Dead time

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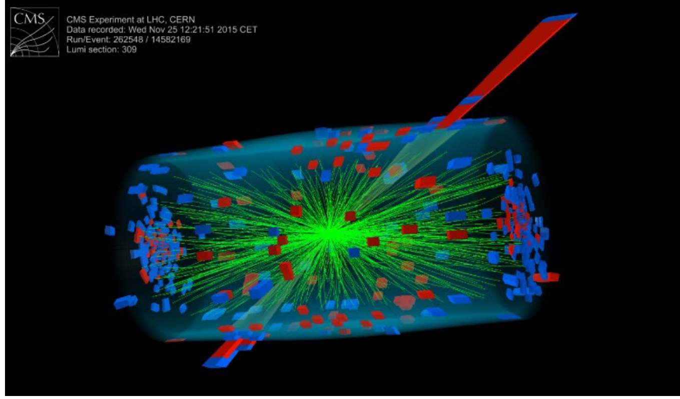
Dead time

- For every readout channel there is a minimum time after registering a signal for which the detector can no longer register a further signal in this channel or at least cannot distinguish a second or third signal from the first one.
- This *dead time* is usually caused by the detector itself since it needs a certain recovery time after sensing a particle and generating a signal.
- Cherenkov detectors or organic scintillators have short dead times ($\sim ns$), whereas Geiger counters have long dead times ($\sim ms$)
- A further dead time is caused by the trigger and readout electronics that need a certain amount of time for signal registration and processing.
- Often no other signal is accepted by the entire detector on any readout channel during this time.

CMS Heavy Ion Event Display
https://cds.cern.ch/record/2105623



Dead time

- In complex detectors the dead time of an individual channel does not necessarily govern the dead time of the entire detector.
- Often an experimental measurement consists of many individual signals which allows accounting for a channel’s dead time as an inefficiency.
- Depending on how long a channel is paralysed by a dead time and how high the signal rate is, this can lead to a significant fraction of time during which the apparatus is not ready for a new measurement.
- The detector dead time must be determined in offline data analysis, the measurements are to be corrected for it.

Equally distributed events: time distribution

- Let's considered events which occur randomly but with equal probability per unit of time, for example in decays of long-lived radioactive isotopes or when observing cosmic rays.
- In these cases, the number of events in a time interval Δt is Poisson distributed.
- With *n* being the event rate, the average number in a fixed time interval Δt is given by $N = n\Delta t$.
- The probability that *k* events appear in this interval is given by the Poisson distribution:

$$P(k|N) = \frac{N^k}{k!}\, e^{-N}$$

Equally distributed events: time distribution

- At any point in time the probability that the next event occurs after a time *t* in the interval *dt* is given by the product of the probability $P(0|N)$ not to see any event in the interval [0, *t*] and the probability ndt to see exactly one event in *dt*:

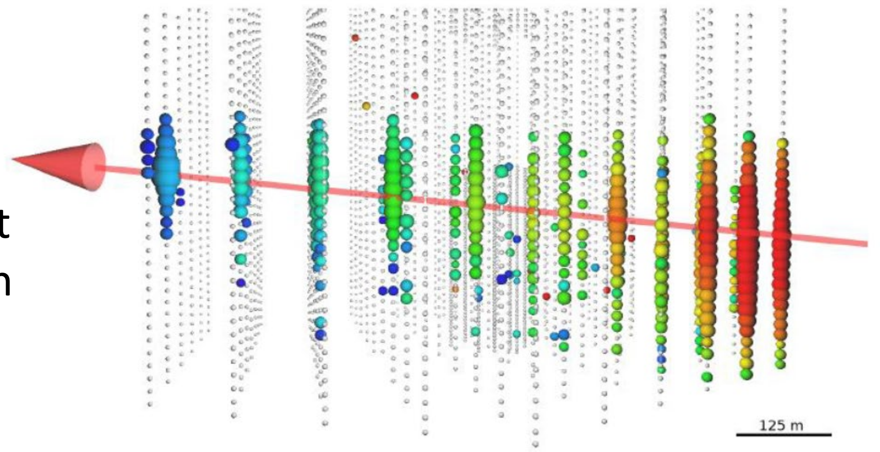
$$dp(t) = P(0|N)\, n\, dt = n\, \mathrm{e}^{-N} dt = n\, \mathrm{e}^{-n\, t} dt$$

