S + B_{H_1} H_1^\dagger H_1 + B_{H_2} H_2^\dagger H_2 + B_{D_R} \tilde{D}_R^\dagger \tilde{D}_R + B_{U_R} \tilde{U}_R^\dagger \tilde{U}_R + \\
&B_Q \tilde{Q}^\dagger \tilde{Q} + B_R \tilde{R}^\dagger \tilde{R} + B_L \tilde{L}^\dagger \tilde{L}) - 6 \frac{c_Y}{M_{St}} \left( \sqrt{2} + 2 \frac{c_B}{M_{St}} \text{Re}b \right) (i\lambda_Y \psi_{\mathbf{b}} + h.c.) + \\
&\left. 2g_B \left( 1 + \sqrt{2} \frac{c_B}{M_{St}} \text{Re}b \right) (-3H_1^\dagger H_1 + 3H_2^\dagger H_2 + 2\tilde{D}_R^\dagger \tilde{D}_R - 4\tilde{U}_R^\dagger \tilde{U}_R + \tilde{Q}^\dagger \tilde{Q} + 6\tilde{R}^\dagger \tilde{R} - 3\tilde{L}^\dagger \tilde{L}) \right\}, \tag{5.28}
\end{aligned}$$

A possible solution is to consider a Lagrangian expanded to zero order with respect to  $\text{Re}b/M_{St}$ . This choice is justified if we are interested in a lagrangian that describes the model at scales sensibly lower than the Stückelberg scale. This is the case, for example, of the neutralino relic density calculation. In fact, we will see that the relevant scale for this process is around  $10^2$  GeV while we will choose much bigger values for the Stückelberg scale.

### 5.1.2 The electroweak gauge sector

The electroweak gauge sector consists of the charged massive gauge boson  $W^\pm$ , the photon  $A$  and two massive neutral gauge bosons,  $Z$  and  $Z'$ . Apart from the  $Z'$ , the composition of the remaining fields in terms of the interaction eigenstates is very similar to the composition obtained in the SM.

The squared mass of the  $W^\pm$  gauge boson is given by

$$m_{W^\pm}^2 = \frac{1}{4} g_2^2 (v_1^2 + v_2^2) \tag{5.29}$$

and the corresponding charged Goldstone is identified in the charged Higgs sector as a massless eigenstate of the mass matrix of that sector.

The neutral sector consists of 3 degrees of freedom,  $(W_\mu^3, A_\mu^Y, B_\mu)$  and the corresponding squared mass matrix is given by

$$M_{ng}^2 = \begin{pmatrix} \frac{1}{4}g_2^2(v_1^2 + v_2^2) & -\frac{1}{4}g_2g_Y(v_1^2 + v_2^2) & \frac{1}{2}g_2g_B(B_{H_1}v_1^2 - B_{H_2}v_2^2) \\ \cdot & \frac{1}{4}g_Y^2(v_1^2 + v_2^2) & \frac{1}{2}g_Yg_B(-B_{H_1}v_1^2 + B_{H_2}v_2^2) \\ \cdot & \cdot & M_{St}^2 + g_B^2(v_1^2B_{H_1}^2 + v_2^2B_{H_2}^2 + v_S^2B_S^2) \end{pmatrix}. \quad (5.30)$$

This matrix has a null eigenvalue corresponding to the SM photon and two eigenvalues corresponding to the squared masses of the  $Z$  and  $Z'$ . The complete analytical expressions for the eigenstates and the eigenvalues for  $Z$  and  $Z'$  are quite complicated but with a simple numerical analysis we can extract the most important features. The light massive eigenstate has a mass and a composition in terms of the fields in the interaction basis which corresponds essentially to the SM  $Z$ . The heavy massive state has a mass that is essentially given by the Stückelberg mass and the corresponding eigenvector is almost completely given by the extra  $U(1)$  gauge field  $B_\mu$ . These results are a consequence of the choice  $M_{St} > v_S$  and we will always consider this situation in the numerical analysis. We do this since, while  $v_S$  is bound to give an effective  $\mu$  parameter in the expected range,  $M_{St}$  is in principle a free parameter of the theory and can assume high values.

The Goldstones needed for these gauge bosons to obtain a mass come from the CP-odd Higgs sector. These can't be simply identified considering the massless states obtained by the diagonalization of the CP-odd mass matrix since  $\text{Im}b$  clearly gives a contribution to the Goldstone of the  $Z'$  but it does not appear in the scalar potential. So the Goldstone directions in the CP-odd sector can be identified using an alternative approach, that is, considering the derivative couplings of the neutral gauge fields with the CP-odd states. The Lagrangian terms that describe such derivative couplings can be expressed as

$$\mathcal{L}_{der.coup.} = g_2W_\mu^3\partial^\mu G_{W^3} + g_YA_\mu^Y\partial^\mu G_Y + g_BB_\mu\partial^\mu G_B \quad (5.31)$$

where

$$\begin{aligned} G_{W^3} &= -\frac{1}{2}v_1\text{Im}H_1^1 + \frac{1}{2}v_2\text{Im}H_2^2 \\ G_Y &= \frac{1}{2}v_1\text{Im}H_1^1 - \frac{1}{2}v_2\text{Im}H_2^2 \\ G_B &= -v_1\text{Im}H_1^1 - v_2\text{Im}H_2^2 - v_S\text{Im}S + M_{St}\text{Im}b. \end{aligned} \quad (5.32)$$

The linear combinations  $G_{W^3}, G_Y, G_B$  define two independent directions in the CP-odd Higgs sector; these directions define the two dimensional neutral Goldstone space. The Goldstones for the  $Z$  and  $Z'$  will be the vectors of an orthonormal basis in this subspace.

In the basis  $(\text{Im}H_1^1, \text{Im}H_2^2, \text{Im}S, \text{Im}b)$  we can express the two Goldstone states as

$$\begin{aligned} G_1^0 &= \frac{\{-v_1, v_2, 0, 0\}}{\sqrt{v_1^2 + v_2^2}} \\ G_2^0 &= \frac{\{g_B B_S v_1 v_2^2, g_B B_S v_1^2 v_2, -g_B B_S v_S (v_1^2 + v_2^2), M_{St} (v_1^2 + v_2^2)\}}{\sqrt{g_B^2 B_S^2 [v_1^2 v_2^2 (v_1^2 + v_2^2) + v_S^2 (v_1^2 + v_2^2)^2] + M_{St}^2 (v_1^2 + v_2^2)^2}}. \end{aligned} \quad (5.33)$$

As we can see in the limit  $M_{St} \gg v_1, v_2, v_S$  the Goldstone  $G_2^0$  coincides essentially with the axion  $\text{Im}b$ . The same situation is obtained if  $B_S = 0$  that is  $B_{H_1} = -B_{H_2}$ ; in this case, since the singlet is not charged under the extra  $U(1)$ , the breaking of the extra  $U(1)$  is due only to the Stückelberg mechanism and obviously the Goldstone corresponding to the  $Z'$  coincides with  $\text{Im}b$ .

### 5.1.3 The Higgs sector

The Higgs sector of the model includes the Higgs doublets  $H_1$  and  $H_2$ , the singlet  $S$  and the  $b$  field. We introduce the following parameterization for these fields

$$\begin{aligned} H_1 &= \frac{1}{\sqrt{2}} \begin{pmatrix} \text{Re}H_1^0 + i \text{Im}H_1^0 \\ \text{Re}H_1^- + i \text{Im}H_1^- \end{pmatrix} & H_2 &= \frac{1}{\sqrt{2}} \begin{pmatrix} \text{Re}H_2^+ + i \text{Im}H_2^+ \\ \text{Re}H_2^0 + i \text{Im}H_2^0 \end{pmatrix} \\ S &= \frac{1}{\sqrt{2}} (\text{Re}S + i \text{Im}S) & b &= \frac{1}{\sqrt{2}} (\text{Re}b + i \text{Im}b) \end{aligned} \quad (5.34)$$

expanded around the vevs of the Higgs fields, of the singlet and of the axion as

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \quad \langle H_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \quad \langle S \rangle = \frac{v_S}{\sqrt{2}} \quad \langle b \rangle = \frac{v_b}{\sqrt{2}}. \quad (5.35)$$

We also define  $\tan\beta = v_2/v_1$ , as usual in the MSSM and in two doublet models.

In order to ensure that the scalar potential has a minimum we have to evaluate its first derivatives with respect to the fields that we have just defined and set these derivatives to zero. We get the following conditions from the derivatives with respect to the neutral real components

$$\begin{aligned} m_{H_1}^2 &= -\frac{1}{2} \lambda^2 (v_2^2 + v_S^2) + \frac{a_\lambda}{\sqrt{2}} \frac{v_S v_2}{v_1} + \frac{1}{8} (g_2^2 + g_Y^2) (v_2^2 - v_1^2) + g_B M_{St} v_b B_{H_1} \\ &\quad - \frac{1}{2} g_B^2 B_{H_1} (v_1^2 B_{H_1} + v_2^2 B_{H_2} + v_S^2 B_S) \end{aligned}$$

$$\begin{aligned}
m_{H_2}^2 &= -\frac{1}{2}\lambda^2(v_1^2 + v_S^2) + \frac{a_\lambda}{\sqrt{2}}\frac{v_S v_1}{v_2} - \frac{1}{8}(g_2^2 + g_Y^2)(v_2^2 - v_1^2) + g_B M_{St} v_b B_{H_2} \\
&\quad - \frac{1}{2}g_B^2 B_{H_2}(v_1^2 B_{H_1} + v_2^2 B_{H_2} + v_S^2 B_S) \\
m_S^2 &= -\frac{1}{2}\lambda^2(v_1^2 + v_2^2) + \frac{a_\lambda}{\sqrt{2}}\frac{v_1 v_2}{v_S} + g_B M_{St} v_b B_S - \frac{1}{2}g_B^2 B_S(v_1^2 B_{H_1} + v_2^2 B_{H_2} + v_S^2 B_S) \\
m_{\text{Re}b}^2 &= -M_{St}^2 + \frac{g_B M_{St}}{2v_b}(v_1^2 B_{H_1} + v_2^2 B_{H_2} + v_S^2 B_S)
\end{aligned} \tag{5.36}$$

while the remaining derivatives are identically zero.

In order to guarantee that the point parametrized by the vev's is a minimum we should ensure that the Hessian of the potential for these scalar fields is positive definite but this can't be done analytically in such a complex case. To address this point we will use an equivalent approach and choose the parameters in the numerical analysis in such a way that the spectrum in the Higgs sector consists of positive squared masses.

The Higgs sector can be divided in three sub-sectors according to the electric charge of the fields and the way in which they transform under CP.

- Charged Higgs sector: this sector consists of the states  $(\text{Re}H_2^1, \text{Re}H_1^2)$ . The mass matrix has a zero eigenvalue corresponding to a charged Goldstone boson and a mass eigenvalue corresponding to the charged Higgs mass

$$m_{H^\pm}^2 = \left(\frac{v_1}{v_2} + \frac{v_2}{v_1}\right) \left(\frac{1}{4}g_2^2 v_1 v_2 - \frac{1}{2}\lambda^2 v_1 v_2 + a_\lambda \frac{v_S}{\sqrt{2}}\right). \tag{5.37}$$

The eigenstates are obtained through a rotation defined by the angle  $\beta$ .

- CP-even sector: this sector consists of the states  $(\text{Re}H_1^1, \text{Re}H_2^2, \text{Re}S, \text{Re}b)$ . We define the squared mass matrix for this sector as  $M_{ev}^2$  and its entries are given by

$$\begin{aligned}
M_{ev11}^2 &= \frac{1}{4}v_1^2 \left(g^2 + 4g_B^2 B_{H_1}^2\right) + \frac{a_\lambda}{\sqrt{2}}\frac{v_2 v_S}{v_1} \\
M_{ev12}^2 &= -\frac{1}{4}v_1 v_2 \left(g^2 - 4g_B^2 B_{H_1} B_{H_2}\right) + \lambda^2 v_1 v_2 - \frac{a_\lambda}{\sqrt{2}}v_S \\
M_{ev13}^2 &= \lambda^2 v_1 v_S - \frac{a_\lambda}{\sqrt{2}}v_2 + g_B^2 v_1 v_S B_{H_1} B_S & M_{ev14}^2 &= -g_B B_{H_1} v_1 M_{St} \\
M_{ev22}^2 &= \frac{1}{4}v_2^2 \left(g^2 + 4g_B^2 B_{H_2}^2\right) + \frac{a_\lambda}{\sqrt{2}}\frac{v_1 v_S}{v_2} \\
M_{ev23}^2 &= \lambda^2 v_2 v_S - \frac{a_\lambda}{\sqrt{2}}v_1 + g_B^2 v_2 v_S B_{H_1} B_S & M_{ev24}^2 &= -g_B B_{H_2} v_2 M_{St} \\
M_{ev33}^2 &= \frac{a_\lambda}{\sqrt{2}}\frac{v_1 v_2}{v_S} + g_B^2 v_S^2 B_S^2 & M_{ev34}^2 &= -g_B B_S v_S M_{St} \\
M_{ev44}^2 &= \frac{g_B M_{St}}{2v_b} \left(v_1^2 B_{H_1} + v_2^2 B_{H_2} + v_S^2 B_S\right)
\end{aligned} \tag{5.38}$$

where  $g = \sqrt{g_2^2 + g_Y^2}$ .

Diagonalizing this matrix we obtain four massive states that we call  $H^i$ ,  $i = 1, \dots, 4$ . Given the complexity of the matrix we are not able to obtain analytical expressions for the eigenstates and eigenvalues. We can assign numerical values to some of the parameters appearing in the matrix using some experimental results: we express  $v_1$  and  $v_2$  in terms of  $v = \sqrt{v_1^2 + v_2^2}$  and  $\tan \beta$ , we fix  $v$  in order to obtain the correct masses for the  $Z$  and the  $W$  gauge boson, we set the gauge couplings  $g_2$  and  $g_Y$  to their low energy values using the experimental values for the weak angle  $\theta_W$  and the fine-structure constant  $\alpha$ .

In the numerical analysis we will make sure that the remaining parameters are chosen in such a way that the squared masses for these states are positive and have a value that satisfies the current experimental bounds.

- CP-odd sector: this sector consists of the fields  $(\text{Im}H_1^1, \text{Im}H_2^2, \text{Im}S, \text{Im}b)$ . The axion field does not appear in the potential so the mass matrix that we can extract from it is a  $4 \times 4$  matrix with no contributions coming from the axion field. It is given by

$$M_{\text{odd}}^2 = \frac{a_\lambda}{\sqrt{2}} \begin{pmatrix} \frac{v_2 v_S}{v_1} & v_S & v_2 & 0 \\ \cdot & \frac{v_1 v_S}{v_2} & v_1 & 0 \\ \cdot & \cdot & \frac{v_1 v_2}{v_S} & 0 \\ \cdot & \cdot & \cdot & 0 \end{pmatrix}. \quad (5.39)$$

From this matrix we get a massive state that we will call  $A^1$ ; this state has a squared mass given by

$$m_{A^1}^2 = \frac{a_\lambda}{\sqrt{2}} \left( \frac{v_1^2 v_2^2 + v_1^2 v_S^2 + v_2^2 v_S^2}{v_1 v_2 v_S} \right) \quad (5.40)$$

and the composition of this state in the basis  $(\text{Im}H_1^1, \text{Im}H_2^2, \text{Im}S, \text{Im}b)$  is given by

$$A^1 = \frac{\{v_2 v_S, v_1 v_S, v_1 v_2, 0\}}{\sqrt{v_2^2 v_S^2 + v_1^2 v_S^2 + v_1^2 v_2^2}}. \quad (5.41)$$

In this sector we expect to find the two neutral Goldstone states needed in the breaking of the electroweak and extra  $U(1)$  symmetry. These states can't be identified from the diagonalization of the mass matrix since this matrix doesn't contain any information on the axion but this state clearly gives a contribution in the definition of the neutral Goldstone subspace. So, in order to identify the remaining states in this sector, we consider the expressions for the Goldstones obtained in Eq. (5.33), the massive state obtained from the diagonalization of the mass matrix,  $A^1$ , and obtain the fourth and last state as the linear

combination that is orthogonal to the other three states. We obtain a massless state that we will call  $A^2$ . In terms of the interaction basis states this is given by

$$A^2 = \frac{\{-M_{St}v_1v_2^2, -M_{St}v_1^2v_2, M_{St}v_Sv^2, g_B B_S (v_1^2v_2^2 + v_S^2v^2)\}}{\sqrt{(v_1^2v_2^2 + v_S^2v^2) [M_{St}^2v^2 + g_B^2 B_S^2 (v_1^2v_2^2 + v_S^2v^2)]}}. \quad (5.42)$$

Given the axion component of this field we will refer to it as to the ‘‘axi-higgs’’. We will discuss about the possibility to introduce a potential and generate a mass for this state in Sec. 5.2.

#### 5.1.4 The neutralino sector

Now we turn to the neutralino sector; we define the mass matrix of this sector,  $M_{\chi^0}$ , in the basis  $(i\lambda_{W^3}, i\lambda_Y, i\lambda_B, \tilde{H}_1^1, \tilde{H}_2^2, \tilde{S}, \psi_b)$  and its non-null entries are given by

$$\begin{aligned} M_{\chi^0}^{11} &= M_W & M_{\chi^0}^{14} &= -\frac{g_2 v_1}{2} & M_{\chi^0}^{15} &= \frac{g_2 v_2}{2} \\ M_{\chi^0}^{22} &= M_Y & M_{\chi^0}^{23} &= M_{YB} & M_{\chi^0}^{24} &= \frac{g_Y v_1}{2} & M_{\chi^0}^{25} &= -\frac{g_Y v_2}{4} \\ M_{\chi^0}^{33} &= M_B & M_{\chi^0}^{34} &= -v_1 g_B B_{H_1} & M_{\chi^0}^{35} &= -v_2 g_B B_{H_2} & M_{\chi^0}^{36} &= -v_S g_B B_S & M_{\chi^0}^{37} &= M_{St} \\ M_{\chi^0}^{45} &= \frac{\lambda v_S}{\sqrt{2}} & M_{\chi^0}^{46} &= \frac{\lambda v_2}{\sqrt{2}} & M_{\chi^0}^{56} &= \frac{\lambda v_1}{\sqrt{2}} & M_{\chi^0}^{77} &= M_{\psi_b} \end{aligned} \quad (5.43)$$

The rotation matrix for this sector is implicitly defined as  $O^{\chi^0}$  and it allows us to express the interaction eigenstates in terms of the mass eigenstates

$$\begin{pmatrix} i\lambda_{w_3} \\ i\lambda_Y \\ i\lambda_B \\ \tilde{H}_1^1 \\ \tilde{H}_2^2 \\ \tilde{S} \\ \psi_b \end{pmatrix} = O^{\chi^0} \begin{pmatrix} \chi_0^0 \\ \chi_1^0 \\ \chi_2^0 \\ \chi_3^0 \\ \chi_4^0 \\ \chi_5^0 \\ \chi_6^0 \end{pmatrix} \quad (5.44)$$

where  $\chi_i^0, i = 0, 1, \dots, 6$  represent the mass eigenstates ordered according to increasing mass. In the numerical analysis we will see that the lightest neutralino represents, for the parameter choice we make, the lightest supersymmetric particle. This, combined with the R-symmetry of the superpotential (see Eq. (5.6) and the discussion that follows it), results in the fact that the lightest neutralino is stable. Since this particle, as all the other neutralinos, has only weak interactions and is massive it is a good candidate as the component of relic cold dark matter.