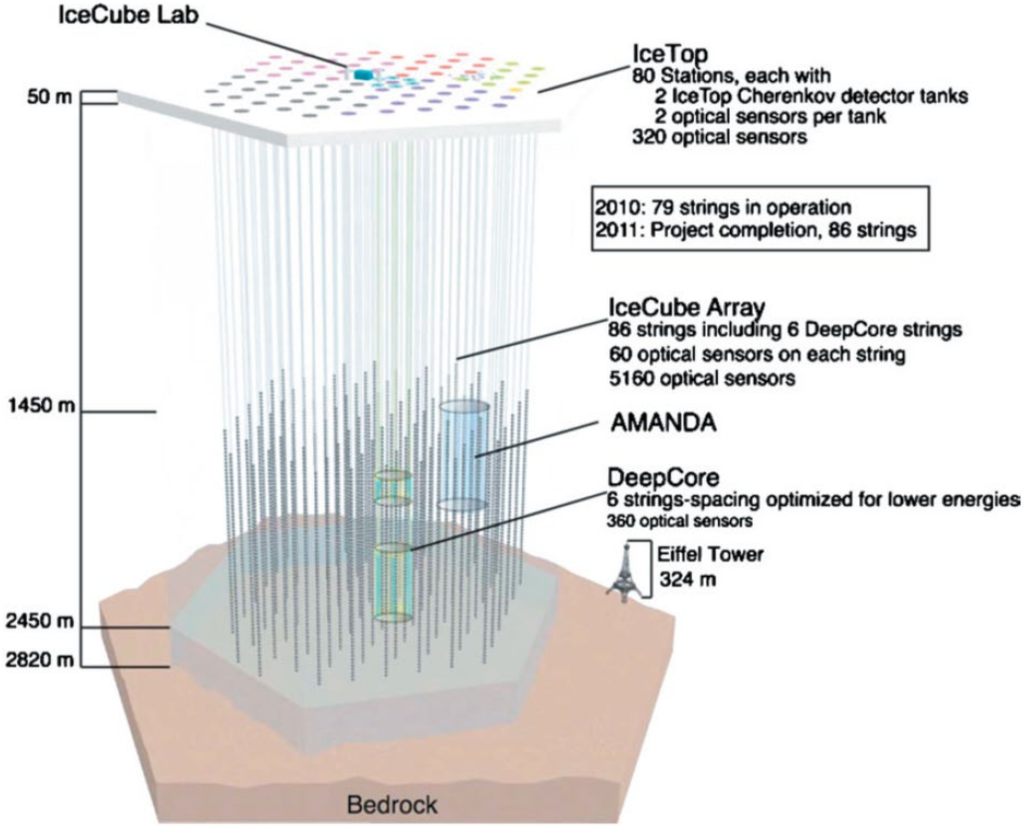
- *dp*(*t*)/*dt* then is the distribution of times after each of which the next event arrives.
- It is also the distribution of time differences Δt between two events (simply consider that the start time *t* = 0 in the above derivation is assigned to the last respective event):

$$\frac{dp(\Delta t)}{d(\Delta t)} = n\, \mathrm{e}^{-n\, \Delta t}$$

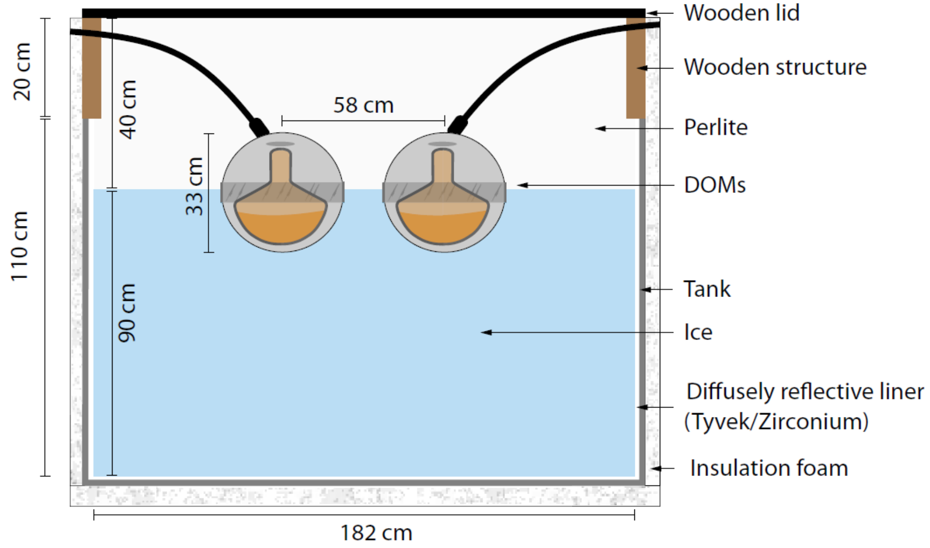
Icecube & IceTop

The high-energy IceCube neutrino event from Sept. 22, 2017
https://www.globalneutrinonetwork.org/sites/site_gnn/content/e227156/e308930/e308934/GNN-News.pdf



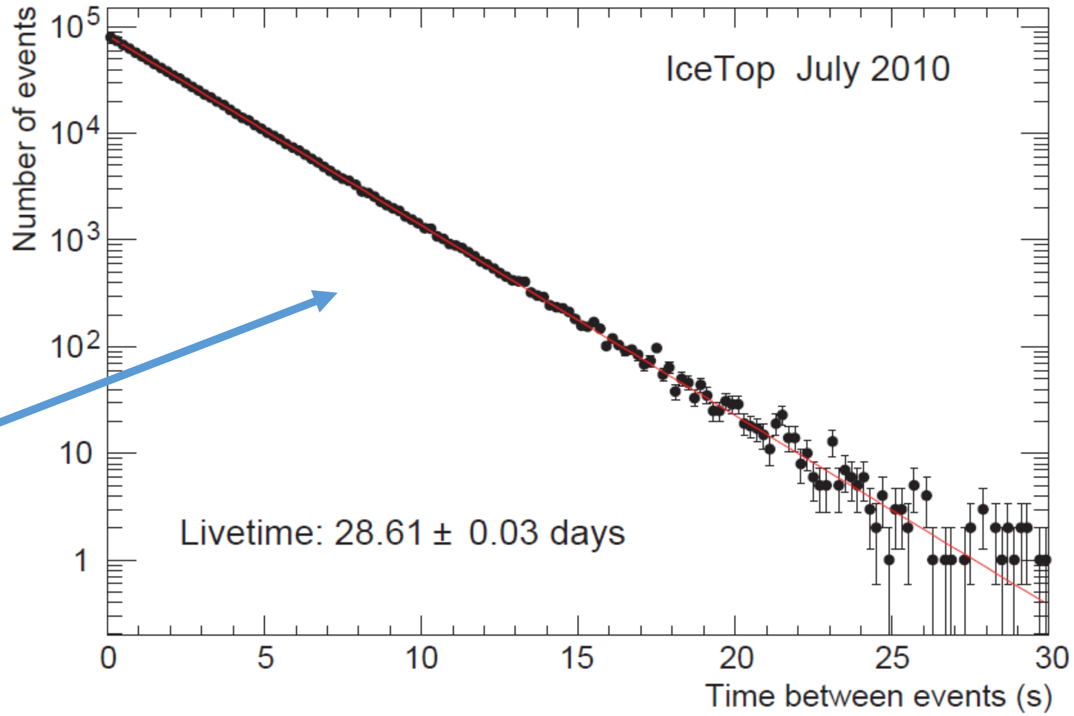


IceTop: https://doi.org/10.1016/j.nima.2012.10.067



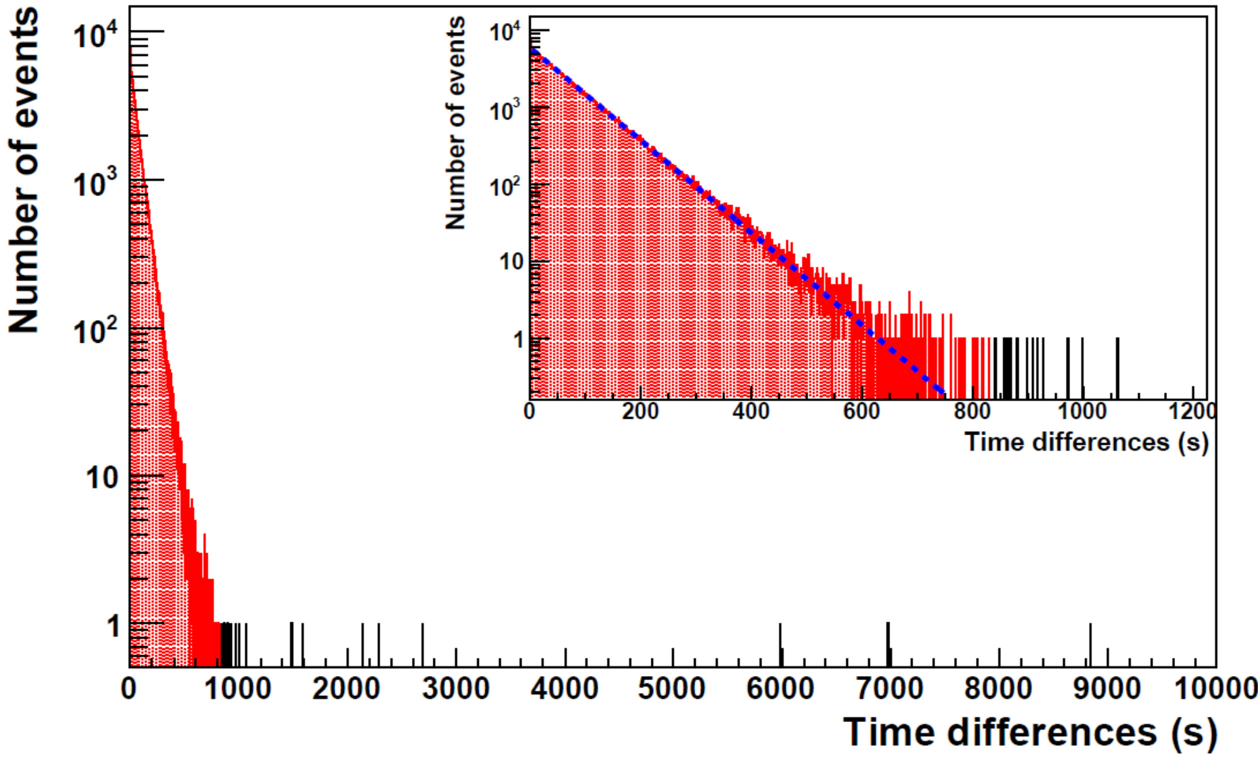
Equally distributed events: time distribution

- Events appearing at random within a time interval (i.e. uniformly distributed in time) therefore follow an exponential distribution in their time differences.
- Such an exponential behaviour of time differences is well evident for air shower events detected by the detector IceTop.