### 5.1.5 The chargino sector

The chargino sector in our model is not different from the same sector in the MSSM. The only difference is that the  $\mu$  parameter is substituted by the analogous parameter obtained using the singlet vev, namely,  $\lambda v_s/\sqrt{2}$ . We describe here the structure of the chargino sector and the diagonalization procedure for completeness. We define

$$\lambda_{w^+} = \frac{1}{\sqrt{2}}(\lambda_{w_1} - i\lambda_{w_2}) \quad \lambda_{w^-} = \frac{1}{\sqrt{2}}(\lambda_{w_1} + i\lambda_{w_2}) \quad (5.45)$$

and in the basis  $(\lambda_{w^+}, \tilde{H}_2^1, \lambda_{w^-}, \tilde{H}_1^2)$  we obtain the mass matrix

$$M_{\chi^\pm} = \begin{pmatrix} 0 & 0 & M_W & \frac{g_2 v_1}{\sqrt{2}} \\ 0 & 0 & \frac{g_2 v_2}{\sqrt{2}} & \frac{\lambda v_s}{\sqrt{2}} \\ M_W & \frac{g_2 v_2}{\sqrt{2}} & 0 & 0 \\ \frac{g_2 v_1}{\sqrt{2}} & \frac{\lambda v_s}{\sqrt{2}} & 0 & 0 \end{pmatrix}. \quad (5.46)$$

From the diagonalization we get the squared eigenvalues

$$m_{\chi_{1,2}^\pm}^2 = \frac{1}{4} \left[ 2M_W^2 + \lambda^2 v_s^2 + g_2^2 v^2 \mp \sqrt{(2M_W^2 + \lambda^2 v_s^2 + g_2^2 v^2)^2 - 4(\sqrt{2}\lambda v_s M_W - g_2^2 v_1 v_2)^2} \right] \quad (5.47)$$

If we define

$$\psi^+ = \begin{pmatrix} \lambda_{w^+} \\ \tilde{H}_2^1 \end{pmatrix} \quad \psi^- = \begin{pmatrix} \lambda_{w^-} \\ \tilde{H}_1^2 \end{pmatrix} \quad (5.48)$$

then the mass eigenstates can be defined as

$$\chi^+ = V\psi^+ \quad \chi^- = U\psi^- \quad (5.49)$$

where  $U$  and  $V$  are two unitary matrices that diagonalize the mass matrix of this sector. If we define

$$X = \begin{pmatrix} M_W & \frac{g_2 v_2}{\sqrt{2}} \\ \frac{g_2 v_1}{\sqrt{2}} & \frac{\lambda v_s}{\sqrt{2}} \end{pmatrix} \quad (5.50)$$

then these unitary matrices are defined in such a way that

$$VX^\dagger XV^{-1} = U^* X X^\dagger U^T = M_{\chi^\pm, \text{diag}}; \quad (5.51)$$

where  $M_{\chi^\pm,diag}^2$  is given by

$$M_{\chi^\pm,diag}^2 = \begin{pmatrix} m_{\chi_1^\pm}^2 & 0 \\ 0 & m_{\chi_2^\pm}^2 \end{pmatrix}. \quad (5.52)$$

### 5.1.6 The sleptons and squarks sector

The sleptons and squarks appear in the model as the supersymmetric partners of leptons and quarks. We have two squarks up, two squarks down and three sleptons for each generation. If we assume that the soft mass parameters for the sleptons and the squarks and the soft trilinear couplings are diagonal in the generation space, then the mass matrices assume a block diagonal form.

In the basis  $(\tilde{Q}_{Li}^1, \tilde{u}_{Ri})$ , where  $i = 1, 2, 3$  is a generation index, we get the mass matrix

$$\begin{aligned} M_{\tilde{u}_i 11}^2 &= \frac{1}{24} \left\{ 3 \left[ g_2^2 (v_1^2 - v_2^2) + 4g_B B_Q \left( -2M_{St} v_b + g_B v_1^2 B_{H_1} + g_B v_2^2 B_{H_2} + g_B v_S^2 B_S \right) \right] + \right. \\ &\quad \left. g_Y^2 (v_2^2 - v_1^2) + 24M_{Q_i}^2 + 12v_2^2 y_{u_i}^2 \right\} \\ M_{\tilde{u}_i 12}^2 &= \frac{v_2 a_{u_i}}{\sqrt{2}} - \frac{1}{2} \lambda v_1 v_S y_{u_i} \\ M_{\tilde{u}_i 22}^2 &= \frac{1}{6} \left[ g_Y^2 (v_2^2 - v_1^2) + 3g_B B_{U_R} \left( -2M_{St} v_b + g_B v_1^2 B_{H_1} + g_B v_2^2 B_{H_2} + g_B v_S^2 B_S \right) + \right. \\ &\quad \left. 6m_{U_i}^2 + 3v_2^2 y_{u_i}^2 \right]. \end{aligned} \quad (5.53)$$

From the diagonalization of this matrix we get six states which are the supersymmetric partners for the up, charm and top quarks.

Analogously, we can build the mass matrix for the superpartners of the down-type quarks. In the basis  $(\tilde{Q}_{Li}^2, \tilde{d}_{Ri})$  this will be given by

$$\begin{aligned} M_{\tilde{d}_i 11}^2 &= \frac{1}{24} \left\{ 3 \left[ g_2^2 (v_2^2 - v_1^2) + 4g_B B_Q \left( -2M_{St} v_b + g_B v_1^2 B_{H_1} + g_B v_2^2 B_{H_2} + g_B v_S^2 B_S \right) \right] + \right. \\ &\quad \left. g_Y^2 (v_2^2 - v_1^2) + 24M_{Q_i}^2 + 12v_1^2 y_{d_i}^2 \right\} \\ M_{\tilde{d}_i 12}^2 &= -\frac{v_1 a_{d_i}}{\sqrt{2}} + \frac{1}{2} \lambda v_2 v_S y_{d_i} \\ M_{\tilde{d}_i 22}^2 &= \frac{1}{12} \left[ g_Y^2 (v_2^2 - v_1^2) + 6g_B B_{D_R} \left( -2M_{St} v_b + g_B v_1^2 B_{H_1} + g_B v_2^2 B_{H_2} + g_B v_S^2 B_S \right) + \right. \\ &\quad \left. 12m_{D_i}^2 + 6v_1^2 y_{d_i}^2 \right] \end{aligned} \quad (5.54)$$

and the corresponding eigenstates describe the supersymmetric partners for the down, strange and bottom squarks.

Regarding the sleptons, in the basis  $(\tilde{L}_i^2, \tilde{e}_{Ri})$  we get the mass matrix

$$\begin{aligned}
M_{\tilde{l}_i 11}^2 &= \frac{1}{8} \left[ (g_Y^2 - g_2^2) (v_1^2 - v_2^2) + 4g_B B_L (-2M_{St} v_b + g_B v_1^2 B_{H_1} + g_B v_2^2 B_{H_2} + g_B v_S^2 B_S) + \right. \\
&\quad \left. 8M_{L_i}^2 + 4v_1^2 y_{l_i}^2 \right] \\
M_{\tilde{l}_i 12}^2 &= -\frac{v_1 a_{l_i}}{\sqrt{2}} + \frac{1}{2} \lambda v_2 v_S y_{l_i} \\
M_{\tilde{l}_i 22}^2 &= \frac{1}{4} \left[ g_Y^2 (v_2^2 - v_1^2) + 2g_B B_R (-2M_{St} v_b + g_B v_1^2 B_{H_1} + g_B v_2^2 B_{H_2} + g_B v_S^2 B_S) + 4m_{R_i}^2 + 2v_1^2 y_{l_i}^2 \right]
\end{aligned} \tag{5.55}$$

that gives the mass for the partners of the electron, muon and tau.

For the superpartners of the neutrinos we have the the mass eigenvalues

$$M_{\tilde{\nu}_i}^2 = \frac{1}{8} \left[ (g_Y^2 + g_2^2) (v_1^2 - v_2^2) + 4g_B B_L (-2M_{St} v_b + g_B v_1^2 B_{H_1} + g_B v_2^2 B_{H_2} + g_B v_S^2 B_S) + 8M_{L_i}^2 \right]. \tag{5.56}$$

The block diagonal form of this mass matrices is spoiled if we take into account the mixing among the down-type quarks parameterized by the Cabibbo-Kobayashi-Maskawa rotation matrix or the neutrino mixing parameterized by the Pontecorvo-Maki-Nakagawa-Sakata matrix. The effect of these mixings is anyway negligible for our purposes in fact, as we will see, in the calculation of the relic densities we will consider a neutralino whose mass is well below the mass of the squarks and of the sleptons.

We also have to point out the fact that the Stückelberg mass appears in the mass matrices for the sfermion sector, in combination with the saxion vev,  $v_b$ . We see that, depending on the charge assignment, the contribution of the Stückelberg mass can be opposite to the contribution coming from the soft breaking terms that, roughly, determine the size of the sfermion masses. Anyway, in the relic density calculation, we will see that the values of the Stückelberg mass that give an acceptable result are such that the spectrum in this sector is positive with eigenvalues well above the mass of the lightest neutralino.

## 5.2 The axi-higgs mass

In Sec. 5.1.3 we have shown how to isolate the physical massless state in the three dimensional CP-odd massless subspace of the Higgs sector. We have called this state  $A^2$ . Now we will comment about the possibility to introduce a scalar potential that generates a mass for this state.

As we have already done in Sec. 4.3, the contributions appearing in the extra potential  $V'$  are defined as the gauge invariant terms that can be build using the axion field  $\text{Im}b$  and the fields in

the Higgs sector. The obtained extra potential is given by

$$V' = \sum_{i=1}^6 V_i \quad (5.57)$$

where

$$\begin{aligned} V_1 &= a_1 S^4 e^{i4g_B B_S \frac{\text{Im}b}{M_{St}}} + h.c. \\ V_2 &= e^{ig_B B_S \frac{\text{Im}b}{M_{St}}} \left( a_2 H_1 \cdot H_2 S^2 + b_2 H_1^\dagger H_1 S + b_3 H_2^\dagger H_2 S + b_4 S^\dagger S^2 + d_1 S \right) + h.c. \\ V_3 &= e^{ig_B 2B_S \frac{\text{Im}b}{M_{St}}} \left( a_3 H_1^\dagger H_1 S^2 + a_4 H_2^\dagger H_2 S^2 + a_5 S^\dagger S^3 + c_1 S^2 \right) + h.c. \\ V_4 &= a_6 (H_1 \cdot H_2)^2 e^{-ig_B 2B_S \frac{\text{Im}b}{M_{St}}} + h.c. \\ V_5 &= b_1 S^3 e^{ig_B 3B_S \frac{\text{Im}b}{M_{St}}} + h.c. \\ V_6 &= c_2 H_1 \cdot H_2 e^{-ig_B B_S \frac{\text{Im}b}{M_{St}}} + h.c. \end{aligned} \quad (5.58)$$

In the expressions above we have grouped together terms that share the same phase factor. Notice that the parameters  $a_i$ ,  $b_j$ ,  $c_k$  and  $d_1$  have different mass dimensions. In analogy with what we have done in Sec. 4.3, we assume that they can be parameterized by suitable powers of the Higgs vev  $v$  that is

$$a_i \sim \lambda_{eff} \quad b_j \sim \lambda_{eff} v \quad c_k \sim \lambda_{eff} v^2 \quad d_1 \sim \lambda_{eff} v^3. \quad (5.59)$$

The extra potential that we have written down is obtained using only the requirement of gauge invariance. This means that we can consider the possibility that  $\lambda_{eff}$  is a parameter accounting for any possible mass generation mechanism for the axion. We also have to point out that this extra potential has not been obtained from the supersymmetric construction considered so far. It is, in fact, difficult to trace the origin of this potential to some lagrangian written in terms of superfields. Nevertheless, we can imagine that such a potential is generated by some mechanism taking place under the supersymmetry breaking scale. Examples of such potentials are the scalar potential generated for a PQ axion at the QCD phase transition scale via non-perturbative effects or the Coleman-Weinberg scalar potentials generated by radiative corrections in supersymmetric inflationary models. In our case, we will assume that the couplings in the potential are suppressed in such a way that their effects can be considered as corrections to the eigenstates and eigenvectors of the CP-odd sector.

If we consider a potential that includes any of the terms in Eq. (5.58), and recompute the CP-odd mass matrix using the new scalar potential we get a modified mass matrix. This matrix still has two massless eigenstates that describe the same subspace defined by the neutral Goldstones identified in Sec. 5.1.3. Furthermore, as we will see in more detail in the next section, we get

two massive states. They can be identified with the physical states defined in Sec. 5.1.3 but now the state that we have defined axi-higgs has a non-zero mass that is proportional to the couplings appearing in the extra potential  $V'$ .

In order to characterize in more detail the potential in Eq. (5.57), we proceed with an analysis of the field dependence of the phase factors in the exponentials. We expect that these factors can be expressed exclusively in terms of the physical fields of the CP-odd sector,  $A^1$  and the axi-higgs  $A^2$ . This is analogous to what we have found in the case of the non-supersymmetric setup (see Eq. (4.51)), where the periodicity has been shown to depend only on the axi-higgs. In order to show the same feature in this case, we introduce the following parameterization of the fields

$$\begin{aligned} H_1^1(x) &= \frac{1}{\sqrt{2}} (\rho_1^1(x) + v_1) e^{i\Phi_1^1(x)/v_1} & H_1^2(x) &= \frac{1}{\sqrt{2}} \rho_1^2(x) e^{i\Phi_1^2(x)} \\ H_2^1(x) &= \frac{1}{\sqrt{2}} \rho_2^1(x) e^{i\Phi_2^1(x)} & H_2^2(x) &= \frac{1}{\sqrt{2}} (\rho_2^2(x) + v_2) e^{i\Phi_2^2(x)/v_2} \\ S(x) &= \frac{1}{\sqrt{2}} (\rho_S(x) + v_S) e^{i\Phi_S(x)/v_S} \end{aligned} \quad (5.60)$$

and select just one of the  $V_i$  in Eq. (5.58) in order to illustrate the behaviour.

Considering only the  $V_1$  term we get the mass matrix,

$$M_{odd}^2 = -\frac{a_\lambda}{\sqrt{2}} \begin{pmatrix} \frac{v_2 v_S}{v_1} & v_S & v_2 & 0 \\ \cdot & \frac{v_1 v_S}{v_2} & v_1 & 0 \\ \cdot & \cdot & \frac{v_1 v_2}{v_S} + 6\sqrt{2} a_1 \frac{v_S^2}{a_\lambda} & 8\sqrt{2} a_1 \frac{g_B B_S v_S^3}{a_\lambda M_{St}^2} \\ \cdot & \cdot & \cdot & 8\sqrt{2} a_1 \frac{g_B^2 B_S^2 v_S^4}{a_\lambda M_{St}^2} \end{pmatrix} \quad (5.61)$$

expressed in the basis  $(\Phi_1^1, \Phi_2^2, \Phi_S, Im b)$ . From this matrix we get two null eigenvalues corresponding to the neutral Goldstones and two eigenvalues which correspond to the masses of the two CP-odd states  $A^1$  and  $A^2$ . This mass eigenvalues take the form

$$\begin{aligned} m_{A^1, A^2}^2 &= \frac{1}{2M_{St} v_1 v_2 v_S} \left( A \pm \sqrt{A^2 - B} \right) \\ A &= 4a_1 v_1 v_2 v_S^3 \left( 4M_{St}^2 + g_B^2 B_S^2 v_S^2 \right) + \sqrt{2} a_\lambda M_{St}^2 \left( v_1^2 v_2^2 + v^2 v_S^2 \right) \\ B &= 16\sqrt{2} a_1 a_\lambda M_{St}^2 v_1 v_2 v_S^5 \left( 4v^2 M_{St}^2 + g_B^2 B_S^2 \left( v_1^2 v_2^2 + v^2 v_S^2 \right) \right). \end{aligned} \quad (5.62)$$

In the limit of vanishing  $a_1$  we obtain a massless state corresponding to the axi-higgs and a massive one corresponding to  $A^1$ . In fact, expanding the expressions above up to first order in  $a_1$

we obtain for the two eigenvalues the approximate forms

$$\begin{aligned} m_{A^1}^2 &\simeq \frac{a_\lambda}{\sqrt{2}} \left( \frac{v_1 v_2}{v_S} + \frac{v_1 v_S}{v_2} + \frac{v_2 v_S}{v_1} \right) + 8a_1 \frac{v_1^2 v_2^2 v_S^2}{v^2 v_S^2 + v_1^2 v_2^2}, \\ m_{A^2}^2 &\simeq \frac{8a_1 v_S^4 \left[ v^2 M_{St}^2 + g_B^2 B_S^2 (v^2 v_S^2 + v_1^2 v_2^2) \right]}{M_{St}^2 (v^2 v_S^2 + v_1^2 v_2^2)}. \end{aligned} \quad (5.63)$$

These relations show that the extra potential induces a correction to the  $A^1$  mass and gives a mass contribution to the previously massless state.

The same result can be obtained from a different perspective. The extra potential  $V_1$  can be rewritten as

$$V_1 = a_1 \frac{1}{2} [\rho_S(x) + v_S]^4 \cos(\bar{\theta}_1) \quad \text{where} \quad \bar{\theta}_1 = \frac{4\Phi_S(x)}{v_S} + \frac{4g_B B_S \text{Im}b(x)}{M_{St}}. \quad (5.64)$$

We rotate this linear combination on the physical basis  $(G_0^1, G_0^2, A^1, A^2)$  using a rotation matrix,  $O^\chi$ , made up by the orthonormal eigenstates obtained in Sec. 5.1.3. After the rotation, we can re-express the angle of misalignment as a linear combination of the mass eigenstate of the CP-odd sector in the form

$$\bar{\theta}_1 = \frac{A^1}{\sigma_{A^1}} + \frac{A^2}{\sigma_{A^2}}. \quad (5.65)$$

As we expect, the terms proportional to the Goldstone bosons cancel and the linear combination obtained is made up only by the physical states. This is expected since the extra potential has been built to be gauge invariant so it shouldn't depend on gauge-dependent fields, like the Goldstones are. The two coefficients defining the obtained linear combination are given by

$$\begin{aligned} \sigma_{A^1} &= \frac{1}{4} \frac{v_S}{v_1 v_2} \sqrt{v_1^2 v_2^2 + v_1^2 v_S^2 + v_2^2 v_S^2} \\ \sigma_{A^2} &= -\frac{1}{4} M_{St} \sqrt{\frac{v_1^2 v_2^2 + v_1^2 v_S^2 + v_2^2 v_S^2}{M_{St}^2 (v_1^2 + v_2^2) + g_B^2 B_S^2 (v_1^2 v_2^2 + v_1^2 v_S^2 + v_2^2 v_S^2)}}. \end{aligned} \quad (5.66)$$

If we expand the cosine in Eq. (5.64) up to second order in the angle  $\theta_1$  we get the mass correction to the heavy CP-odd state and the mass contribution to the axi-higgs. The result coincides with the result in Eq. (5.63). We can obtain a numerical estimate of the axi-higgs mass; we have

$$m_{A^2}^2 = \frac{a_1}{2} \frac{v_S^4}{\sigma_{A^2}^2}. \quad (5.67)$$

If we consider the parameter values

$$g_B = .1 \quad B_S = 1 \quad M_{St} = 5\text{TeV} \quad v_S = 1\text{TeV} \quad \tan \beta = 10 \quad (5.68)$$

and take  $a_1 \simeq 10^{-24}$ , we get  $m_{A^2}^2 \simeq 1\text{eV}$ .

The analysis of the extra potential holds, in principle, for an axi-higgs of any mass, although we will not explicitly study an axion whose mass goes beyond the MeV region. To have an axion which is long-lived, the true discriminant of our study is the axion mass and for this reason we are going to present a study of the decay rates of this particle keeping the mass as a free parameter varying in the “meV–MeV” interval.

### 5.3 Decay of a gauged axion

In this section we compute the decay rate of the axi-higgs  $A^2$  into two-photons mediated both by the direct Wess-Zumino interaction and by the fermion loop. The two contributions are shown in Fig. 5.1. This study is needed since we are going to ask ourselves if we can obtain the observed dark matter density (or, at least, a fraction of it) from an axi-higgs condensate generated through the misalignment mechanism.

We will refer to the axi-higgs also as to  $\chi$  to make explicit the analogy with the axi-higgs in the MLSOM (see Ch. 4).