Equally distributed events: time distribution

- Distribution of time differences between events in 2008 for Auger.
- The exponential fit ($P(t) = e^{-t/\tau}$) is shown as a dashed line in the inset where the histograms are zoomed ($\tau = 72.4\ s$).
- If interval occurrence Poisson probability is less than 10^{-5}, they are indicated in black and considered detector "dead time" (software updates mostly).



https://doi.org/10.1016/j.nima.2009.11.018

Dead times for events uniformly distributed in time

- On every event there follows a time interval (the system dead time τ) during which the detector cannot register new events.
- The system dead time is to be distinguished from the dead-time interval Δt (also called the *dead time*) which depends on additional conditions to be seen in the followings slides.

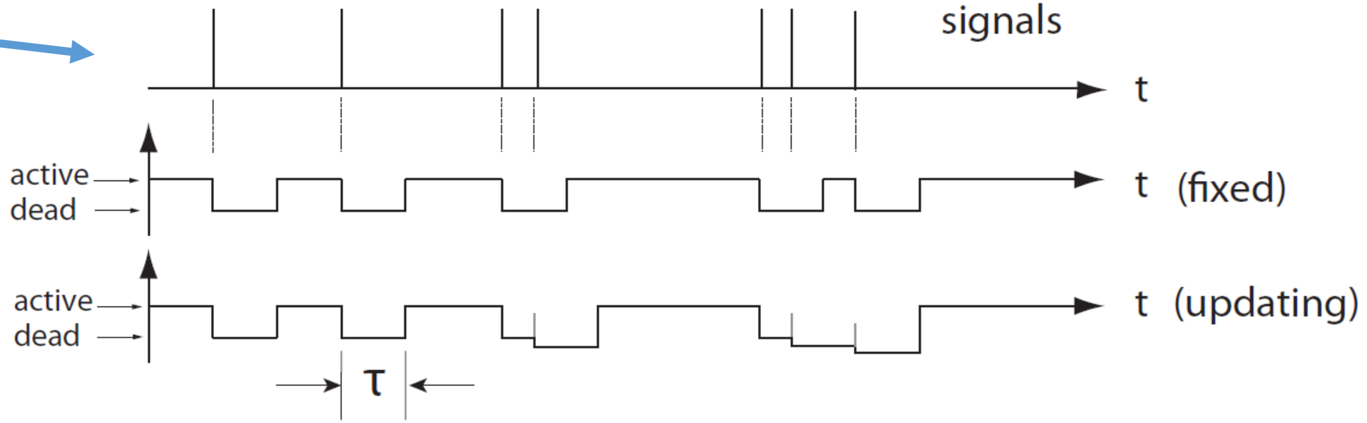
Dead times for events uniformly distributed in time

- There are two main cases:

1. *Detector with a fixed dead-time interval (non-paralysable detector):* The detector system generates a time interval $\Delta t = \tau$ after every registered event, within which the detector is not sensitive to register further events. This time interval is fixed and is not updated if further events appear during this time.
2. *Detector with updating dead-time interval (paralysable detector):* if a further event appears during the time the system is dead, Δt will be updated by τ . Since this can happen repeatedly, Δt can in principle become infinitely long (at high rates).

Dead times for events uniformly distributed in time

- Time sequence of signal events;
- response of a detector with ***fixed dead-time interval*** after a registered event;
- response of a detector with ***updating dead-time*** interval after a registered event.



Of the seven signal events five are registered in the case of a fixed dead time and four in the case of updating dead time.

Fixed dead-time interval

- Let n be the true average event rate (N events per time t) and m be the measured average rate (M measured events per time t).
- The average number of events that lie in a dead-time interval τ , and hence are not registered, is $n\tau$.
- Since the dead-time intervals appear with rate m the rate of non-registered events is:

$$\frac{N-M}{t} = n - m = mn\tau$$

Fixed dead-time interval

- Solving for *n* yields the true rate obtained from the measured rate with knowledge of the system dead time τ:

$$n = \frac{m}{1 - m\tau}$$

- Conversely the measured rate as a function of the true rate is:

$$m = \frac{n}{1 + n\tau}$$

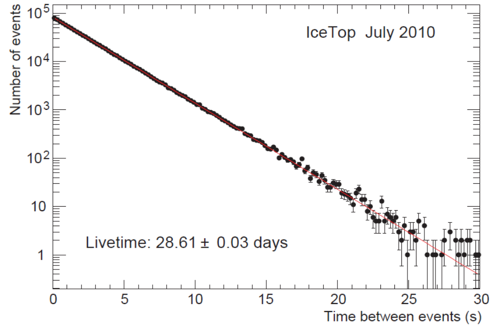
- and the fractional dead time of the total time of data taking is:

$$\frac{n - m}{n} = \frac{n\tau}{1 + n\tau} .$$

Fixed dead-time interval

- Another way to see the previous results is that the apparatus is dead for a fixed time τ after each recorded event.
- If the observed counting rate is ‘m’, then the fraction of time during which the apparatus is dead is mτ.
- And the fraction of time during which the apparatus is sensitive is 1−mτ.
- Thus, the fraction of true number of events that can be recorded is given as:

$$\frac{M}{N} = \frac{m}{n} = 1 - m\tau \rightarrow n = \frac{m}{1 - m\tau}$$



Updating dead-time interval

- In this case the dead-time interval is extended if during its duration another event occurs.
- Events are only registered for which the time distance to the previous event is larger than the system dead time τ .
- The probability for this to happen follows from:

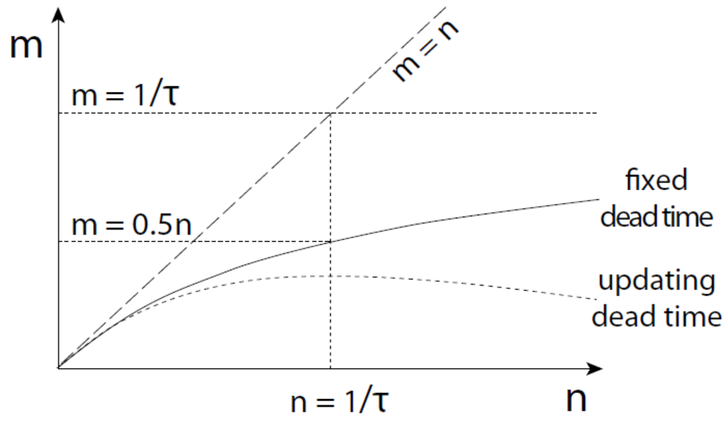
$$p(\Delta t > \tau) = \int_{\tau}^{\infty} n\,\mathrm{e}^{-n\,\Delta t}\,d(\Delta t) = \mathrm{e}^{-n\tau}$$

- The measured average rate *m* for which $\Delta t \ > \tau$ is valid is the product of the previous equation with average rate *n*:

$$m = n\,e^{-n\tau}$$

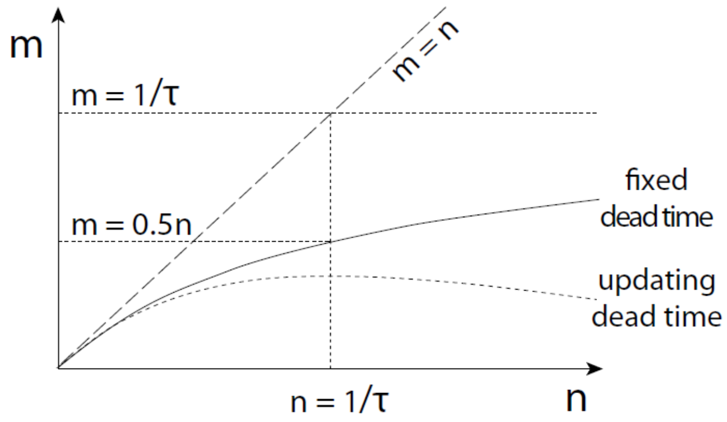
- It cannot be explicitly solved for *n*.
- It must be solved either numerically or can be obtained for example by fitting the distribution of time distances between successive events

Dead time types comparison



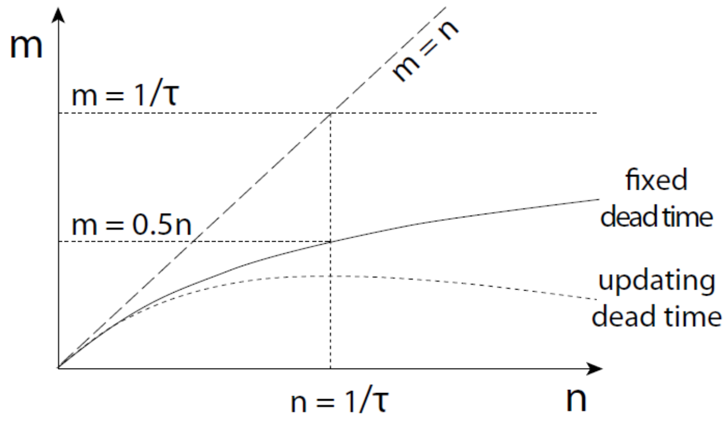
- Measured count rate *m* as a function of the true rate *n* for detectors with fixed dead-time interval (solid line), updating dead-time interval (dotted line) and without dead time (dashed line).
- **<u>For small rates</u>** ($n \ll 1/\tau$) both curves start tangentially to the diagonal *m* = *n*.
- **<u>For high event rates</u>**, that is, when the influence of dead time becomes stronger and stronger, the measurable rate *m* for a system with fixed dead time asymptotically approaches $m = 1/\tau$.
- This means that after the end of every dead-time interval a new event immediately appears leading to a new dead-time interval.