The WZ interaction in Fig. 4.2 is given by

$$\mathcal{M}_{WZ}^{\mu\nu}(\chi \rightarrow \gamma\gamma) = 4g_{\gamma\gamma}^\chi \varepsilon[\mu, \nu, k_1, k_2], \quad (5.69)$$

where  $g_{\gamma\gamma}^\chi$  is the coupling, defined via the relations

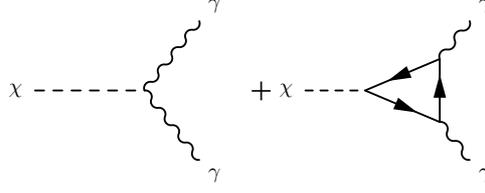
$$g_{\gamma\gamma}^\chi = -\frac{g_B B_S (4c_Y g_2^2 + c_W g_Y^2)}{16g^2 M_{St}} \sqrt{\frac{v^2 v_S^2 + v_1^2 v_2^2}{M_{St}^2 v^2 + g_B^2 B_S^2 (v^2 v_S^2 + v_1^2 v_2^2)}} \quad (5.70)$$

obtained from the rotation of the WZ vertices on the physical basis. The decay rate coming from the WZ vertex is given by

$$\Gamma_{WZ}(\chi \rightarrow \gamma\gamma) = \frac{m_\chi^3}{4\pi} (g_{\gamma\gamma}^\chi)^2. \quad (5.71)$$

Combining the tree level decay with the 1-loop amplitude, we have the total amplitude

$$\mathcal{M}^{\mu\nu}(\chi \rightarrow \gamma\gamma) = \mathcal{M}_{WZ}^{\mu\nu} + \mathcal{M}_f^{\mu\nu}. \quad (5.72)$$



**Figure 5.1:** Contributions to the axi-higgs decay  $A^2 \rightarrow \gamma\gamma$ .

The amplitude mediated by the fermion loop is given by the expression

$$\mathcal{M}_f^{\mu\nu}(\chi \rightarrow \gamma\gamma) = \sum_f N_c(f) i C_0(m_\chi^2, m_f) c_{\gamma\gamma}^{\chi,f} \varepsilon[\mu, \nu, k_1, k_2] \quad f = \{u, d, e\} \quad (5.73)$$

where  $N_c(f)$  is the color factor for the fermions species. In the domain  $0 < m_\chi < 2m_f$ , which is the relevant domain for our study, the pseudoscalar triangle when both photons are on mass-shell is given by the expression

$$C_0(m_\chi^2, m_f) = -\frac{m_f}{\pi^2 m_\chi^2} \arctan^2 \left( \left( \frac{4m_f^2}{m_\chi^2} - 1 \right)^{-1/2} \right) = -\frac{m_f}{\pi^2 m_\chi^2} \eta(\tau_f) \quad (5.74)$$

where we have defined

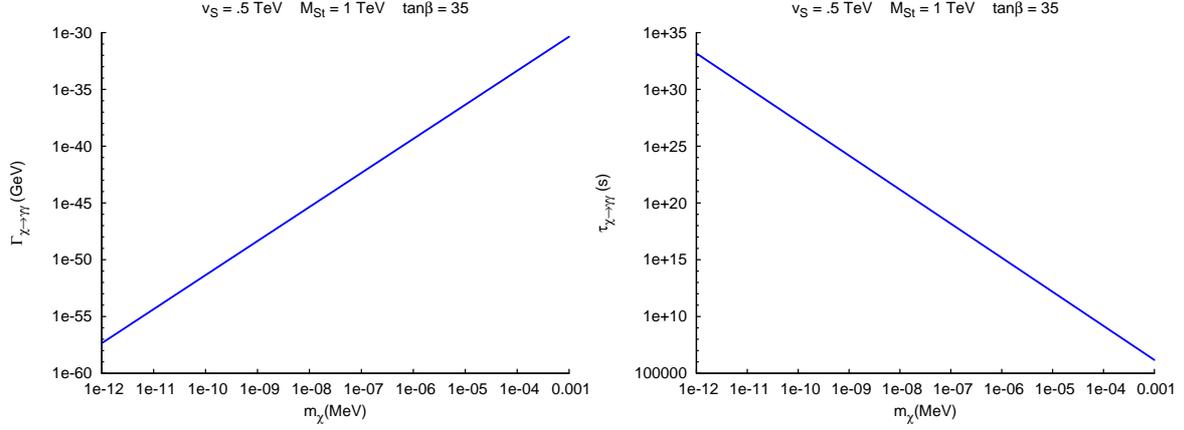
$$\eta(\tau_f) = \arctan^2 \left( \left( -\rho_{f\chi}^2 \right)^{-1/2} \right) \quad \rho_{f\chi} = \sqrt{1 - \tau_f} \quad \tau_f = 4m_f^2/m_\chi^2. \quad (5.75)$$

The coefficient  $c_{\gamma\gamma}^{\chi,f}$  is the factor for the vertex between the axi-higgs and the fermion current. This vertex comes from the Yukawa couplings of the Higgs components of the axi-higgs. These factors are given by

$$\begin{aligned} c_{\gamma\gamma}^{\chi,qu} &= -\frac{i\sqrt{2}y_u M_{St} v_1^2 v_2}{\sqrt{(v^2 v_S^2 + v_1^2 v_2^2) [M_{St}^2 v^2 + g_B B_S (v^2 v_S^2 + v_1^2 v_2^2)]}}, \\ c_{\gamma\gamma}^{\chi,qd} &= -\frac{v_2 y_d}{v_1 y_u} c_{\gamma\gamma}^{\chi,qu}, \\ c_{\gamma\gamma}^{\chi,l} &= \frac{y_e}{y_d} c_{\gamma\gamma}^{\chi,qd}. \end{aligned} \quad (5.76)$$

We obtain the following expression for the decay amplitude

$$\Gamma_\chi \equiv \Gamma(\chi \rightarrow \gamma\gamma) = \frac{m_\chi^3}{32\pi} \left\{ 8(g_{\gamma\gamma}^\chi)^2 + \frac{1}{2} \left| \sum_f N_c(f) i \frac{\tau_f \eta(\tau_f)}{4\pi^2 m_f} e^2 Q_f^2 c_{\gamma\gamma}^{\chi,f} \right|^2 \right\} +$$



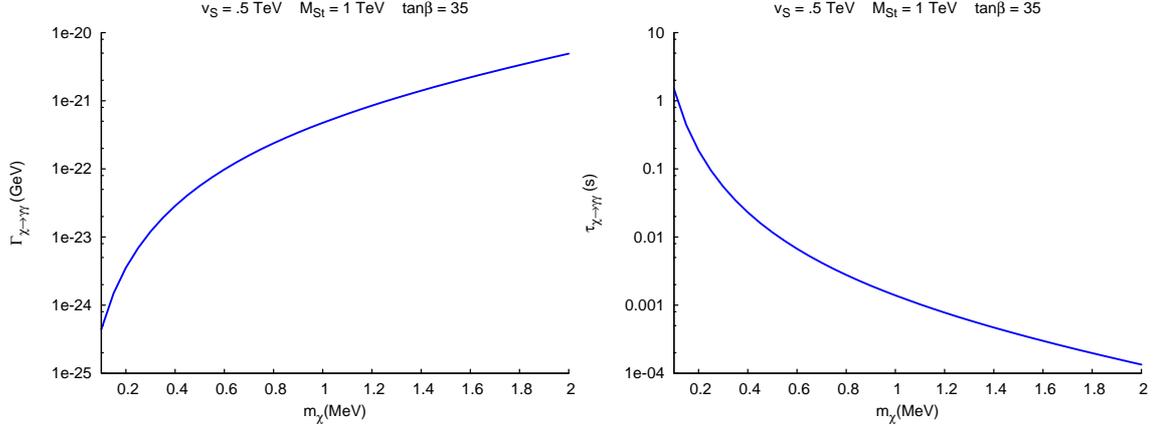
**Figure 5.2:** Decay amplitude (left panel) and mean lifetime (right panel) for  $\chi \rightarrow \gamma\gamma$  as a function of the axi-Higgs mass.

$$4g_{\gamma\gamma}^{\chi} \sum_f N_c(f) i \frac{\tau_f \eta(\tau_f)}{4\pi^2 m_f} e^2 Q_f^2 c_{\gamma\gamma}^{\chi,f} \Bigg\}, \quad (5.77)$$

where the three terms correspond, respectively, to the point-like WZ term, to the 1-loop contribution and to their interference.

Notice that in the expression of this decay rate, both the direct ( $\sim (g_{\gamma\gamma}^{\chi})^2$ ) and the interference ( $\sim g_{\gamma\gamma}^{\chi}$ ) contributions are suppressed as inverse powers of the Stückelberg mass, that we will take equal to 1 TeV. For  $v_S$  we have chosen the value of 500 GeV. The Yukawa couplings have been set to give the right Standard Model fermion masses and we have chosen  $g_B = 0.1$  and  $B_S = 4$ .

We show in Figs. 5.2, 5.3 the results obtained from the numerical evaluation of the decay amplitude as a function of the mass of the axion  $m_{\chi}$ . It is clear that the decay rates are very small for a milli-eV particle. We conclude that a light axi-higgs is indeed long-lived and so it could contribute, in principle, to the relic densities of dark matter. Instead, for an axion with a mass in the MeV region the decay is rather quick.



**Figure 5.3:** Decay amplitude and mean lifetime for  $\chi \rightarrow \gamma\gamma$  as a function of the axi-Higgs mass for an axion whose mass in the MeV range.

## 5.4 Cold dark matter by misalignment of the axion field

As we have already done in the case of the MLSOM, we can ask ourselves if the axi-higgs potential is such that we can have dark matter production through the misalignment mechanism (See Sec. 4.5). In the case of the supersymmetric model we are considering the discussion goes along the same lines.

We define the abundance of  $\chi$  at the oscillation temperature  $T_i$  as

$$Y_{A^2}(T_i) \equiv \frac{n_{A^2}}{s} \Big|_{T_i} \quad (5.78)$$

and we have to consider the oscillation condition

$$m_{A^2}(T_i) = 3H(T_i), \quad (5.79)$$

where  $m_{A^2}(T_i)$  is the mass of  $A^2$  and  $H(T_i)$  is the Hubble constant at the oscillation temperature  $T_i$ . We will consider the case in which the extra potential is given by  $V_1$ , so the oscillation temperature will be equal or smaller than the singlet vev,  $v_S$ , since the mass of the axi-higgs is proportional to this vev. Re-expressing the Hubble constant in Eq. (5.79) in terms of  $g_{*,T_i}$ , the number of effectively massless degrees of freedom at the oscillation temperature, and using the expression for the mass of  $A^2$  in Eq. (5.67), we get

$$\frac{a_1}{2} \frac{v_S^4}{\sigma_{A^2}^2} = \frac{4}{5} \pi^3 g_{*,T_i} \frac{T_i^4}{M_P^2}. \quad (5.80)$$

So, if we want the oscillations to start at the temperature  $T_i = v_S$ , we must have

$$a_1 = \frac{8}{5} \pi^3 g_{*,T_i} \frac{\sigma_{A^2}^2}{M_P^2} \quad (5.81)$$

In order to obtain an numerical estimate we can take

$$g_B = .1 \quad B_S = 1 \quad M_{St} = 5\text{TeV} \quad v_S = 1\text{TeV} \quad \tan \beta = 10 \quad (5.82)$$

The number of effectively massless degrees of freedom at the chosen  $v_S$  is  $\approx 100$  and we have  $v \approx 246 \text{ GeV}$ ,  $M_P = 2.43 \times 10^{18} \text{ GeV}$ . These values give us  $a_1 \simeq 10^{-29}$ .

Following the same reasoning as in the case of the MLSOM, we get

$$Y_\chi(T_i) = \frac{45 \sigma_{A^2}^2 \theta_i^2}{2 \sqrt{5\pi g_{*,T_i}} T_i M_P}. \quad (5.83)$$

Using the conservation of the abundance  $Y_{\chi 0} = Y_\chi(T_i)$ , the expression of the contribution to the energy density today as a fraction of the total energy density is given by

$$\Omega_\chi^{mis} = Y_\chi(T_i) m_\chi \frac{s_0}{\rho_c} = \frac{n_\chi}{s} \Big|_{T_i} m_\chi \frac{s_0}{\rho_c} = \frac{45 \theta_i^2}{2 \sqrt{5\pi g_{*,T_i}} M_P} \frac{\sigma_{A^2}^2 m_{A^2}}{T_i} \frac{s_0}{\rho_c}. \quad (5.84)$$

We have  $\sigma_{A^2}^2 m_{A^2} = \sqrt{a_1/2} v_S^2 \sigma_{A^2}$  so

$$\Omega_\chi^{mis} = \sqrt{\frac{a_1}{2}} \frac{45 \theta_i^2}{2 \sqrt{5\pi g_{*,T_i}} M_P} \frac{s_0}{\rho_c} \frac{v_S^2 \sigma_{A^2}}{T_i}. \quad (5.85)$$

We remind that the values of the critical energy density ( $\rho_c$ ) and the entropy density today are estimated as [70, 71]

$$\rho_c = 5.3 \cdot 10^{-6} \text{ GeV/cm}^3 \quad s_0 = 2970 \text{ cm}^{-3}. \quad (5.86)$$

We also assume that the initial misalignment angle is  $\theta_i \simeq 1$ .

If we take  $T_i = v_S$  then we have

$$\Omega_\chi^{mis} = 2.31 \times 10^{-5} \sqrt{a_1}. \quad (5.87)$$

So, even in this case, we cannot have a dark matter population made of long lived particles. This is clear considering the results for the decay rate of the axi-higgs in Sec. 5.3 and the fact that the required lifetime is of the order of the age of the Universe,  $t_0 = 4.3 \times 10^{17} \text{ s}$  [13].

## 5.5 Numerical study of the CP-even sector

The next point that we will address is the possibility of having thermal dark matter production via the decoupling of the lightest neutralino. In the numerical analysis for the neutralino relic densities we also have to make sure that the choice of the parameters gives positive mass squared values in the CP-even sector and that these values are compatible with the current bounds. We remind that we were not able to obtain analytical expressions for the eigenstates and eigenvalues for the CP-even sector given the complexity of the mass matrix. The analytical results combined with the requirement that the squared mass matrix is positive definite would give us some conditions on the parameters defining that matrix.

From Sec. 5.1.3, we recall that the parameters appearing in the CP-even mass matrix that can't be fixed using known experimental values are  $(\tan\beta, \lambda, v_S, g_B, M_{St}, v_b)$  and two among the charges  $(B_{H_1}, B_{H_2}, B_S)$ , since the third is fixed by the requirement that the superpotential is gauge invariant. We start by setting the couplings to natural values,  $g_B = 0.4$  and  $\lambda = 0.5$ . We will consider small variations of these parameters and check how they affect the CP-even spectrum.

We remind that the factor  $\lambda v_S / \sqrt{2}$  has to be in the range  $10^2 - 10^3$  GeV since it has to reproduce the  $\mu$  parameter of the MSSM and this range has been determined to be the phenomenologically viable one. So, given our choice of  $\lambda$ ,  $v_S$  should be in the range  $2\sqrt{2} \times (10^2 - 10^3)$ .

In principle the Stückelberg mass  $M_{St}$  is a free parameter even if, in the limit  $M_{St} \rightarrow \infty$ , from the expressions obtained in Sec. 5.1.3, we get that the higgs-axion mixing in the CP-odd sector disappears. Moreover, in this limit, the whole axion supermultiplet becomes so massive that it can be decoupled from the rest of the model. So, in order to stay in the “interesting region” for Higgs-axion mixing, we will consider the range 2 – 22 TeV for the Stückelberg mass.

The remaining unconstrained parameters are  $\tan\beta$  and the saxion vev  $v_b$ . For these we will consider the values (10, 40) and (10, 20, 30, 40) GeV respectively and we will refer to the case  $\tan\beta = 10$  as to the “low  $\tan\beta$  case” and to the case  $\tan\beta = 40$  as to the “high  $\tan\beta$  case”. We will see that these values are such that they don't restrict too much the allowed parameter space.

We will also consider two scenarios for the Higgs charges under the extra  $U(1)$ . In the first case we will take  $B_{H_1} = -2$  and  $B_{H_2} = -1$  and this choice gives a positive charge for  $B_S$  ( $B_S = 3$ ). Then we will take  $B_{H_1} = 2$  and  $B_{H_2} = -1$  in order to consider the case of a negative  $B_S$  ( $B_S = -1$ ). We do this since in the mass matrix we repeatedly find the combination  $v_S B_S$ , so, since we are interested in the  $v_S$  dependence, we want to analyse the effect of the sign of  $B_S$  on the results.

We will sort the eigenvalues from the smaller to the bigger in absolute value, refer to them as to  $(r_1, r_2, r_3, r_4)$  and plot their values only in the regions where these values are above 90 GeV. This value should be consistent with the current bounds for the CP-even Higgs mass once one takes into account the radiative corrections on the mass lightest CP-even state. The calculation of the radiative corrections on the Higgs potential, in the case of the USSM, can be found in [76].

### 5.5.1 Results

In what follows we summarize the results obtained in the analysis of the CP-even sector spectrum. We will define a region of the parameter space in which we get an acceptable spectrum for this sector and we will use this information in the relic density calculation of the LSP.

- **Positive  $B_S$ , low  $\tan\beta$**

In this region the heaviest state is always well above the bound and it grows with  $v_S$  and  $M_{St}$ . Looking at the components of this mass eigenstate we find that, as expected, it coincides almost completely with the saxion and has negligible components in terms of the remaining states of the CP-even sector.

It is more interesting to consider the behaviour of the remaining states, shown in Fig. 5.4. We can clearly see that, combining the exclusion regions coming from the three lightest eigenvalues for a fixed value of  $v_b$ , we obtain an exclusion region in the low  $v_S$ -high  $M_{St}$  region of the scanned parameter space. We also notice that this exclusion region enlarges when  $v_b$  gets larger values.

- **Positive  $B_S$ , high  $\tan\beta$**

In this region, for the heaviest state we get the same results obtained in the low  $\tan\beta$  region and, once again, the interesting part is given by the lightest states. The results in this case are given in Fig. 5.5 and the same comments as for the case of low  $\tan\beta$  apply. We conclude that the result, in the parameter region we are considering, depends only mildly on  $\tan\beta$ , while we have a significant dependence on the saxion vev,  $v_b$ .

- **Negative  $B_S$**

In this case, for the parameter range considered, we don't get any "allowed region" since some of the eigenvalues are always negative.

- **$\lambda$  dependence**

In order to analyse the dependence of the CP-even spectrum on the coupling  $\lambda$ , we consider a "central" region with respect to the parameters considered so far ( $\tan\beta = 25$  and  $v_b = 25$  GeV) and we set  $B_S$  to the positive value. We take three different values for  $\lambda$ , (0.2, 0.5, 0.8) and for the three lightest states we get the result given in Fig. 5.6. We can see that bigger values of  $\lambda$  broaden the "forbidden" region.

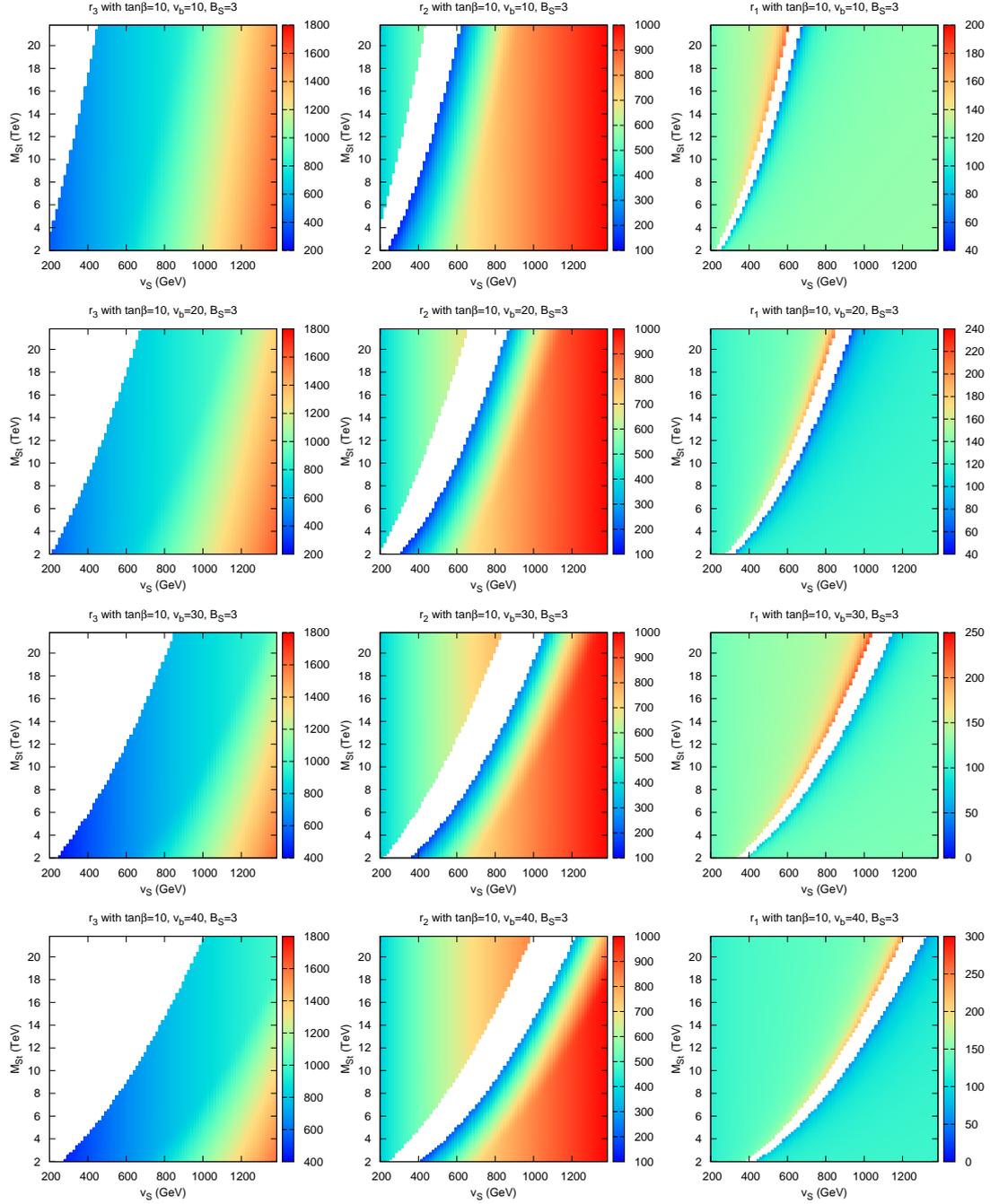
- **$g_B$  dependence**

As we have just done for the parameter  $\lambda$ , we take "central" values for  $\tan\beta$  and  $v_b$  and consider a varying  $g_B$  (0.2, 0.4, 0.6). Again, we consider the three lightest eigenvalues and the result is given in Fig. 5.7. In this case we see that smaller values of the coupling  $g_B$  broaden the "forbidden" region.