Dead time types comparison



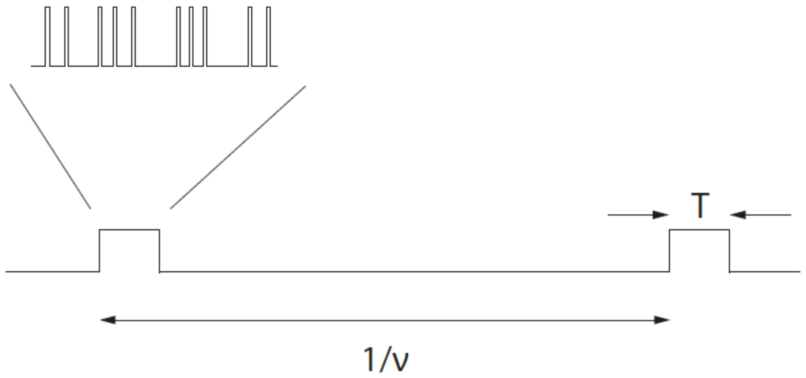
- By contrast the curve for updating dead time runs through a maximum at $n = 1/\tau$ with $m = n/e = 0.37n$ and then decreases afterwards, whereas for fixed dead time at $n = 1/\tau$ the measured rate reaches the value *m* = 0.5 *n*.
- For the updating system and high input rate, the measured rate approaches the zero line because due to the permanently updating dead time the detector can hardly register new events.
- Given the *"parabolic"* shape two true rates can the be assigned to a measured rate *m*, a low and a high one, which can lead to misinterpretations.

Dead time types comparison



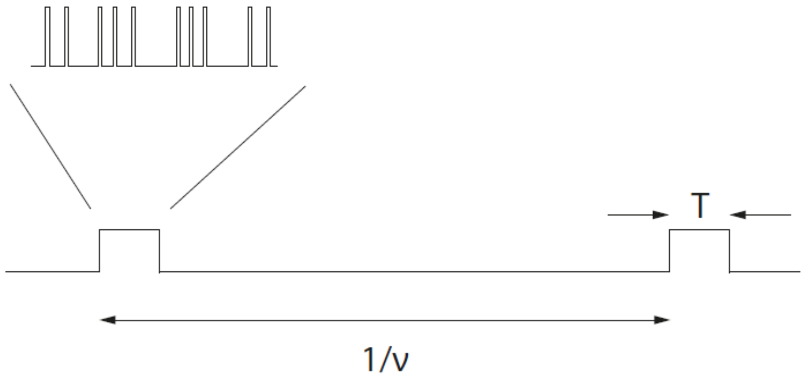
- For both dead time models the true event rate should be small compared to $1/\tau$ in order that the measured rate does not differ too much from the true rate.
- For updating dead-time intervals the detected rate *m* even decreases for $n > 1/\tau$ below the rates achievable at lower input rate *n*.
- These limits can be overcome by parallel processing and buffering of data. This, however, requires significant additional circuit complexity.

Dead time for pulsed event generation



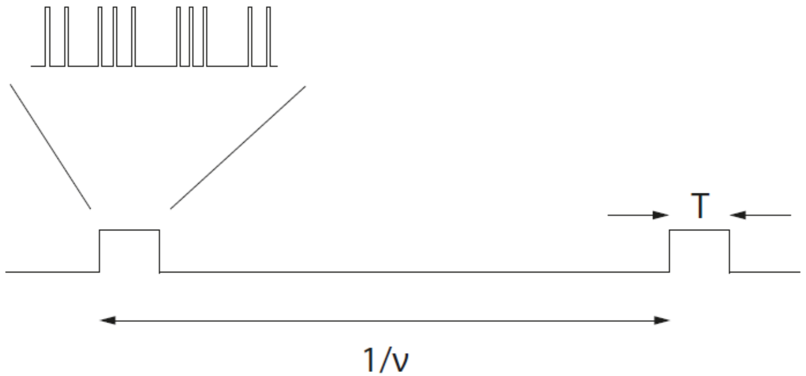
- If the events to be measured by a detector do not occur in random order but are pulsed in ***bunch trains*** of duration *T* (within T several events are observed), appearing equidistant in time with repetition rate ν then the previous analysis may not be strictly applicable.
- Within a pulse train there can be additional time structures, as is the case for instance in packet structures of accelerator beams.
- We assume the substructures negligible relative to the pulse duration, for the rest of the analysis
- The quantities to consider are (as seen previously) the true rate *n* and the measured rate *m* (each taken with respect to the total period $1/\nu$) and the corresponding number of events per pulse $N = n/\nu$ and $M = m/\nu$.

Dead time for pulsed event generation



- One can then distinguish the following cases:
- $\boldsymbol{\tau \ll T}$: If the detector's system dead time is much smaller than the duration of the beam pulses *T*, then the fact that a coarse bunch structure exists has little effect and the results obtained for uniformly distributed events can be used.
- $\boldsymbol{\tau > T \ \&\& \ \tau < 1/\nu - T}$: If τ is larger than *T* but smaller than the time between the beam pulses $(1/\nu - T)$, then *at most one event per pulse (bunch train) can be detected*.