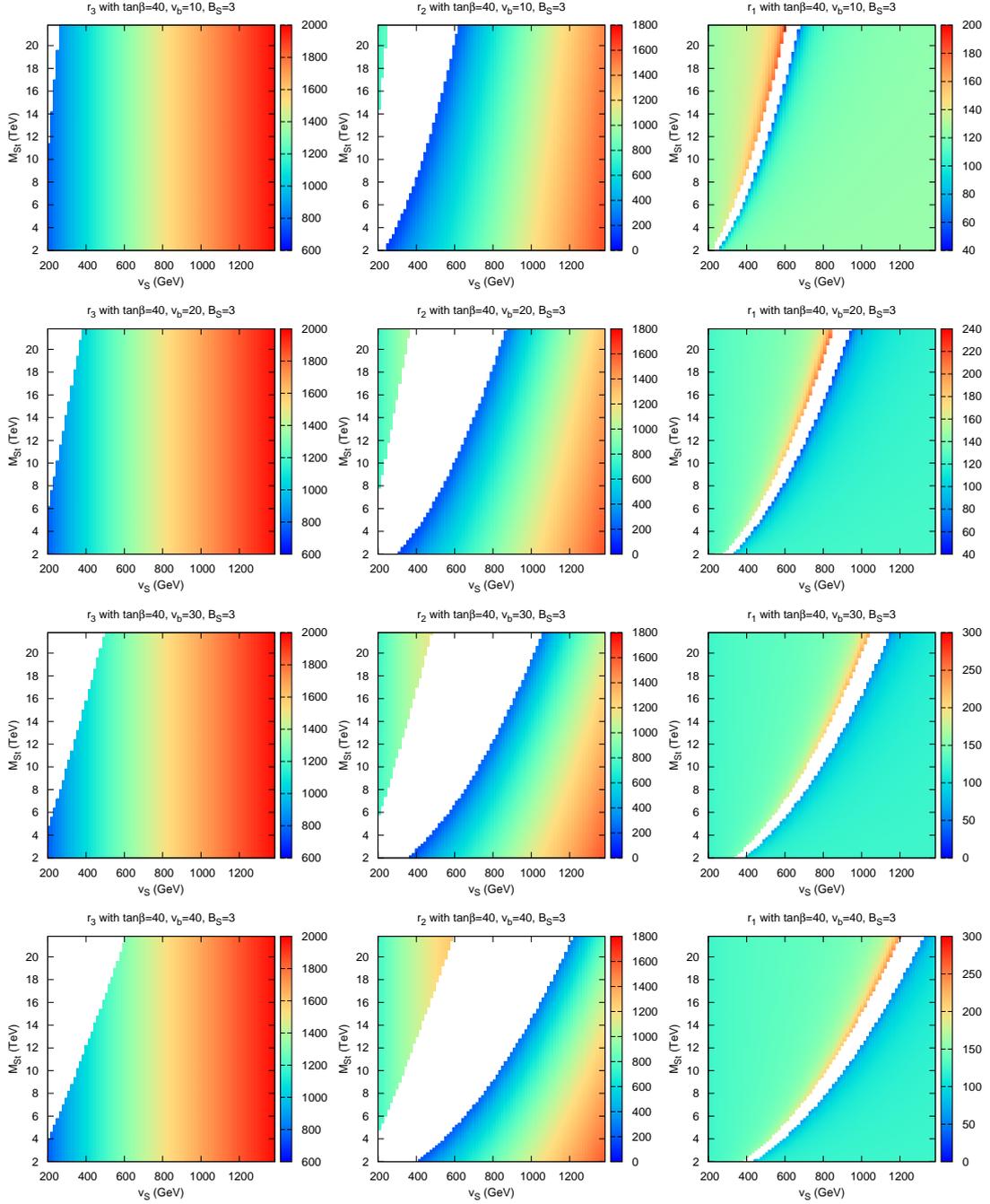
### 5.5.2 Summary

The analysis of the CP-even sector masses restricts the parameter space we are considering. We have seen that smaller values of the coupling  $g_B$  and of the vev  $v_b$  broaden the forbidden region. The same effect is obtained considering bigger values of the coupling  $\lambda$ . On the contrary, we've seen no significant dependence on the value of  $\tan\beta$ . We've also found out that negative values of  $B_S$  don't result in any allowed region.

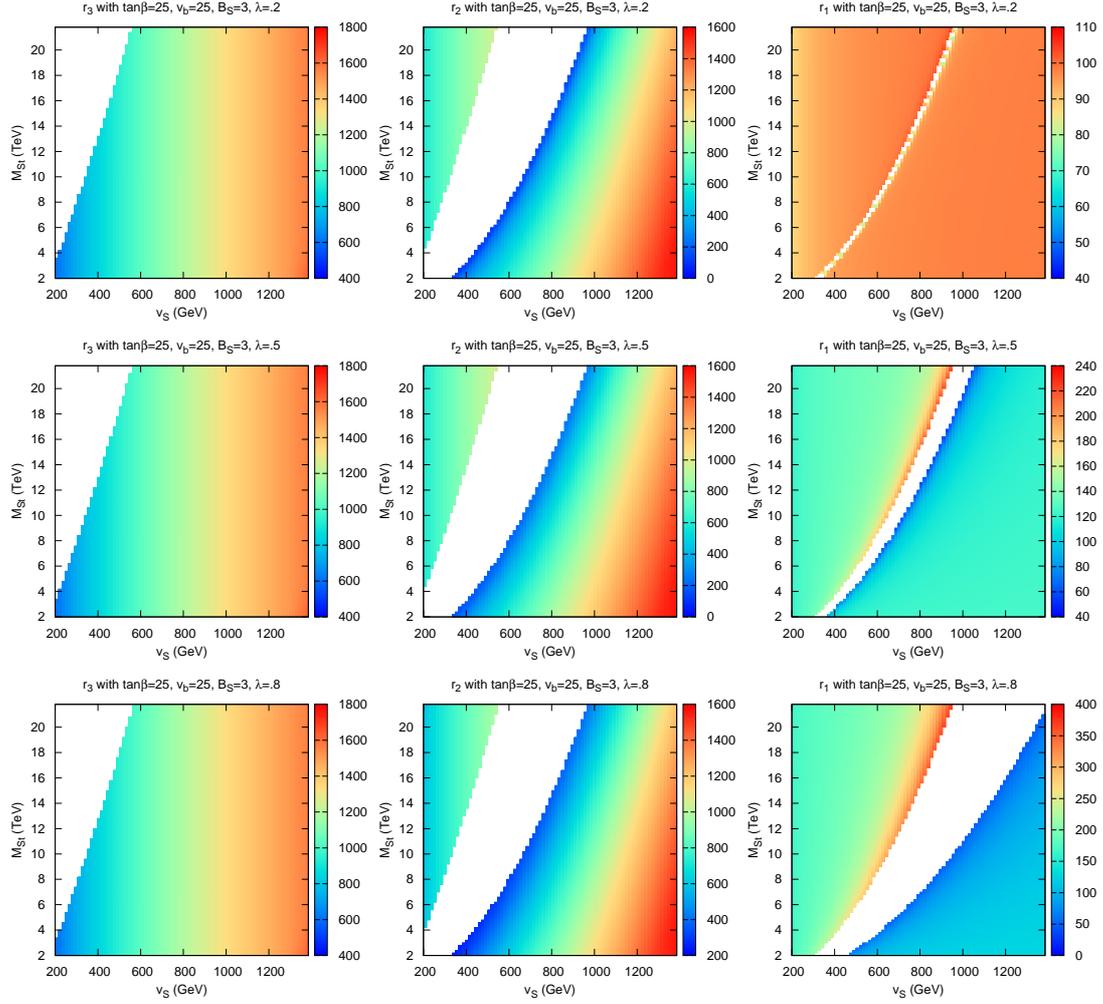
With regard to  $v_S$  and  $M_{S_t}$ , the result is that the low  $v_S$ -high  $M_{S_t}$  region is forbidden by the requirement of having an acceptable spectrum in the CP-even sector. We will take this into account in the numerical analysis of the LSP relic densities.



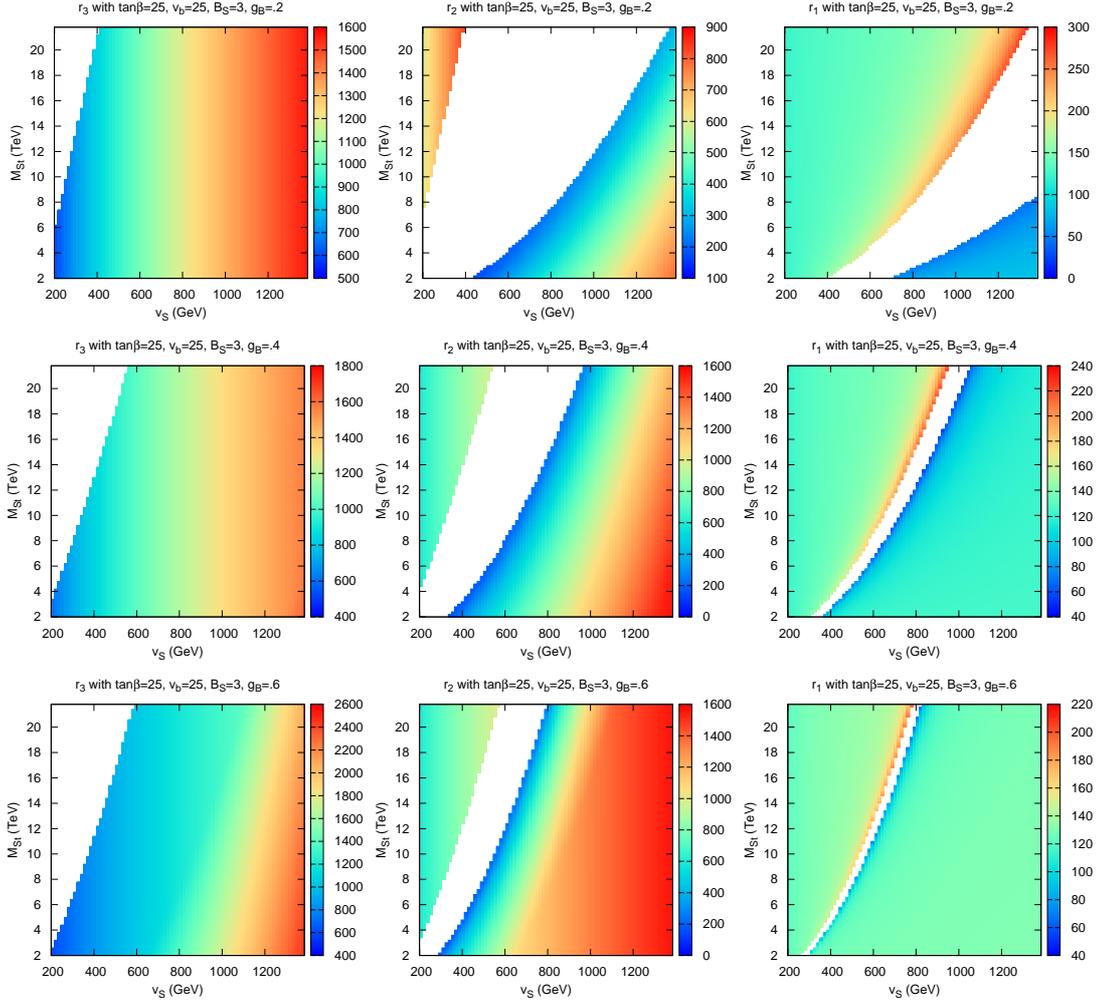
**Figure 5.4:** CP-even sector mass eigenvalues as a function of  $v_S$  and  $M_{S_t}$  in the low  $\tan\beta$  region. Along the rows the result for increasing values of  $v_b$  (10, 20, 30, 40) GeV is shown while the columns correspond to the 3 lightest states, ordered according to decreasing mass.



**Figure 5.5:** CP-even sector mass eigenvalues as a function of  $v_S$  and  $M_{St}$  in the high  $\tan\beta$  region. Along the rows the result for increasing values of  $v_b$  (10, 20, 30, 40) GeV is shown while the columns correspond to the 3 lightest states, ordered according to decreasing mass.



**Figure 5.6:** CP-even sector lightest mass eigenvalues as a function of  $v_S$  and  $M_{St}$  for “central” values of  $\tan\beta$  and  $v_b$  and varying  $\lambda$  (0.2, 0.5, 0.8).



**Figure 5.7:** CP-even sector lightest mass eigenvalues as a function of  $v_S$  and  $M_{St}$  for “central” values of  $\tan\beta$  and  $v_b$  and varying  $g_B$  (0.2, 0.4, 0.6).

## 5.6 Numerical study of the neutralino sector

As already mentioned in Sec. 5.1.4, the neutralino sector consists of the eigenstates of the space spanned by the neutral fields  $(i\lambda_{W_3}, i\lambda_Y, i\lambda_B, \tilde{H}_1^1, \tilde{H}_2^2, \tilde{S}, \psi_{\mathbf{b}})$ , which involve the three neutral gauginos, the two Higgsinos, the singlino (the fermion component of the singlet superfield) and the axino (the fermion component of the Stückelberg supermultiplet). We have denoted with  $M_{\chi^0}$  the corresponding mass matrix. The neutralino eigenstates of this mass matrix are labeled as  $\chi_i^0$  ( $i = 0, \dots, 6$ ) and can be formally expressed in the basis  $\{i\lambda_{W_3}, i\lambda_Y, i\lambda_B, \tilde{H}_1^1, \tilde{H}_2^2, \tilde{S}, \psi_{\mathbf{b}}\}$  as

$$\chi_i^0 = a_{i1} i\lambda_{W_3} + a_{i2} i\lambda_Y + a_{i3} i\lambda_B + a_{i4} \tilde{H}_1^1 + a_{i5} \tilde{H}_2^2 + a_{i6} \tilde{S} + a_{i7} \psi_{\mathbf{b}} \quad (5.88)$$

in which the coefficients  $a_{ij}$  can be identified with the elements of the rotation matrix  $O^{\chi^0}$  defined in Sec. 5.1.4 in the following way

$$a_{ij} = O_{j,i+1}^{\chi^0} \quad (5.89)$$

We will also use the notation  $c_{ij} = |a_{ij}|^2$ .

The neutralino mass eigenstates  $\chi_i^0$  are ordered in mass with the lightest eigenstate corresponding to  $i = 0$ . The complexity of the mass matrix of this sector is such that it's not possible to obtain an analytic expression for the neutralino masses, so we will study the parameter dependence numerically.

In order to perform the numerical analysis of the neutralino mass matrix and relic densities we need to fix some of the parameters, first of all requiring the consistency of the choice with the measured SM parameters. For this purpose, the Higgs vev's  $v_1$  and  $v_2$  are constrained in order to generate the correct mass values for the SM massive gauge bosons  $W^\pm$  and  $Z$ , whose masses depend on  $v^2 = v_1^2 + v_2^2$ , the Yukawa couplings are fixed in order to give the correct masses of the SM fermions and the  $SU(2)$  and  $U(1)_Y$  couplings are set to their experimental values. We also set the saxion vev  $v_b$  to 20 GeV.

Following[75], the gauge mass terms parameters relative to the SM gauge group have been selected according to the relation

$$M_Y : M_W : M_G = 1 : 2 : 6; \quad (5.90)$$

this is an approximate relation that is obtained in the case of the MSSM, if we impose the equality of the gaugino masses at the Grand Unification scale. We have set

$$M_Y = 500 \text{ GeV} \quad M_W = 1 \text{ TeV} \quad M_G = 2 \text{ TeV}. \quad (5.91)$$

We have to choose the values of the sfermion masses and the trilinear coupling terms. We

set most of the parameters defining these terms introducing the common values  $M_0$  and  $a_0$ . Our choice is the following

$$\begin{aligned}
M_L = M_Q = m_R = m_D = m_U = M_0 &= 1 \text{ TeV} \\
a_e = a_d = a_u = a_0 &= 1 \text{ TeV} \\
a_\lambda &= -100 \text{ GeV},
\end{aligned} \tag{5.92}$$

where  $M_L$ ,  $M_Q$ ,  $m_R$ ,  $m_D$  and  $m_U$  are the scalar mass terms for the sleptons and the squarks, assumed to be equal for all the 3 generations,  $a_e$ ,  $a_u$  and  $a_d$  are the trilinear terms coupling corresponding to the Yukawa interactions in the superpotential and  $a_\lambda$  is the coupling of the soft breaking term corresponding to the superpotential term  $\lambda SH_1 \cdot H_2$ .

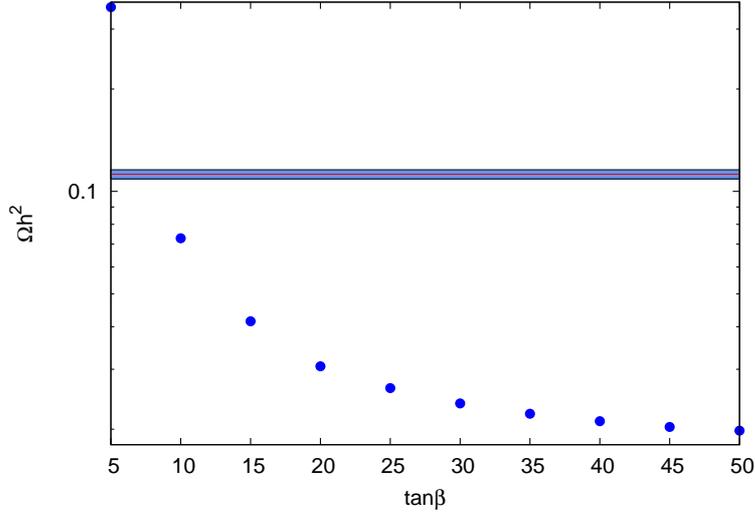
With these choices we are left with a few more parameters. We have some SUSY breaking parameters, i.e. the gaugino mass for  $\lambda_B$ , denoted by  $M_B$ , together with the mixed gauge mass term parameter  $M_{YB}$  and the axino and saxion mass parameters,  $M_{\psi_b}$  and  $M_{\text{Reb}}$ . We set these parameter to the usual SUSY breaking scale

$$M_B = 1 \text{ TeV} \quad M_{YB} = 1 \text{ TeV} \quad M_{\psi_b} = 1 \text{ TeV} \quad M_{\text{Reb}} = 1 \text{ TeV}. \tag{5.93}$$

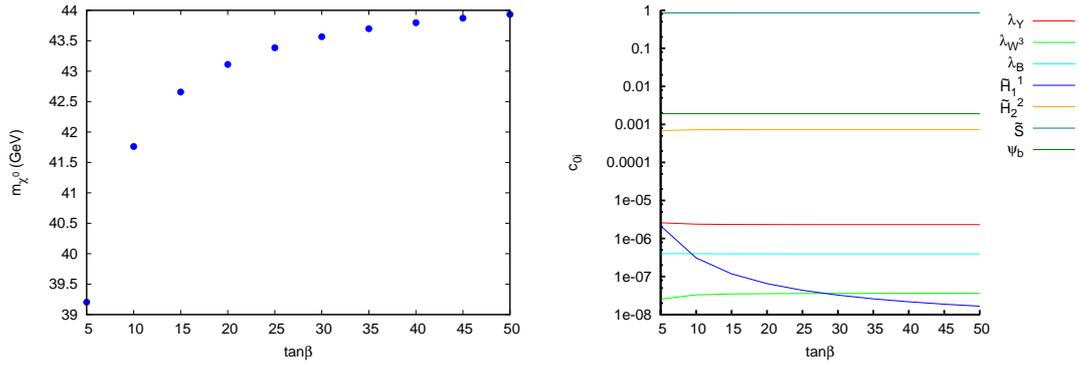
We will set the coupling constant for the extra  $U(1)$  gauge group,  $g_B$ , to 0.4. This is the same value that we have used in the numerical analysis of the CP-even Higgs sector (see Sec. 5.5) in which we have also considered the effects of taking a smaller or bigger parameter. From previous investigations such values of the anomalous coupling are known to be compatible with LEP data at the  $Z$  resonance[62, 84]. In particular, a suitable choice of the Stückelberg mass parameter gives a mass for the extra  $Z'$  that satisfies the current constraints and makes it compatible with actual searches for extra neutral currents.

The charges of the Higgs fields under the extra  $U(1)$  will be chosen as  $B_{H_1} = -2$ ,  $B_{H_2} = -1$  and, consequently,  $B_S = 3$ . In the numerical analysis of the CP-even sector we have seen that this choice leaves a wide parameter space for  $v_S$  and  $M_{S_t}$ .

Having fixed all these parameters we are left with 3 parameters,  $\tan\beta$ ,  $M_{S_t}$  and  $v_S$ . Let's start by considering the case  $v_S = 1.45 \text{ TeV}$ ,  $M_{S_t} = 8 \text{ TeV}$  and  $\tan\beta$  varying in the range 5 – 50. We use micrOMEGAs (see Sec. A.3) for the relic density calculation. With this parameter choice, the LSP is the lightest neutralino and the result for the relic densities calculation is shown in Fig. 5.8, while in Fig. 5.9 we plot the mass of the LSP and the squared components of the corresponding eigenvector, in order to clarify the composition of the lightest neutralino in terms of the interaction basis fields. With this choice of the parameters we get a lightest neutralino with a mass around 40 GeV for different values of  $\tan\beta$ . We also see that the lightest neutralino is essentially given by the fermion component of the singlet superfield  $\hat{S}$ . We see that we get bigger values for the relic density in the case of lower  $\tan\beta$ . The output of the code also tells us



**Figure 5.8:** Relic density as a function of  $\tan\beta$  for  $v_S = 1.45$  TeV and  $M_{St} = 8$  TeV. The horizontal line represents the measured value of the dark matter relic density,  $\Omega h^2 = 0.1123 \pm 0.0035$  [13]



**Figure 5.9:** Mass and components of the lightest neutralino as a function of  $\tan\beta$  for  $v_S = 1.45$  TeV and  $M_{St} = 8$  TeV.

that the main depletion channels are the annihilation of neutralinos in down-type quarks, up and charm quarks, neutrinos and leptons and this happens for every value of  $\tan\beta$  (see Sec. A.3.1 for a sample output). Using CalcHEP (see Sec. A.2), we can inspect the process that gives the main annihilation channel for the lightest neutralino, that is the annihilation in a  $d\bar{d}$  pair (the remaining down-type quarks contribute in a similar way). The tree level amplitude for this process receives several contributions that we can collect in two main classes

- s-channel diagrams: these diagrams contain the neutral gauge bosons ( $A, Z, Z'$ ), the CP-even Higgses ( $H_0^1, H_0^2, H_0^3, H_0^4$ ) and the CP-odd Higgses ( $A^1, A^2$ );
- t-channel diagrams: these diagrams contain the scalar partners of the fermions produced in the specific process.

By calculating the annihilation cross section for the lightest neutralino, setting the momentum of the incoming particle to  $m_{\chi^0}/20 \simeq 2$  GeV we see that the contribution to the cross section coming from the s-channel diagram in which the  $Z$  gauge boson appears gives almost all of the cross section. The choice for the momentum of the incoming particle has been made taking into account the fact that the typical values for the freeze-out temperature for a non-relativistic weakly interacting particle are typically around  $m_{CDM}/20$ . This holds also in our case; the value of the freeze-out temperature can be derived from the value of the variable  $Xf$  appearing in the program output and defined as  $Xf = m_{CDM}/T_{f.o.}$  where  $m_{CDM}$  represents the mass of the lightest neutralino and  $T_{f.o.}$  is the freeze-out temperature (see Sec. 3.1 for a discussion in the general case).

Since the s-channel diagram with the  $Z$  is by far the most important one, it's interesting to extract the two vertices appearing in this diagram and analyze their dependence on the free parameters. The  $\chi^0 \chi^0 Z$  vertex is given by

$$V_{\chi^0 \chi^0 Z}^\mu = \frac{1}{2} \gamma^\mu \gamma^5 \left\{ g_Y O_{ng}^{22} \left[ \left( O_{\chi^0}^{14} \right)^2 - \left( O_{\chi^0}^{13} \right)^2 \right] + g_2 O_{ng}^{21} \left[ \left( O_{\chi^0}^{13} \right)^2 - \left( O_{\chi^0}^{14} \right)^2 \right] + 2g_B O_{ng}^{23} \left[ B_{H_1} \left( O_{\chi^0}^{13} \right)^2 + B_{H_2} \left( O_{\chi^0}^{14} \right)^2 + B_S \left( O_{\chi^0}^{15} \right)^2 \right] \right\}, \quad (5.94)$$

while for the vertex  $Z d \bar{d}$  we have

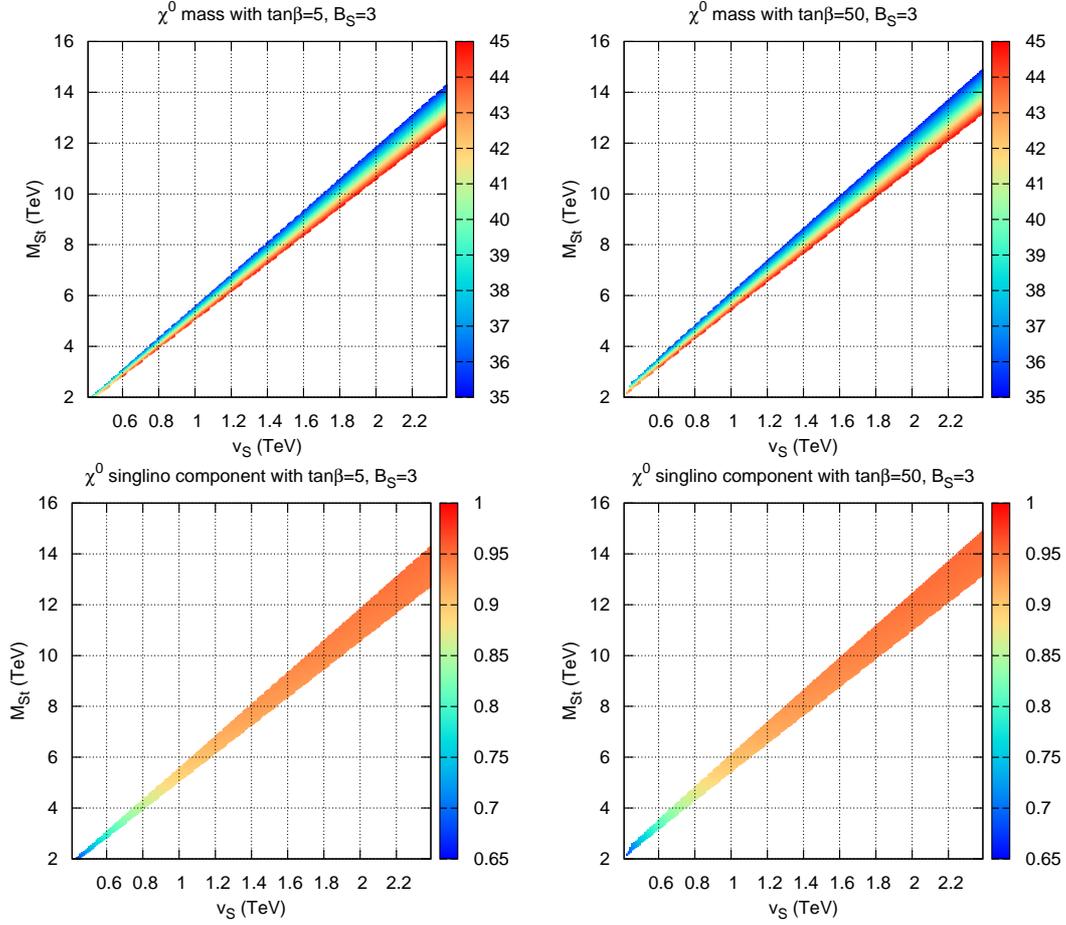
$$V_{Z d \bar{d}}^\mu = \frac{1}{12} \gamma^\mu \left\{ -g_Y O_{ng}^{22} (1 + 3\gamma^5) - 3g_2 O_{ng}^{21} (1 - \gamma^5) - 6g_B O_{ng}^{23} [B_{D_R} (1 + \gamma^5) - B_Q (1 - \gamma^5)] \right\}. \quad (5.95)$$

In the expressions of the vertices  $O_{ng}^{ij}$ , with  $i, j = 1, 2, 3$ , represent the rotation matrix of the neutral gauge sector. The matrix element  $O_{ng}^{23}$  represents the projection of the mass eigenstate  $Z$  on the interaction eigenstate  $B$ , that is on the extra gauge boson associated to the anomalous symmetry. This projection is suppressed since it depends on the ratio of the scales  $v/M_{St}$ ; as an example, for the specific choice of parameters that we are considering, we have  $O_{ng}^{23} \approx 10^{-4}$ . On the other hand, the other neutral gauge rotation matrix elements appearing in the expressions of the vertices can be expressed in terms of the weak angle  $\theta_w$  as

$$O_{ng}^{21} \simeq \cos \theta_w \quad O_{ng}^{22} \simeq -\sin \theta_w \quad (5.96)$$

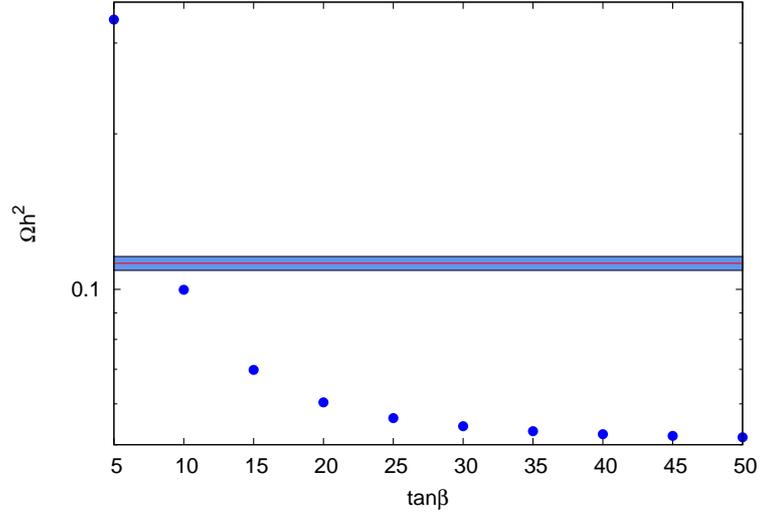
with  $\sin^2 \theta_w \approx 0.23$ . This means that we don't have any dependence on  $\tan \beta$  for the vertex  $Z d \bar{d}$ .

The components of the rotation matrix  $O_{\chi^0}$  appearing in the vertex  $V_{\chi^0 \chi^0 Z}^\mu$  are those relative to the higgsino and singlino components of the lightest neutralino. We notice that the dominant component, which is the singlino component (see Fig. 5.9), appears in combination with the suppressed rotation matrix element  $O_{ng}^{23}$ , while the higgsino component appear in combination

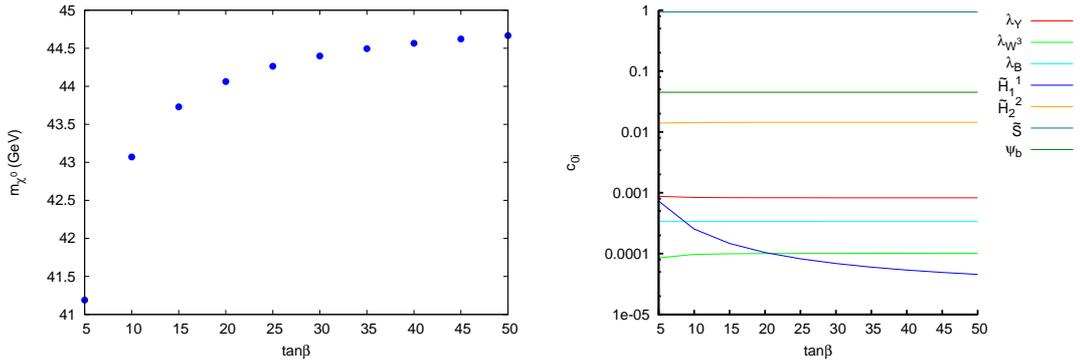


**Figure 5.10:** Mass of the lightest neutralino and singlino squared component as a function of  $v_S$  and  $M_{St}$  for  $\tan \beta = 5$  and  $\tan \beta = 50$ .

with the unsuppressed matrix elements  $O_{ng}^{21}$  and  $O_{ng}^{22}$ . Regarding the dependence on  $\tan \beta$ , we see from Fig. 5.9 that one of the higgsino components, namely the  $\tilde{H}_2^2$  component, is dominant with respect to the other higgsino component for every chosen  $\tan \beta$ . So we have a negligible dependence of this vertex on  $\tan \beta$ . We conclude then that the only important dependence on  $\tan \beta$  is on the mass of the lightest neutralino (see Sec. 3.1 for a discussion on general features of thermal production of cold dark matter). We expect that choosing  $v_S$ ,  $M_{St}$  and  $\tan \beta$  in order to obtain a neutralino with a mass around 40 GeV, we would obtain dark matter densities very close to the ones obtained so far. In Fig. 5.10 we plot the mass of the lightest neutralino for  $\tan \beta = 5$  and  $\tan \beta = 50$  as a function of  $v_S$  and  $M_{St}$  restricting the plotted values to those in the range 35 – 45 GeV in order to highlight the region that, as we have seen, gives a neutralino relic density in the desired amount. We also plot the squared singlino component for the same state. We notice that the variation of  $\tan \beta$  doesn't affect the result strongly. If we take  $v_S = 2$  TeV and  $M_{St} = 11$  TeV and consider again  $\tan \beta$  varying in the range 5 – 50, we get the relic densities



**Figure 5.11:** Relic density as a function of  $\tan\beta$  for  $\nu_S = 2$  TeV and  $M_{St} = 11$  TeV. The horizontal line represents the measured value of the dark matter relic density,  $\Omega h^2 = 0.1123 \pm 0.0035$ [13]



**Figure 5.12:** Mass and components of the lightest neutralino as a function of  $\tan\beta$  for  $\nu_S = 2$  TeV and  $M_{St} = 11$  TeV.

plotted in Fig. 5.11. The corresponding masses and components for the lightest neutralino are plotted in Fig. 5.12. We clearly see that the result is completely analogous to the result obtained for the other point in the parameter space we have chosen.

## 5.7 Conclusions

We believe that the investigation of the phenomenological role played by models containing anomalous gauge interactions from abelian extensions of the Standard Model will receive further attention in the future. These studies can be motivated within several scenarios, including string and supergravity theories, in which gauged axionic symmetries are introduced for anomaly cancellation. In turn, these modified mechanisms of cancellation of the anomalies, which involve

an anomalous fermion spectrum and an axion, are essentially connected with the UV completion of these field theories, which in a string framework is realized by the Green-Schwarz mechanism.

The model that we have investigated summarizes the most salient physical features of these types of constructions, where a Stückelberg supermultiplet is associated to an anomalous abelian structure in order to restore the gauge invariance of the anomalous effective action. We have also pointed out how the axion field, that is essential in the anomaly cancellation mechanism, can appear as a component of a physical field and is not necessarily just a Goldstone state.

In this work we have tried to characterize in detail some among the main phenomenological implications of these models, which are particularly interesting for cosmology.

We have presented a study of the neutralino relic densities, showing that the Stückelberg mass value is constrained by the requirement of a consistent mass spectrum and by the experimental value for the dark matter abundance. Only in this case the WMAP constraints[85] on the contribution to the energy density coming from dark matter can be satisfied. Thus, in these models, the allowed value of the Stückelberg scale is positively correlated with the value of  $\tan \beta$  and  $v_S$ .

The calculation of the relic density we have made considers only a small portion of the parameter space of the model. Exploring the whole parameter space is an impossible task, as usual in supersymmetric extensions of the SM. Following the approach considered in the case of the MSSM, we can think to reduce the number of free parameters setting them to some common values at a certain high energy scale and then evolve them through the renormalization group equations of the model in order to obtain their low energy values. This procedure would give us a definitely smaller parameter space and the possibility to obtain more significant result.

We also point out that the codes we have developed allows to extend this analysis, with little effort, to the direct and indirect detection of dark matter.

## *Codes and program outputs*

### **A.1 FeynRules model file**

In this section we give some details on the implementation of the lagrangian in FeynRules[86]. FeynRules is a Mathematica package that allows to implement the lagrangian of a particle physics model and calculate its Feynman Rules. Furthermore it also allows to “translate” the obtained Feynman rules in order to create input files for several Feynman diagrams calculators. In particular, we were interested in the possibility to generate CalcHEP model files (for more informations on CalcHEP see Sec. A.2). These are, in fact, the input files required by micrOMEGAs (see Sec. A.3) in order to perform a relic density calculation since it uses CalcHEP internally for the calculation of the cross sections.

In order to implement a lagrangian in FeynRules the gauge group, the fields and the parameters of the corresponding model have to be defined in a model file.

The latest version of FeynRules (1.6.0 as of today) contains a superfield module[87] that allows to implement a supersymmetric lagrangian directly in terms of superfields. The package’s website also hosts the model files for some of the most known supersymmetric extensions of the SM. So, in order to write the model file for the supersymmetric extension we are considering, we started from the available MSSM model file and modified it for our purpose.

The model file contains a definition of the gauge group of the model together with the gauge coupling constants and the vector superfields associated to the different group factors. Then a definition of a series of useful indices follows.

After this the superfields of the model are introduced specifying their names, their gauge quantum numbers and the names for their bosonic and fermionic components. Then the interaction and mass basis fields are defined together with the rules that allow to express the interaction basis eigenstates in terms of the mass eigenstates. This is done by using implicitly defined rotation matrices or rotation angles in the case of a  $2 \times 2$  sector. Furthermore, Dirac fields are defined in terms of the Weyl fields appearing as the components of the different superfields. For the mass

eigenstates we also define the decay width, the mass, the Particle Data Group (PDG) code and additional informations that can be needed to produce input files for other programs. The mass and the decay width are defined implicitly and they will be provided at a second stage, after all the parameters have been given a numerical value. For what regards the decay width, it can be left unspecified and micrOMEGAs will take care of calculating it at run-time.