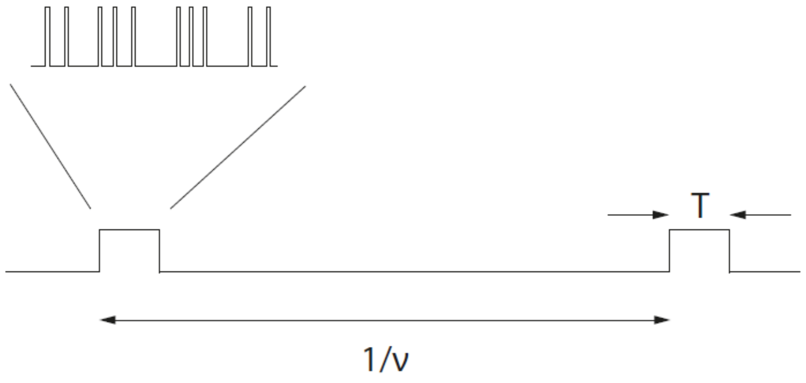
Dead time for pulsed event generation



- The detector is always ready for detection at the beginning of a new beam.
- The measured event rate m can at most be equal to the pulse frequency, that is, $m \leq \nu$.
- Thus, the average number of events per pulse is $M = \frac{m}{\nu} \leq 1$, whereas the average number of true events $N = n/\nu$ can be larger than one.
- The Poisson distribution determines again the probability that a true event per pulse occurs:

$$P(N > 0) = 1 - P(0) = 1 - e^{-n/\nu}$$

Dead time for pulsed event generation



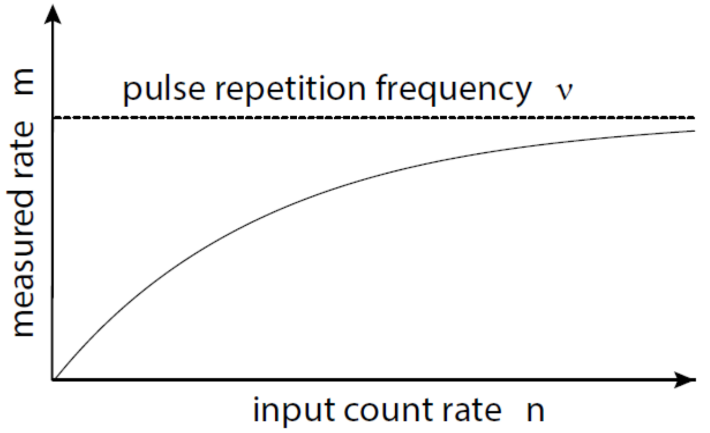
- Since the detector is always ready at the beginning of a beam pulse it will register an event when at least one occurs. Hence:

$$\frac{m}{\nu} = 1 - e^{-\frac{n}{\nu}}$$

- The measured rate approaches the pulse repetition frequency.
- Solving for *n* we obtain:

$$n = \nu \ln\left(\frac{\nu}{\nu - m}\right)$$

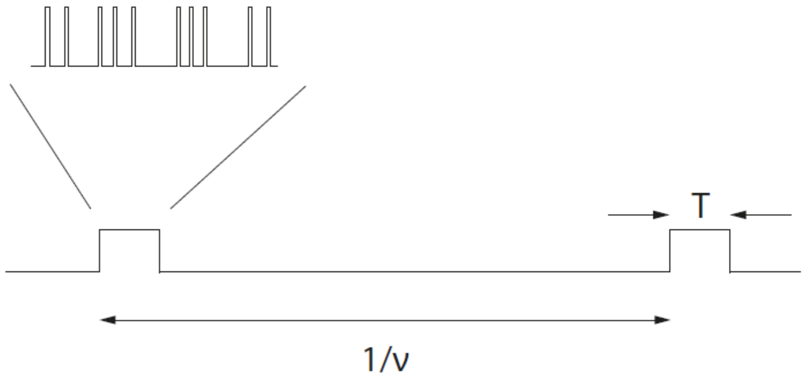
- Neither the exact value of the system dead time nor the dead time behaviour (fixed or updating) play a role.