Then, in the “Parameters” section the different parameters appearing in the lagrangian are defined, including the elements of the rotation matrices for the different sectors, the CKM and PMNS matrices for the down-quark and neutrino mixing, the electroweak and strong couplings, the Higgs vevs, the Stückelberg mass, the couplings appearing in the superpotential and the soft breaking parameters. We list the content of the model file below.



```

1 (* ***** Information ***** *)
M$modelName = "AnUSSM";
M$Information = { };

(* ***** Flags ***** *)
6 $CKMdiag = False; (* CKM = identity or not *)
MNSdiag = False; (* PMNS = identity or not *)

(* ***** Gauge groups ***** *)
M$GaugeGroups = {
11 U1X == { Abelian->True, CouplingConstant->gx, Superfield->XSF, Charge->Xc,
          GUTNormalization->3/5},
      U1Y == { Abelian->True, CouplingConstant->gp, Superfield->BSF, Charge->Y,
          GUTNormalization->3/5},
      SU2L == { Abelian->False, CouplingConstant->gw, Superfield->WSF,
16          StructureConstant->ep, Representations->{Ta,SU2D},
          Definitions->{Ta[a_]->PauliSigma[a]/2, ep->Eps}},
      SU3C == { Abelian->False, CouplingConstant->gs, Superfield->GSF,
21          StructureConstant->f, Representations->{{T,Colour},{Tb,Colourb}},
          DTerm->dSUN} };

(* ***** Indices ***** *)
IndexRange[Index[SU2W]] = Unfold[Range[3]]; IndexStyle[SU2W,j];
IndexRange[Index[SU2D]] = Unfold[Range[2]]; IndexStyle[SU2D,k];
IndexRange[Index[Gluon]] = NoUnfold[Range[8]]; IndexStyle[Gluon, a];
26 IndexRange[Index[Colour]] = NoUnfold[Range[3]]; IndexStyle[Colour, m];
IndexRange[Index[Colourb]] = NoUnfold[Range[3]]; IndexStyle[Colourb, m];
IndexRange[Index[NEU]] = Range[7]; IndexStyle[NEU, i];
IndexRange[Index[CHA]] = Range[2]; IndexStyle[CHA, i];
IndexRange[Index[GEN]] = Range[3]; IndexStyle[GEN, f];
31 IndexRange[Index[SCA]] = Range[6]; IndexStyle[SCA, i];
IndexRange[Index[Odd]] = Range[4]; IndexStyle[Odd, i];
IndexRange[Index[Even]] = Range[4]; IndexStyle[Even, i];
IndexRange[Index[Gau]] = Range[3]; IndexStyle[Gau, i];

36 (* ***** Superfields ***** *)
M$Superfields = {
VSF[1] == {ClassName->BSF, GaugeBoson->B, Gaugino->bow},

```

```

VSF [2] == {ClassName->WSF, GaugeBoson->Wi, Gaugino->wow, Indices->{Index[SU2W]}},
VSF [3] == {ClassName->GSF, GaugeBoson->G, Gaugino->gow, Indices->{Index[Gluon]}},
41 VSF [4] == {ClassName->XSF, GaugeBoson->X, Gaugino->xow},
CSF [1] == {ClassName->HU, Chirality->Left, Weyl->huw, Scalar->hus,
QuantumNumbers->{Y-> 1/2, Xc-> XHU}, Indices->{Index[SU2D]}},
CSF [2] == {ClassName->HD, Chirality->Left, Weyl->hdw, Scalar->hds,
QuantumNumbers->{Y->-1/2, Xc-> XHD}, Indices->{Index[SU2D]}},
46 CSF [3] == {ClassName->LL, Chirality->Left, Weyl->LLw, Scalar->LLs,
QuantumNumbers->{Y->-1/2, Xc-> XLL},
Indices->{Index[SU2D], Index[GEN]}},
CSF [4] == {ClassName->ER, Chirality->Left, Weyl->ERw, Scalar->ERs,
QuantumNumbers->{Y-> 1, Xc-> XER}, Indices->{Index[GEN]}},
51 CSF [5] == {ClassName->VR, Chirality->Left, Weyl->VRw, Scalar->VRs,
QuantumNumbers->{Xc-> XVR}, Indices->{Index[GEN]}},
CSF [6] == {ClassName->QL, Chirality->Left, Weyl->QLw, Scalar->QLs,
QuantumNumbers->{Y-> 1/6, Xc-> XQL},
Indices->{Index[SU2D], Index[GEN], Index[Colour]}},
56 CSF [7] == {ClassName->UR, Chirality->Left, Weyl->URw, Scalar->URs,
QuantumNumbers->{Y->-2/3, Xc-> XUR}, Indices->{Index[GEN],
Index[Colourb]}},
CSF [8] == {ClassName->DR, Chirality->Left, Weyl->DRw, Scalar->DRs,
QuantumNumbers->{Y-> 1/3, Xc-> XDR}, Indices->{Index[GEN],
61 Index[Colourb]}},
CSF [9] == {ClassName->AX, Chirality->Left, Weyl->axw, Scalar->axs,
QuantumNumbers->{Y-> 0, Xc-> 0}},
CSF [10] == {ClassName->SS, Chirality->Left, Weyl->ssw, Scalar->sss,
QuantumNumbers->{Y-> 0, Xc-> XS}}};

66 (* ***** Fields ***** *)
M$ClassesDescription = {
(* Gauge bosons: unphysical vector fields *)
V [11] == {ClassName->B, Unphysical->True, SelfConjugate->True,
71 Definitions-> {B[mu_]-> Oga[1,2] A[mu] + Oga[2,2] Z[mu] +
Oga[3,2] Zp[mu] }},
V [12] == {ClassName->Wi, Unphysical->True, SelfConjugate->True,
Indices->{Index[SU2W]}, FlavorIndex->SU2W,
Definitions-> {Wi[mu_,1]-> (Wbar[mu]+W[mu])/Sqrt[2],
76 Wi[mu_,2]->(Wbar[mu]-W[mu])/(I*Sqrt[2]),
Wi[mu_,3]-> Oga[1,1] A[mu] + Oga[2,1] Z[mu] +
Oga[3,1] Zp[mu]}}},
V [13] == {ClassName->X, Unphysical->True, SelfConjugate->True,
Definitions-> {X[mu_]-> Oga[1,3] A[mu] + Oga[2,3] Z[mu] +
81 Oga[3,3] Zp[mu] }},

(* Gauge bosons: physical vector fields *)
V [1] == {ClassName->A, SelfConjugate->True, Mass->0, Width->0, ParticleName->"a",
PDG->22, PropagatorLabel->"A", PropagatorType->Sine,
86 PropagatorArrow->None},
V [2] == {ClassName->Z, SelfConjugate->True, Mass->MZ, Width->WZ, ParticleName->"Z",
PDG->23, PropagatorLabel->"Z", PropagatorType->Sine,
PropagatorArrow->None},
V [3] == {ClassName->W, SelfConjugate->False, Mass->MW, Width->WW,
91 ParticleName->"W+"},

```

```

        AntiParticleName->"W-", QuantumNumbers->{Q->1}, PDG->24,
        PropagatorLabel->"W", PropagatorType->Sine, PropagatorArrow->Forward},
V[4] == {ClassName->G, SelfConjugate->True, Indices->{Index[Gluon]}, Mass->0,
        Width->0, ParticleName->"g", PDG->21, PropagatorLabel->"G",
96     PropagatorType->C, PropagatorArrow->None },
V[5] == {ClassName->Zp, SelfConjugate->True, Mass->MZp, Width->WZp,
        ParticleName->"Zp", PDG->25, PropagatorLabel->"Zp",
        PropagatorType->Sine, PropagatorArrow->None},

101 (* Gauginos: unphysical Weyls *)
W[20] == {ClassName->bow, Unphysical->True, Chirality->Left, SelfConjugate->False,
        Definitions->{bow[s_]:>Module[{i}, -I*Conjugate[NN[i,1]]*neuw[s,i]}}},
W[21] == {ClassName->wow, Unphysical->True, Chirality->Left, SelfConjugate->False,
        Indices->{Index[SU2W]}, FlavorIndex->SU2W,
106     Definitions->{
        wow[s_,1]:>Module[{i}, (Conjugate[UU[i,1]]*chmw[s,i]+
        Conjugate[VV[i,1]]*chpw[s,i])/(I*sqrt[2])],
        wow[s_,2]:>Module[{i}, (Conjugate[UU[i,1]]*chmw[s,i]-
        Conjugate[VV[i,1]]*chpw[s,i])/(-sqrt[2])],
111     wow[s_,3]:>Module[{i}, -I*Conjugate[NN[i,2]]*neuw[s,i]}}},
W[22] == {ClassName->gow, Unphysical->True, Chirality->Left, SelfConjugate->False,
        Indices->{Index[Gluon]}, Definitions->{gow[inds_]->-I*gow[inds]}},
W[27] == {ClassName->xow, Unphysical->True, Chirality->Left, SelfConjugate->False,
        Definitions->{xow[s_]:>Module[{i}, -I*Conjugate[NN[i,5]]*neuw[s,i]}}},
116

(* Higgsinos: unphysical Weyls *)
W[23] == {ClassName->huw, Unphysical->True, Chirality->Left, SelfConjugate->False,
        Indices->{Index[SU2D]}, FlavorIndex->SU2D,
        QuantumNumbers->{Y->1/2, Xc->XHU},
121     Definitions->{
        huw[s_,1]:>Module[{i}, Conjugate[VV[i,2]]*chpw[s,i]],
        huw[s_,2]:>Module[{i}, Conjugate[NN[i,4]]*neuw[s,i]}}},
W[24] == {ClassName->hdw, Unphysical->True, Chirality->Left, SelfConjugate->False,
        Indices->{Index[SU2D]}, FlavorIndex->SU2D,
126     QuantumNumbers->{Y->-1/2, Xc->XHD},
        Definitions->{
        hdw[s_,1]:>Module[{i}, Conjugate[NN[i,3]]*neuw[s,i]],
        hdw[s_,2]:>Module[{i}, Conjugate[UU[i,2]]*chmw[s,i]}}},
W[28] == {ClassName->axw, Unphysical->True, Chirality->Left, SelfConjugate->False,
        QuantumNumbers->{Y->0, Xc->0},
131     Definitions->{axw[s_]:>Module[{i}, Conjugate[NN[i,6]]*neuw[s,i]}}},
W[29] == {ClassName->ssw, Unphysical->True, Chirality->Left, SelfConjugate->False,
        QuantumNumbers->{Y->0, Xc->XS},
        Definitions->{ssw[s_]:>Module[{i}, Conjugate[NN[i,7]]*neuw[s,i]}}},
136

(* Gauginos/Higgsinos: physical Weyls *)
W[1] == {ClassName->neuw, Unphysical->True, Chirality->Left, SelfConjugate->False,
        Indices->{Index[NEU]}, FlavorIndex->NEU },
W[2] == {ClassName->chpw, Unphysical->True, Chirality->Left, SelfConjugate->False,
141     Indices->{Index[CHA]}, FlavorIndex->CHA,
        QuantumNumbers->{Q->1} },
W[3] == {ClassName->chmw, Unphysical->True, Chirality->Left, SelfConjugate->False,
        Indices->{Index[CHA]}, FlavorIndex->CHA,

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QuantumNumbers->{Q->-1} } ,
146 W[4] == {ClassName->goww, Unphysical->True, Chirality->Left, SelfConjugate->False,
Indices->{Index[Gluon]} },

(* Gauginos/Higgsinos: physical Diracs *)
F[1] == {ClassName->neu, SelfConjugate->True, Indices->{Index[NEU]},
151 FlavorIndex->NEU,
WeylComponents->neu,
ParticleName->{"~n1", "~n2", "~n3", "~n4", "~n5", "~n6", "~n7"},
ClassMembers->{neu1, neu2, neu3, neu4, neu5, neu6, neu7},
156 Mass->{Mneu, Mneu1, Mneu2, Mneu3, Mneu4, Mneu5, Mneu6, Mneu7},
Width->{Wneu, Wneu1, Wneu2, Wneu3, Wneu4, Wneu5, Wneu6, Wneu7},
PDG->{1000022, 1000023, 1000025, 1000035, 1000045, 1000055, 1000065},
PropagatorLabel->{"neu", "neu1", "neu2", "neu3", "neu4", "neu5", "neu6", "neu7"},
PropagatorType->Straight, PropagatorArrow->None },
F[2] == {ClassName->ch, SelfConjugate->False, Indices->{Index[CHA]},
161 FlavorIndex->CHA,
WeylComponents->{chpw, chmwb}, ParticleName->{"~x1+", "~x2+"},
AntiParticleName->{"~x1-", "~x2-"}, QuantumNumbers->{Q->1},
ClassMembers->{ch1, ch2}, Mass->{Mch, Mch1, Mch2}, Width->{Wch, Wch1, Wch2},
166 PDG->{1000024, 1000037}, PropagatorLabel->{"ch", "ch1", "ch2"},
PropagatorType->Straight, PropagatorArrow->Forward },
F[3] == {ClassName->go, SelfConjugate->True, Indices->{Index[Gluon]},
WeylComponents->goww, Mass->Mgo, Width->Wgo, ParticleName->"~go",
171 PDG->1000021, PropagatorLabel->"go", PropagatorType->Straight,
PropagatorArrow->None},

(* Higgs: unphysical scalars *)
S[21] == {ClassName->hus, Unphysical->True, SelfConjugate->False,
Indices->{Index[SU2D]}, FlavorIndex->SU2D,
176 QuantumNumbers->{Y-> 1/2, Xc->XHU},
Definitions->{
hus[1]-> Cos[beta]*H + Sin[beta]*GP,
hus[2]:> Module[{i, j}, 1/Sqrt[2]*(vu+Conjugate[Oev[i, 2]]*heven[i] +
I*Conjugate[Ood[j, 2]]*hodd[j])]}},
S[22] == {ClassName->hds, Unphysical->True, SelfConjugate->False,
181 Indices->{Index[SU2D]}, FlavorIndex->SU2D,
QuantumNumbers->{Y->-1/2, Xc->XHD},
Definitions->{
hds[1]:> Module[{i, j}, 1/Sqrt[2]*(vd+Conjugate[Oev[i, 1]]*heven[i] +
I*Conjugate[Ood[j, 1]]*hodd[j])},
186 hds[2]-> Sin[beta]*Hbar - Cos[beta]*GPbar}},
S[29] == {ClassName->axs, Unphysical->True, SelfConjugate->False,
QuantumNumbers->{Y-> 0, Xc->0},
Definitions->{
191 axs:> Module[{i, j}, 1/Sqrt[2]*( vb + Conjugate[Oev[i, 3]]*heven[i] +
I*Conjugate[Ood[j, 3]]*hodd[j])]}},

S[30] == {ClassName->sss, Unphysical->True, SelfConjugate->False,
QuantumNumbers->{Y-> 0, Xc->XS},
Definitions->{
196 sss:> Module[{i, j}, 1/Sqrt[2]*( vs + Conjugate[Oev[i, 4]]*heven[i] +
I*Conjugate[Ood[j, 4]]*hodd[j])]}},

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(* Higgs: physical fields and Goldstones *)
S[1] == {ClassName->heven, SelfConjugate->True, Indices->{Index[Even]},
201   FlavorIndex->Even, ParticleName->{"h01","h02","h03","h04"},
   ClassMembers->{h01,h02,h03,h04}, Mass->{Mh01,Mh02,Mh03,Mh04},
   Width->{Wh01,Wh02,Wh03,Wh04}, PDG->{27,35,45,55},
   PropagatorLabel->{"h0","h01","h02","h03","h04"},
   PropagatorType->ScalarDash, PropagatorArrow->None},
206 S[2] == {ClassName->hodd, SelfConjugate->True, Indices->{Index[Odd]},
   FlavorIndex->Odd, ParticleName->{"A1","A2","G01","G02"},
   ClassMembers->{A1,A2,G01,G02}, Mass->{MA1,MA2,0,0},
   Width->{WA1,WA2,WG01,WG02}, PDG->{26,36,46,56},
   PropagatorLabel->{"odd","A1","A2","G01","G02"},
211   PropagatorType->ScalarDash, PropagatorArrow->None},
S[3] == {ClassName->H, SelfConjugate->False, QuantumNumbers->{Q-> 1},
   Mass->MH,Width->WH,ParticleName->"H+",AntiParticleName->"H-",PDG->39,
   PropagatorLabel->"H",PropagatorType->ScalarDash,
   PropagatorArrow->Forward},
216 S[4] == {ClassName->GP,SelfConjugate->False,QuantumNumbers->{Q-> 1},Mass->MW,
   Width->WGP, Goldstone->W, ParticleName->"G+", AntiParticleName->"G-",
   PDG->251,PropagatorLabel->"GP",PropagatorType->D,
   PropagatorArrow->None},

221 (* Fermions: unphysical Weyls *)
W[25] == {ClassName->LLw,Unphysical->True,Chirality->Left,SelfConjugate->False,
   Indices->{Index[SU2D],Index[GEN]}, FlavorIndex->SU2D,
   QuantumNumbers->{Y->-1/2,Xc->XLL},
   Definitions->{LLw[s_,1,ff_]:>Module[{ff2}, PMNS[ff,ff2]*vLw[s,ff2]],
226   LLw[s_,2,ff_]->eLw[s,ff]}},
W[26] == {ClassName->QLw,Unphysical->True,Chirality->Left,SelfConjugate->False,
   Indices->{Index[SU2D],Index[GEN],Index[Colour]},FlavorIndex->SU2D,
   QuantumNumbers->{Y->1/6,Xc->XQL},
   Definitions->{QLw[s_,1,ff_,cc_]->uLw[s,ff,cc],
231   QLw[s_,2,ff_,cc_]:>Module[{ff2},CKM[ff,ff2] dLw[s,ff2,cc]}},

(* Fermions: physical Weyls *)
W[5] == {ClassName->vLw,Unphysical->True,Chirality->Left,SelfConjugate->False,
   Indices->{Index[GEN]}, FlavorIndex->GEN },
236 W[6] == {ClassName->eLw,Unphysical->True,Chirality->Left,SelfConjugate->False,
   Indices->{Index[GEN]}, FlavorIndex->GEN },
W[7] == {ClassName->VRw,Unphysical->True,Chirality->Left,SelfConjugate->False,
   Indices->{Index[GEN]}, FlavorIndex->GEN, QuantumNumbers->{Xc->XVR}},
W[8] == {ClassName->ERw,Unphysical->True,Chirality->Left,SelfConjugate->False,
241   Indices->{Index[GEN]},FlavorIndex->GEN,QuantumNumbers->{Y->1,Xc->XER}},
W[9] == {ClassName->uLw,Unphysical->True, Chirality->Left, SelfConjugate->False,
   Indices->{Index[GEN],Index[Colour]}, FlavorIndex->GEN },
W[10]== {ClassName->dLw,Unphysical->True,Chirality->Left,SelfConjugate->False,
   Indices->{Index[GEN],Index[Colour]}, FlavorIndex->GEN },
246 W[11]== {ClassName->URw,Unphysical->True,Chirality->Left,SelfConjugate->False,
   Indices->{Index[GEN],Index[Colourb]}, FlavorIndex->GEN,
   QuantumNumbers->{Y->-2/3,Xc->XUR}},
W[12]== {ClassName->DRw,Unphysical->True,Chirality->Left,SelfConjugate->False,
   Indices->{Index[GEN],Index[Colourb]}, FlavorIndex->GEN,

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251     QuantumNumbers ->{Y-> 1/3,Xc->XDR}},

(* Fermions: physical Dirac *)
F[4] == {ClassName->v1,SelfConjugate->False,Indices->{Index[GEN]},
        FlavorIndex->GEN,WeylComponents->{vLw,VRwbar},
256     ParticleName->{"ve","vm","vt"},AntiParticleName->{"ve~","vm~","vt~"},
        ClassMembers->{ve,vm,vt},Mass->{Mv1,Mve,Mvm,Mvt}, Width->0,
        PDG->{12,14,16},PropagatorLabel->{"v","ve","vm","vt"},
        PropagatorType->Straight,PropagatorArrow->Forward},
F[5] == {ClassName->l,SelfConjugate->False,Indices->{Index[GEN]},
261     FlavorIndex->GEN,WeylComponents->{eLw,ERwbar},QuantumNumbers->{Q->-1},
        ParticleName->{"e-","mu-","tau-"},
        AntiParticleName->{"e+","mu+","tau+"},ClassMembers->{e,m,ta},
        Mass->{Ml,Me,Mm,Mta}, Width->0,PDG->{11,13,15},
        PropagatorLabel->{"l","e","mu","tau"},
266     PropagatorType->Straight, PropagatorArrow->Forward},
F[6] == {ClassName->uq,SelfConjugate->False,Indices->{Index[GEN],Index[Colour]},
        FlavorIndex->GEN,WeylComponents->{uLw,URwbar},QuantumNumbers->{Q->2/3},
        ParticleName->{"u","c","t"}, AntiParticleName->{"u~","c~","t~"},
        ClassMembers->{u,c,t}, Mass->{Muq,MU,MC,MT}, Width->{Wuq,0,0,WT},
271     PDG->{2,4,6},PropagatorLabel->{"uq","u","c","t"},
        PropagatorType->Straight,PropagatorArrow->Forward},
F[7]=={ClassName->dq, SelfConjugate->False, Indices->{Index[GEN],Index[Colour]},
        FlavorIndex->GEN,WeylComponents->{dLw,DRwbar},QuantumNumbers->{Q->-1/3},
        ParticleName->{"d","s","b"}, AntiParticleName->{"d~","s~","b~"},
276     ClassMembers->{d,s,b}, Mass->{Mdq,MD,MS,MB}, Width->0,PDG->{1,3,5},
        PropagatorLabel->{"dq","d","s","b"},PropagatorType->Straight,
        PropagatorArrow->Forward},

(* Sfermion: unphysical scalars *)
281 S[23] == {ClassName->LLs, Unphysical->True, SelfConjugate->False,
        Indices->{Index[SU2D], Index[GEN]}, FlavorIndex->SU2D,
        QuantumNumbers->{Y->-1/2,Xc->XLL},
        Definitions->{LLs[1,ff_]:>Module[{ff2,ff3},
286             Conjugate[Rn[ff3,ff2]]*PMNS[ff,ff2]*sn[ff3]],
            LLs[2,ff_]:>Module[{ff2},Conjugate[R1L[ff2,ff]]*sl[ff2]]}},
S[24] == {ClassName->ERs, Unphysical->True, SelfConjugate->False,
        Indices->{Index[GEN]},FlavorIndex->GEN,QuantumNumbers->{Y->1,Xc->XER},
        Definitions->{ ERs[ff_] :> Module[{ff2}, slbar[ff2]*R1R[ff2,ff]] } },
S[25] == {ClassName->VRs, Unphysical->True, SelfConjugate->False,
291     Indices->{Index[GEN]}, FlavorIndex->GEN, QuantumNumbers->{Xc->XVR},
        Definitions->{ VRs[_] -> 0 } },
S[26] == {ClassName->QLs, Unphysical->True, SelfConjugate->False,
        Indices->{Index[SU2D], Index[GEN],Index[Colour]}, FlavorIndex->SU2D,
        QuantumNumbers->{Y->1/6,Xc->XQL},
296     Definitions->{QLs[1,ff_,cc_]:>Module[{ff2},
            Conjugate[RuL[ff2,ff]]*su[ff2,cc]],
            QLs[2,ff_,cc_]:>Module[{ff2,ff3},
            Conjugate[RdL[ff2,ff3]]*CKM[ff,ff3]*sd[ff2,cc]]}},
S[27] == {ClassName->URs, Unphysical->True, SelfConjugate->False,
301     Indices->{Index[GEN],Index[Colourb]}, FlavorIndex->GEN,
        QuantumNumbers->{Y->-2/3,Xc->XUR},
        Definitions->{ URs[ff_,cc_]:>Module[{ff2},subar[ff2,cc]*RuR[ff2,ff]]}},

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S[28] == {ClassName->DRs, Unphysical->True, SelfConjugate->False,
  Indices->{Index[GEN],Index[Colourb]}, FlavorIndex->GEN,
306   QuantumNumbers->{Y-> 1/3,Xc->XDR},
   Definitions->{ DRs[ff_,cc_]:>Module[{ff2},sdbar[ff2,cc]*RdR[ff2,ff]}},

(* Sfermion: physical scalars *)
S[11] == {ClassName->sn, SelfConjugate->False, Indices->{Index[GEN]},
311   FlavorIndex->GEN, ParticleName->{"~sv1","~sv2","~sv3"},
   AntiParticleName->{"~Sv1","~Sv2","~Sv3"},
   ClassMembers->{sn1, sn2, sn3},Mass->{Msn,Msn1,Msn2,Msn3},
   Width->{Wsn,Wsn1,Wsn2,Wsn3}, PDG->{1000012,1000014,1000016},
   PropagatorLabel->{"sn","sn1","sn2","sn3"}, PropagatorType->ScalarDash,
316   PropagatorArrow->Forward },

S[12] == {ClassName->s1, SelfConjugate->False, Indices->{Index[SCA]},
  FlavorIndex->SCA, QuantumNumbers->{Q->-1},
  ParticleName->{"~s11","~s12","~s13","~s14","~s15","~s16"},
  AntiParticleName->{"~S11","~S12","~S13","~S14","~S15","~S16"},
321   ClassMembers->{s11,s12,s13,s14,s15,s16},
   Mass->{Msl,Msl1,Msl2,Msl3,Msl4,Msl5,Msl6},
   Width->{Wsl,Wsl1,Wsl2,Wsl3,Wsl4,Wsl5,Wsl6},
   PDG->{1000011,1000013,1000015,2000011,2000013,2000015},
   PropagatorLabel->{"s1","s11","s12","s13","s14","s15","s16"},
326   PropagatorType->ScalarDash, PropagatorArrow->Forward},

S[13]=={ClassName->su,SelfConjugate->False,Indices->{Index[SCA],Index[Colour]},
  FlavorIndex->SCA, QuantumNumbers->{Q-> 2/3},
  ParticleName->{"~su1","~su2","~su3","~su4","~su5","~su6"},
  AntiParticleName->{"~Su1","~Su2","~Su3","~Su4","~Su5","~Su6"},
331   ClassMembers->{su1,su2,su3,su4,su5,su6},
   Mass->{Msu,Msu1,Msu2,Msu3,Msu4,Msu5,Msu6},
   Width->{Wsu,Wsu1,Wsu2,Wsu3,Wsu4,Wsu5,Wsu6},
   PDG->{1000002,1000004,1000006,2000002,2000004,2000006},
   PropagatorLabel->{"su","su1","su2","su3","su4","su5","su6"},
336   PropagatorType->ScalarDash, PropagatorArrow->Forward},

S[14]=={ClassName->sd,SelfConjugate->False,Indices->{Index[SCA],Index[Colour]},
  FlavorIndex->SCA, QuantumNumbers->{Q->-1/3},
  ParticleName->{"~sd1","~sd2","~sd3","~sd4","~sd5","~sd6"},
  AntiParticleName->{"~Sd1","~Sd2","~Sd3","~Sd4","~Sd5","~Sd6"},
341   ClassMembers->{sd1,sd2,sd3,sd4,sd5,sd6},
   Mass->{Msd,Msd1,Msd2,Msd3,Msd4,Msd5,Msd6},
   Width->{Wsd,Wsd1,Wsd2,Wsd3,Wsd4,Wsd5,Wsd6},
   PDG->{1000001,1000003,1000005,2000001,2000003,2000005},
   PropagatorLabel->{"sd","sd1","sd2","sd3","sd4","sd5","sd6"},
346   PropagatorType->ScalarDash, PropagatorArrow->Forward}
};

(* ***** Parameters ***** *)
M$Parameters = {
351 (* Mixing: external parameters *)
  RMNS== { ParameterType->External, ComplexParameter->False,
    Indices->{Index[GEN],Index[GEN]}, BlockName->UPMNS,
    Description->"Neutrino PMNS mixing matrix (real part)"},
  IMNS== { ParameterType->External, ComplexParameter->False,
356   Indices->{Index[GEN],Index[GEN]}, BlockName->IMUPMNS,

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        Description->"Neutrino PMNS mixing matrix (imaginary part)"},
RCKM== { ParameterType->External, ComplexParameter->False,
        Indices->{Index[GEN],Index[GEN]}, BlockName->VCKM,
        Description->"CKM mixing matrix (real part)"},
361 ICKM== { ParameterType->External, ComplexParameter->False,
        Indices->{Index[GEN],Index[GEN]}, BlockName->IMVCKM,
        Description->"CKM mixing matrix (imaginary part)"},
RNN == {ParameterType->External, ComplexParameter->False,
        Indices->{Index[NEU],Index[NEU]}, BlockName->NMIX,
366  Description->"Neutralino mixing matrix (real part)"},
INN == {ParameterType->External, ComplexParameter->False,
        Indices->{Index[NEU],Index[NEU]}, BlockName->IMNMIX,
        Description->"Neutralino mixing matrix (imaginary part)"},
alp == {TeX->\[Alpha], ParameterType->External, ComplexParameter->False,
371  BlockName->FRALPHA, Description-> "Neutral Higgses mixing angle"},

(* Mixing: internal parameters *)
cw =={TeX->Subscript[c,w],ParameterType->Internal,ComplexParameter->False,
        Value->MW/MZ, Description->"Cosine of the weak angle"},
376 sw =={TeX->Subscript[s,w], ParameterType->Internal, ComplexParameter->False,
        Value->Sqrt[1-cw^2], Description->"Sine of the weak angle"},
Oev =={TeX->Superscript[0,even],ParameterType->External,ComplexParameter->False,
        Indices->{Index[Even],Index[Even]}, Unitary->True,
        Description-> "Neutral even sector rotation matrix"},
381 Ood =={TeX->Superscript[0,odd],ParameterType->External,ComplexParameter->False,
        Indices->{Index[Odd],Index[Odd]}, Unitary->True,
        Description-> "Neutral odd sector rotation matrix"},
Oga=={TeX->Superscript[0,gauge],ParameterType->External,ComplexParameter->False,
        Indices->{Index[Gau],Index[Gau]}, Unitary->True,
386  Description-> "Neutral gauge sector rotation matrix"},
PMNS== { TeX->Superscript[U,pmns], ParameterType->Internal,
        ComplexParameter->True, Indices->{Index[GEN],Index[GEN]},
        Unitary->True,If[ $\$MNSDiag$ , Definitions:>{PMNS[i_,j_]:>0 /;(i!=j),
        PMNS[i_,j_]:>1/(i==j)},Value->{PMNS[i_,j_]:>RMNS[i,j]+I*IMNS[i,j]}],
391  Description-> "Neutrino PMNS mixing matrix"},
CKM =={TeX->Superscript[V,ckm],ParameterType->Internal,ComplexParameter->True,
        Indices->{Index[GEN],Index[GEN]}, Unitary->True,
        If[ $\$CKMDiag$ ,Definitions:>{CKM[i_,j_]:>0 /;(i!=j),CKM[i_,j_]:>1/(i==j)},
        Value->{CKM[i_,j_]:>RCKM[i,j]+I*RCKM[i,j]}],
396  Description-> "CKM mixing matrix"},
NN =={TeX->N, ParameterType->Internal, ComplexParameter->True,
        Indices->{Index[NEU],Index[NEU]}, Unitary->True,
        Value->{NN[i_,j_]->RNN[i,j]+I*INN[i,j]},
        Description-> "Neutralino mixing matrix"},
401 UU =={TeX->U, ParameterType->Internal, ComplexParameter->False,
        Indices->{Index[CHA],Index[CHA]}, Unitary->True,
        Description-> "Chargino mixing matrix U"},
VV =={TeX->V, ParameterType->Internal, ComplexParameter->False,
        Indices->{Index[CHA],Index[CHA]}, Unitary->True,
406  Description-> "Chargino mixing matrix V"},
Rl =={TeX->Superscript[R,l], ParameterType->External, ComplexParameter->False,
        Indices->{Index[SCA],Index[SCA]}, Unitary->True,
        Description-> "Slepton mixing matrix"},

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Rn =={TeX->Superscript[R,n], ParameterType->External, ComplexParameter->False,
411 Indices->{Index[GEN],Index[GEN]}, Unitary->True,
Description->"Sneutrino mixing matrix"},
Ru =={TeX->Superscript[R,u], ParameterType->External, ComplexParameter->False,
Indices->{Index[SCA],Index[SCA]}, Unitary->True,
Description->"Up squark mixing matrix"},
416 Rd =={TeX->Superscript[R,d], ParameterType->External, ComplexParameter->False,
Indices->{Index[SCA],Index[SCA]}, Unitary->True,
Description->"Down squark mixing matrix"},

(* Left and right parts of the sfermion mixing matrices *)
421 RlL == {TeX->Superscript[RL,l],ParameterType->External,ComplexParameter->False,
Indices->{Index[SCA],Index[GEN]}, Unitary->False,
Definitions->{RlL[i_,j_]:>Rl[i,j]/;NumericQ[j]},
Description->"Slepton mixing matrix - first three columns"},
RlR == {TeX->Superscript[RR,l],ParameterType->External,ComplexParameter->False,
426 Indices->{Index[SCA],Index[GEN]}, Unitary->False,
Definitions->{RlR[i_,j_]:>Rl[i,j+3]/;NumericQ[j]},
Description->"Slepton mixing matrix - last three columns"},
RuL == {TeX->Superscript[RL,u],ParameterType->External,ComplexParameter->False,
Indices->{Index[SCA],Index[GEN]}, Unitary->False,
431 Definitions->{RuL[i_,j_]:>Ru[i,j]/;NumericQ[j]},
Description->"Up squark mixing matrix - first three columns"},
RuR == {TeX->Superscript[RR,u],ParameterType->External,ComplexParameter->False,
Indices->{Index[SCA],Index[GEN]}, Unitary->False,
Definitions->{RuR[i_,j_]:>Ru[i,j+3]/;NumericQ[j]},
436 Description->"Up squark mixing matrix - last three columns"},
RdL == {TeX->Superscript[RL,d],ParameterType->External,ComplexParameter->False,
Indices->{Index[SCA],Index[GEN]}, Unitary->False,
Definitions->{RdL[i_,j_]:>Rd[i,j]/;NumericQ[j]},
Description->"Down squark mixing matrix - first three columns"},
441 RdR == {TeX->Superscript[RR,d],ParameterType->External,ComplexParameter->False,
Indices->{Index[SCA],Index[GEN]}, Unitary->False,
Definitions->{RdR[i_,j_]:>Rd[i,j+3]/;NumericQ[j]},
Description->"Down squark mixing matrix - last three columns"},

446 (* Couplings constants: external parameters *)
aEWM1 == {TeX->Subsuperscript[\[Alpha],w,-1], ParameterType->External,
ComplexParameter->False, BlockName->SMINPUTS, OrderBlock->1,
InteractionOrder->{QED,-2},
Description->"Inverse of the EW coupling constant at the Z pole"},
451 aS=={TeX->Subscript[\[Alpha],s],ParameterType->External,ComplexParameter->False,
BlockName->SMINPUTS, OrderBlock->3, InteractionOrder->{QCD,2},
Description->"Strong coupling constant at the Z pole."},

(* Couplings constants: internal parameters *)
456 ee == {TeX->e, ParameterType->Internal, ComplexParameter->False,
Value->Sqrt[4 Pi / aEWM1], InteractionOrder->{QED,1},
Description->"Electric coupling constant"},
gs == {TeX->Subscript[g,s], ParameterType->Internal, ComplexParameter->False,
Value->Sqrt[4 Pi aS], InteractionOrder->{QCD,1}, ParameterName->G,
461 Description->"Strong coupling constant"},
gp == {TeX->g', ParameterType->External, ComplexParameter->False,

```

```

        InteractionOrder->{QED,1},
        Description->"Hypercharge coupling constant at the Z pole"},
gw == {TeX->Subscript[g,w], ParameterType->External, ComplexParameter->False,
466   InteractionOrder->{QED,1},
        Description->"Weak coupling constant at the Z pole"},
gx == {TeX->Subscript[g,x], ParameterType->External, ComplexParameter->False,
        Description->"Extra U(1) coupling"},

471 (* Higgs sector: external parameters *)
tb=={TeX->Subscript[t,b], ParameterType->External, ComplexParameter->False,
        BlockName->HMIX,OrderBlock->2,Description->"Ratio of the two Higgs vevs"},

(* Higgs sector: internal parameters *)
476 beta == {TeX->\[Beta], ParameterType->Internal, ComplexParameter->False,
        Value->ArcTan[tb],
        Description->"Arctan of the ratio of the two Higgs vevs"},
vev == {TeX->v, ParameterType->Internal, ComplexParameter->False,
        Value->2*MZ*sw*cw/ee,
481   InteractionOrder->{QED,-1},
        Description->"Higgs vacuum expectation value"},
Mst == {TeX->MSt, ParameterType->External, ComplexParameter->False,
        Description->"Stueckelberg mass"},
vd == {TeX->Subscript[v,d], ParameterType->Internal, ComplexParameter->False,
486   Value->vev*Cos[beta], InteractionOrder->{QED,-1},
        Description->"Down-type Higgs vacuum expectation value"},
vu == {TeX->Subscript[v,u], ParameterType->Internal, ComplexParameter->False,
        Value->vev*Sin[beta], InteractionOrder->{QED,-1},
        Description->"Up-type Higgs vacuum expectation value"},
491 vs == {TeX->Subscript[v,S], ParameterType->External, ComplexParameter->False,
        Description->"Singlet Higgs vacuum expectation value"},
vb == {TeX->Subscript[v,b], ParameterType->External, ComplexParameter->False,
        Description->"Saxion vacuum expectation value"},

496 (* Superpotential: internal parameters *)
yu == {TeX->Superscript[y,u], ParameterType->External, ComplexParameter->False,
        Indices->{Index[GEN],Index[GEN]},
        Definitions:>{yu[i_,j_]:>0 /;(i!=j)}, InteractionOrder->{QED,1},
        Description->"Up-type quark Yukawa matrix"},
501 yd == {TeX->Superscript[y,d], ParameterType->External, ComplexParameter->False,
        Indices->{Index[GEN],Index[GEN]},
        Definitions:>{yd[i_,j_]:>0 /;(i!=j)}, InteractionOrder->{QED,1},
        Description->"Down-type quark Yukawa matrix"},
ye == {TeX->Superscript[y,e], ParameterType->External, ComplexParameter->False,
506   Indices->{Index[GEN],Index[GEN]},
        Definitions:>{ye[i_,j_]:>0 /;(i!=j)}, InteractionOrder->{QED,1},
        Description->"Charged lepton Yukawa matrix"},
lam == {TeX->\[Lambda], ParameterType->External, ComplexParameter->False,
        Description->"S Hu Hd coupling"},
511 alam == {TeX->Subscript[a,\[Lambda]], ParameterType->External,
        ComplexParameter->False,
        Description->"S Hu Hd breaking term"},

(* Soft terms: external parameters *)

```

```

516 mHu2=={TeX->Subsuperscript[m,Subscript[H,u],2], ParameterType->External,
      ComplexParameter->False, BlockName->MSOFT, OrderBlock->21,
      Description->"Up-type Higgs squared mass"},
mHd2=={TeX->Subsuperscript[m,Subscript[H,d],2], ParameterType->External,
      ComplexParameter->False, BlockName->MSOFT, OrderBlock->22,
521      Description->"Down-type Higgs squared mass"},
mS2== {TeX->Subsuperscript[m,S,2], ParameterType->External,
      ComplexParameter->False,
      BlockName->MSOFT, OrderBlock->22,
      Description->"Singlet Higgs squared mass"},
526 mb2== {TeX->Subsuperscript[m,Reb,2], ParameterType->External,
      ComplexParameter->False,
      BlockName->MSOFT, OrderBlock->22, Description->"Saxion squared mass"},
Mx1== {TeX->Subscript[M,Y], ParameterType->External, ComplexParameter->False,
      Description->"Bino mass"},
531 Mx2== {TeX->Subscript[M,W], ParameterType->External, ComplexParameter->False,
      Description->"Wino mass"},
Mx3== {TeX->Subscript[M,G], ParameterType->External, ComplexParameter->False,
      Description->"Gluino mass"},
Mx4 =={TeX->Subscript[M,B], ParameterType->External, ComplexParameter->False,
536      Description->"Xino mass"},
Mx5 =={TeX->Subscript[M,YB], ParameterType->External, ComplexParameter->False,
      Description->"Xino-Bino mass"},
Mxa =={TeX->Subscript[M,ax], ParameterType->External, ComplexParameter->False,
      Description->"Axino mass"},
541 (* Soft terms: internal parameters *)
mL2 == { TeX->Subsuperscript[m,OverTilde[L],2], ParameterType->External,
      ComplexParameter->False, Indices->{Index[GEN],Index[GEN]},
      Definitions:>{mL2[i_,j_]:>0 /(i!=j)},
      Description-> "Left-handed slepton squared mass matrix"},
546 mE2 == { TeX->Subsuperscript[m,OverTilde[E],2], ParameterType->External,
      ComplexParameter->False, Indices->{Index[GEN],Index[GEN]},
      Definitions:>{mE2[i_,j_]:>0 /(i!=j)},
      Description-> "Right-handed slepton squared mass matrix"},
mQ2 == { TeX->Subsuperscript[m,OverTilde[Q],2], ParameterType->External,
551      ComplexParameter->False, Indices->{Index[GEN],Index[GEN]},
      Definitions:>{mQ2[i_,j_]:>0 /(i!=j)},
      Description-> "Left-handed squark squared mass matrix"},
mU2 == { TeX->Subsuperscript[m,OverTilde[U],2], ParameterType->External,
      ComplexParameter->False, Indices->{Index[GEN],Index[GEN]},
556      Definitions:>{mU2[i_,j_]:>0 /(i!=j)},
      Description-> "Right-handed up-type squark squared mass matrix"},
mD2 == { TeX->Subsuperscript[m,OverTilde[D],2], ParameterType->External,
      ComplexParameter->False, Indices->{Index[GEN],Index[GEN]},
      Definitions:>{mD2[i_,j_]:>0 /(i!=j)},
561      Description-> "Right-handed down-type squark squared mass matrix"},
te == { TeX->Subscript[T,e], ParameterType->External, ComplexParameter->False,
      Indices->{Index[GEN],Index[GEN]},
      Definitions:>{te[i_,j_]:>0 /(i!=j)}, InteractionOrder->{QED,1},
      Description-> "Charged slepton trilinear coupling"},
566 tu == { TeX->Subscript[T,u], ParameterType->External, ComplexParameter->False,
      Indices->{Index[GEN],Index[GEN]},
      Definitions:>{tu[i_,j_]:>0 /(i!=j)}, InteractionOrder->{QED,1},

```

```

        Description-> "Up-type squark trilinear coupling"},
571 td == { TeX->Subscript[T,d], ParameterType->External, ComplexParameter->False,
        Indices->{Index[GEN],Index[GEN]}, InteractionOrder->{QED,1},
        Definitions:>{td[i_,j_]:>0 /;(i!=j)},
        Description-> "Down-type squark trilinear coupling"},

(* Extra U(1) charges *)
576 XHU == { TeX->XHu, ParameterType->External, ComplexParameter->False,
        Description-> "Higgs up doublet extra U(1) charge"},
XHD == { TeX->XHd, ParameterType->External, ComplexParameter->False,
        Description-> "Higgs down doublet extra U(1) charge"},
581 XLL == { TeX->XLl, ParameterType->External, ComplexParameter->False,
        Description-> "Lepton doublet extra U(1) charge"},
XER == { TeX->XEr, ParameterType->External, ComplexParameter->False,
        Description-> "Lepton singlet extra U(1) charge"},
XQL == { TeX->XQl, ParameterType->External, ComplexParameter->False,
        Description-> "Quark doublet extra U(1) charge"},
586 XUR == { TeX->XUr, ParameterType->External, ComplexParameter->False,
        Description-> "Up singlet extra U(1) charge"},
XDR == { TeX->XDr, ParameterType->External, ComplexParameter->False,
        Description-> "Down singlet extra U(1) charge"},
XVR == { TeX->XVr, ParameterType->External, ComplexParameter->False,
601 Description-> "Neutrino extra U(1) charge"},
XAX == { TeX->XAx, ParameterType->External, ComplexParameter->False,
        Description-> "Axion extra U(1) charge"},
XS == { TeX->XSs, ParameterType->External, ComplexParameter->False,
        Description-> "Singlet extra U(1) charge"},
596 bG == { TeX->Subscript[b,G], ParameterType->External, ComplexParameter->False,
        Description-> ""},
bW == { TeX->Subscript[b,W], ParameterType->External, ComplexParameter->False,
        Description-> ""},
bY == { TeX->Subscript[b,Y], ParameterType->External, ComplexParameter->False,
601 Description-> ""},
bX == { TeX->Subscript[b,X], ParameterType->External, ComplexParameter->False,
        Description-> ""},
bXY== { TeX->Subscript[b,XY], ParameterType->External, ComplexParameter->False,
        Description-> ""}
606 };

```



Once all these definitions are made we can define the Lagrangian. We can define the product of superfields that appear in the lagrangian and FeynRules is able to handle the algebra of the Grassmannian superspace coordinates  $\theta$  and  $\bar{\theta}$  and to split the result according to the various combination of Grassmannian variables that appear in the result. Then, performing the Berezin integral in order to obtain the desired lagrangian density consists in selecting the appropriate component of the result. This is done through the following commands.



```

(*****)
Print["Define the vector lagrangian"];
LVector := Module[{}],

```

```

4 Plus @@ (Module[{tmp}, tmp = SF2Components[#];
Expand[tmp[[2, 5]]+tmp[[2, 6]]] & /@ (List @@ VSFkineticTerms[]));

(*****)
Print["Define the Wess-Zumino lagrangian"];
9 LWZ = -1/2 bY ((AX Ueps[al, be] SuperfieldStrengthL[BSF, al] \
SuperfieldStrengthL[BSF, be] // SF2Components)[[2, 5]]) -
1/2 bX ((AX Ueps[al, be] SuperfieldStrengthL[XSF, al] \
SuperfieldStrengthL[XSF, be] // SF2Components)[[2, 5]]) -
14 1/2 bXY ((AX Ueps[al, be] SuperfieldStrengthL[XSF,al] \
SuperfieldStrengthL[BSF, be] // SF2Components)[[2, 5]]) -
1/2 bG 1/(16 gs^2) 1/2 ((AX Ueps[al, be] SuperfieldStrengthL[GSF, al,ga] \
SuperfieldStrengthL[GSF, be, ga] // SF2Components)[[2, 5]]) -
1/2 bW 1/(16 gw^2) 1/2 ((AX Ueps[al, be] SuperfieldStrengthL[WSF, al, ga] \
SuperfieldStrengthL[WSF, be, ga] // SF2Components)[[2, 5]]);
19 LWZ = LWZ + HC[LWZ]//Expand//Simplify//Expand;

(*****)
Print["Define the Stueckelberg lagrangian"];
24 LSt = 1/2 ((AX+HC[AX]+Sqrt[2] Mst XSF)^2//SF2Components)[[2,9]]//Expand;

(*****)
Print["Define the chiral lagrangian"];
(* The kinetic terms for the axion superfield are contained in LSt *)
29 LChiral = Plus @@ (Theta2Thetabar2Component[#] & /@ (List @@ (CSFKineticTerms[]
- CSFKineticTerms[AX]) ) );

(*****)
Print["Define the superpotential and the corresponding lagrangian"];
34 SPot := Module[{ff1, ff2, ff3, cc1}, yu[ff1, ff2] UR[ff1, cc1]
(QL[1, ff2, cc1] HU[2] - QL[2, ff2, cc1] HU[1]) +
yd[ff1, ff3] Conjugate[CKM[ff2, ff3]] DR[ff1, cc1]
(QL[1, ff2, cc1] HD[2] - QL[2, ff2, cc1] HD[1]) +
ye[ff1, ff2] ER[ff1] (LL[1, ff2] HD[2] - LL[2, ff2] HD[1]) +
39 lam SS (HU[1] HD[2] - HU[2] HD[1]);

LSuperW = (Plus @@ (Module[{tmp}, tmp = SF2Components[#];
tmp[[2, 5]] + tmp[[2, 6]]]& /@ (List @@ Expand[SPot + HC[SPot]])) \
/.Conjugate[CKM[a_, b_]]*CKM[a_, c_] -> IndexDelta[b, c];
44

(*****)
Print["Define the soft breaking terms"];
LSoft = Module[{Mino, MSca, Tri, Bil},
(*Gaugino mass terms*)
49 Mino := Module[{s, gl}, -Mx1*bow[s].bow[s] - Mx4*xow[s].xow[s] -
Mx5*(bow[s].xow[s] + xow[s].bow[s]) -
Mx2*wow[s, gl].wow[s, gl] - Mx3*gow[s, gl].gow[s, gl] +
Mxa*axw[s].axw[s]];
(*Scalar mass terms*)
54 MSca := Module[{ii, ff1, ff2, ff3, ff4, cc1},
-mb2 1/4 (axs + axsbar)^2 - mS2*HC[sss]*sss -
mHu2*HC[hus[ii]]*hus[ii] - mHd2*HC[hds[ii]]*hds[ii] -

```

```

mL2[ff1, ff2]*HC[LLs[ii, ff1]]*LLs[ii, ff2] -
mE2[ff1, ff2]*HC[ERs[ff1]]*ERs[ff2] -
59 CKM[ff1, ff2]*mQ2[ff2, ff3]*Conjugate[CKM[ff4, ff3]]*
HC[QLs[ii, ff1, cc1]]*QLs[ii, ff4, cc1] -
mU2[ff1, ff2]*HC[URs[ff1, cc1]]*URs[ff2, cc1] -
mD2[ff1, ff2]*HC[DRs[ff1, cc1]]*DRs[ff2, cc1]];
(*Trilinear couplings*)
64 Tri := -tu[ff1, ff2]*URs[ff1, cc1] (QLs[1, ff2, cc1] hus[2] -
      QLs[2, ff2, cc1] hus[1]) - Conjugate[CKM[ff3, ff2]]*td[ff1, ff2]*
      DRs[ff1, cc1] (QLs[1, ff3, cc1] hds[2] - QLs[2, ff3, cc1] hds[1]) -
      te[ff1, ff2]*ERs[ff1] (LLs[1, ff2] hds[2] - LLs[2, ff2] hds[1]) -
      alam sss (hus[1] hds[2] - hus[2] hds[1]);
69 (*Everything together*)
(Mino + HC[Mino])/2 + MSca + Tri + HC[Tri]];

```



After defining the lagrangian we calculate the equations of motion for the  $D$  and  $F$  auxiliary fields (these are automatically defined according to the definitions of the superfields of the theory) and substitute them back in the lagrangian. Then we can express the lagrangian in terms of mass eigenstates applying the definitions declared in the model file. These definitions usually contain rotation matrix elements whose analytic expression we're not able to obtain since the corresponding mass matrices are quite complicated (for example, we have a  $4 \times 4$  mass matrix in the CP-even Higgs sector or a  $7 \times 7$  matrix in the neutralino sector). In the process of writing the CalcHEP model files these matrix elements, according to the definitions made in the model file, are treated as numerical variables and we take care to give them the correct value, depending on the chosen parameters of the model. In order to do this, we calculate the analytic expressions of the mass matrices of the model and store them for later use. Then the CalcHEP interface function is called and the CalcHEP model files are written. These are five files containing the vertices of the theory, a series of automatically defined variables used in the definition of the vertices, a list of the particles appearing in the vertices and of their properties (such as mass, width, PDG code) and a list of the parameters of the model with their value. In particular, the parameters of the model are written in a file called "vars1.mdl" and, according to the definitions we have made in the model file, it also contains the rotation matrix elements. Clearly, this file has to be rewritten in order to include the correct numerical values for the rotation matrix elements and the mass eigenvalues that we obtain once a specific choice of the free parameters is made and it has to be rewritten every time we change the value of any parameter. In order to do this we wrote a Mathematica package that loads the definitions of the mass matrices of the model (that we have obtained from the lagrangian and stored in a text file) and the numerical values of the parameters (stored in another text file), substitutes the numerical values for the parameters and evaluates the eigenvalues and eigenvectors of the mass matrices, thus defining the numerical value of the elements of the rotation matrices. The results obtained are then used to rewrite the "vars1.mdl" file in which all of the parameters appearing in the vertex definitions and the masses

of the particles are assigned a numerical value. At this stage everything is set and we can go on with the calculation of the relic density or with the numerical calculation of any tree level cross section or decay amplitude in the model using micrOMEGAs or CalcHEP

## A.2 CalcHEP

CalcHEP[88] is a program that allows one to perform cross section and decay rate calculations at tree-level in a generic particle physics model. In order to do this, the vertices of the model and its particle content have to be defined in some model files. Once this is done the package is able to generate the tree-level Feynman diagrams for a given process (also allowing to exclude the diagrams where some selected particles appear) and then write the necessary code for the numerical evaluation of the result. This code can be written in different languages in order to allow further manipulations but the files used in the numerical calculation made within CalcHEP are written in C. Then the code is able to evaluate the cross section or decay rate for the required process. For more informations about the capabilities of the package we refer the reader to the [package website](#).

## A.3 micrOMEGAs

micrOMEGAs[89] is a package that allows to calculate the properties of a cold dark matter candidate (that is a massive and stable particle) in a generic supersymmetric particle physics model. It allows to calculate the relic density for a given dark matter candidate (including all the annihilation and co-annihilation channels) and the direct and indirect detection rates. The necessary cross sections are evaluated using CalcHEP so the specific model has to be defined in terms of the CalcHEP model files. The package can be extended invoking external programs such as programs that perform the renormalization group evolution of the parameters or codes for the calculation of critical observables; this is done for the MSSM case, which is included in the program bundle, in which the interface with several spectrum calculator is defined.

For further informations we refer the reader to the [program website](#).

### A.3.1 Sample micrOMEGAs output

Here we list a sample output from micrOMEGAs obtained using the generated CalcHEP model files for our model (see A.1). In this case we have set  $v_S = 1.45$  TeV,  $M_{St} = 8$  TeV and  $\tan\beta = 10$ . These values, together with the remaining ones defined in Sec. 5.6, have been used to numerically evaluate the eigenvalues and eigenvectors of the mass matrices and we have written the obtained values, together with all the values for the remaining parameters of the model, in the file format

required by CalcHEP



```
Dark matter candidate is '~n1' with spin=1/2

=== MASSES OF HIGGS AND ODD PARTICLES: ===
4 Higgs masses and widths
PROCESS: h01->2*x
  h01  124.12 5.37E-03
PROCESS: h02->2*x
  h02  1017.37 6.99E+00
9 PROCESS: h03->2*x
  h03  1624.90 4.28E+01
PROCESS: h04->2*x
  h04  22362.38 9.90E+02
PROCESS: A1->2*x
14  A1  1017.77 7.12E+00
PROCESS: A2->2*x
asks 1->3 for A2
  A2    0.00 0.00E+00
PROCESS: H+>2*x
19  H+  1017.06 6.86E+00

Masses of odd sector Particles:
~n1  : Mneu1 = 41.8 || ~n2  : Mneu2 = 484.4 || ~x1+ : Mch1  = 509.9
~n3  : Mneu3 = 522.0 || ~n4  : Mneu4 = 540.8 || ~su1 : Msu1  = 727.7
24 ~su2 : Msu2  = 738.8 || ~su3 : Msu3  = 756.4 || ~sd1 : Msd1  = 1000.0
~sd2 : Msd2  = 1000.1 || ~sd3 : Msd3  = 1000.3 || ~x2+ : Mch2  = 1007.8
~n5  : Mneu5 = 1007.8 || ~s11 : Ms11  = 1191.8 || ~s12 : Ms12  = 1192.0
~s13 : Ms13  = 1195.6 || ~sv1 : Msn1  = 1196.5 || ~sv2 : Msn2  = 1196.5
~sv3 : Msn3  = 1196.5 || ~s14 : Ms14  = 1202.5 || ~s15 : Ms15  = 1206.1
29 ~s16 : Ms16  = 1206.3 || ~su4 : Msu4  = 1208.0 || ~su5 : Msu5  = 1382.1
~su6 : Msu6  = 1528.7 || ~sd4 : Msd4  = 2295.8 || ~sd5 : Msd5  = 2775.6
~go  : Mgo   = 3000.0 || ~sd6 : Msd6  = 3114.7 || ~n6  : Mneu6 = 7277.8
~n7  : Mneu7 = 9224.9 ||

34 ===== Calculation of relic density =====
WMAP measure Omega h^2 = 1.123e-01
PROCESS: ~n1,~n1->2*x
  excluding from final state: ~n1,~n2,~n3,~n4,~n5,~n6,~n7,~x1+,~x1-,~x2+,
  ~x2-,~go,~sv1,~Sv1,~sv2,~Sv2,~sv3,~Sv3,~s11,~S11,~s12,~S12,~s13,~S13,~s14,
39  ~S14,~s15,~S15,~s16,~S16,~su1,~Su1,~su2,~Su2,~su3,~Su3,~su4,~Su4,~su5,
  ~Su5,~su6,~Su6,~sd1,~Sd1,~sd2,~Sd2,~sd3,~Sd3,~sd4,~Sd4,~sd5,~Sd5,~sd6,~Sd6
PROCESS: Z->2*x
PROCESS: Zp->2*x
Xf=2.46e+01 Omega h^2=7.28e-02

44 Channels which contribute to 1/(omega) more than 1%.
Relative contributions in % are displayed
15% ~n1 ~n1 ->s s~
15% ~n1 ~n1 ->d d~
49 15% ~n1 ~n1 ->b b~
12% ~n1 ~n1 ->u u~
```

```

12% ~n1 ~n1 ->c c~
7% ~n1 ~n1 ->vm vm~
7% ~n1 ~n1 ->ve ve~
54 7% ~n1 ~n1 ->vt vt~
3% ~n1 ~n1 ->e- e+
3% ~n1 ~n1 ->mu- mu+
3% ~n1 ~n1 ->tau- tau+

59 ===== Indirect detection =====
      Channel          vcs[cm^3/s]
=====
      ~n1,~n1 -> b b~      3.80E-29
      ~n1,~n1 -> tau- tau+ 2.28E-30
64  ~n1,~n1 -> c c~      4.24E-31
sigmav=4.07E-29[cm^3/s]
Photon flux for angle of sight f=0.10[rad]
and spherical region described by cone with angle 0.10[rad]
Photon flux = 1.55E-15[cm^2 s GeV]^{-1} for E=20.9[GeV]
69 Positron flux = 3.20E-13[cm^2 sr s GeV]^{-1} for E=20.9[GeV]
Antiproton flux = 1.19E-12[cm^2 sr s GeV]^{-1} for E=20.9[GeV]

===== RESET_FORMFACTORS =====
protonFF (default) d 3.30E-02, u 2.35E-02, s 2.59E-01
74 neutronFF(default) d 4.24E-02, u 1.82E-02, s 2.59E-01
protonFF (new)      d 4.33E-02, u 2.92E-02, s 4.54E-01
neutronFF(new)      d 5.27E-02, u 2.39E-02, s 4.53E-01

===== Calculation of CDM-nucleons amplitudes =====
79 PROCESS: QUARKS,~n1->QUARKS,~n1{~go,u,u~,c,c~,t,t~,d,d~,s,s~,b,b~,
      ~su1,~Su1,~su2,~Su2,~su3,~Su3,~su4,~Su4,~su5,~Su5,~su6,~Su6,
      ~sd1,~Sd1,~sd2,~Sd2,~sd3,~Sd3,~sd4,~Sd4,~sd5,~Sd5,~sd6,~Sd6
Delete diagrams with S0!=1,V5,a
PROCESS: QUARKS,~n1->QUARKS,~n1{u,u~,d,d~,s,s~
84 Delete diagrams with V5!=1,S0,a
PROCESS: t->2*x
PROCESS: ~su1->2*x
PROCESS: ~su2->2*x
PROCESS: ~su3->2*x
89 PROCESS: ~su4->2*x
PROCESS: ~su5->2*x
PROCESS: ~su6->2*x
PROCESS: ~sd1->2*x
PROCESS: ~sd2->2*x
94 PROCESS: ~sd3->2*x
PROCESS: ~sd4->2*x
PROCESS: ~sd5->2*x
PROCESS: ~sd6->2*x
CDM[antiCDM]-nucleon micrOMEGAS amplitudes:
99 proton: SI -2.273E-09 [-2.273E-09] SD 1.490E-07 [1.490E-07]
neutron: SI -2.333E-09 [-2.333E-09] SD -1.318E-07 [-1.318E-07]
CDM[antiCDM]-nucleon cross sections[pb]:
proton SI 2.160E-09 [2.160E-09] SD 2.786E-05 [2.786E-05]
neutron SI 2.276E-09 [2.276E-09] SD 2.180E-05 [2.180E-05]

```

104

```
=====  
=====  
73Ge: Total number of events=1.52E-03 /day/kg  
Number of events in 10 - 50 KeV region=6.84E-04 /day/kg  
131Xe: Total number of events=2.50E-03 /day/kg  
109 Number of events in 10 - 50 KeV region=7.58E-04 /day/kg  
23Na: Total number of events=5.21E-04 /day/kg  
Number of events in 10 - 50 KeV region=2.63E-04 /day/kg  
I127: Total number of events=2.48E-03 /day/kg  
Number of events in 10 - 50 KeV region=7.78E-04 /day/kg
```



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