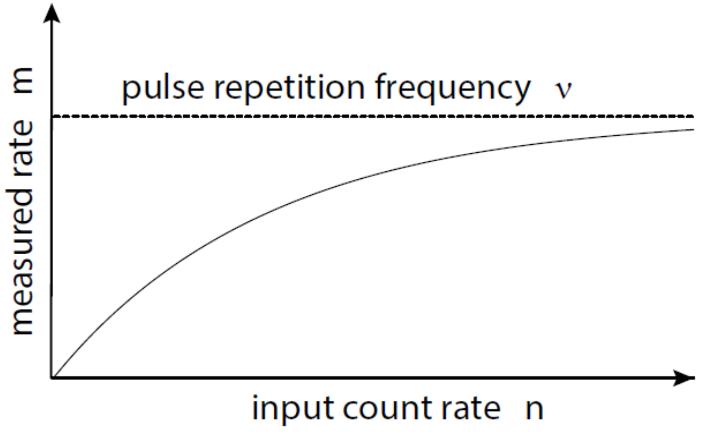
Saturation behaviour of the measured event rate in the case of pulsed event generation

For $n \to \infty, \ m_{max} \to \nu$

Dead time for pulsed event generation



- The maximally measurable counting rate m_{max} in this case is equal to ν , the pulse repetition rate, since at most one event can be recorded per pulse.
- Thus, it does not pay off to increase the input rate n to values much larger than ν by increasing the pulse intensity.
- The measured rate reaches 63% of the maximum rate already at $n = \nu$, 86% at $n = 2\nu$ and 95% at $n = 3\nu$.

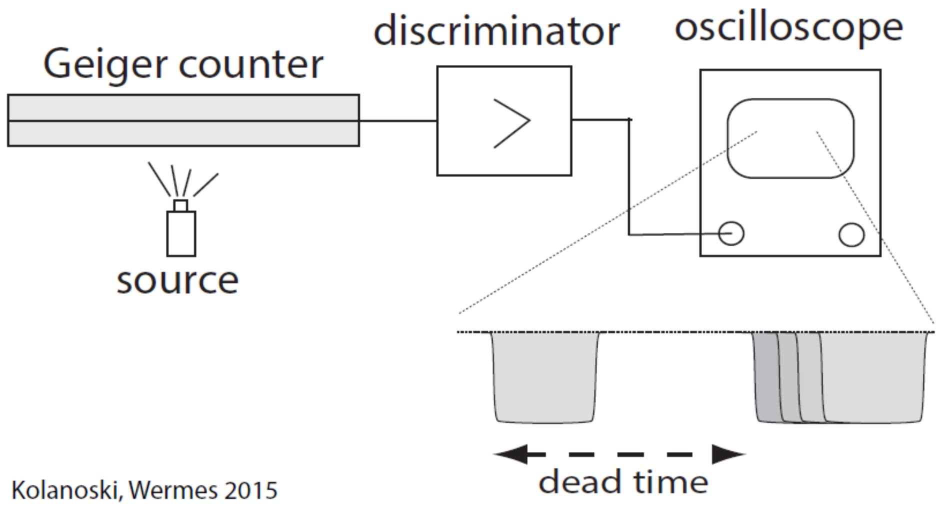


$\boldsymbol{\tau \lesssim T}$: For the case that detector dead time and pulse duration are of the same order a more complex treatment is needed

Methods of dead time determination for uniformly distributed events

- The dead time of detectors must be considered in quantitative analyses of experimental data by a dead-time correction
- Since in most cases the dead time is not known and can also not be estimated with sufficient precision, it must be measured.
- We will see:
  - Dead time estimation by use of an oscilloscope.
  - Dead time determination from decay curve.
  - Dead time determination from time difference distributions.
  - Electronic determination of the inactive time of a large detector system

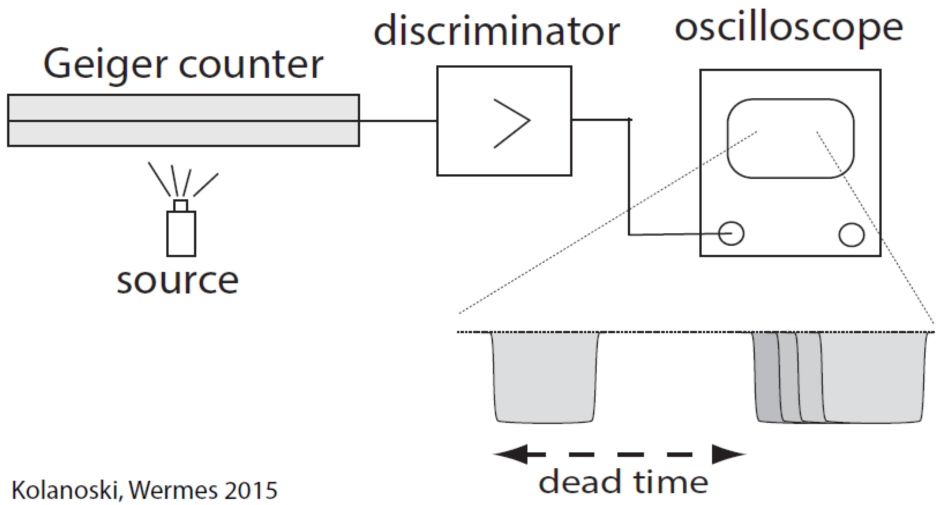
Dead time estimation by use of an oscilloscope.



Kolanoski, Wermes 2015

- The dead time of a simple counter system, for example a Geiger counter with a fixed dead time, can be measured very easily with an oscilloscope.
- A counting tube registers the radiation of a radioactive source.
- By a discriminator those signals with pulse heights above a certain threshold are converted into logic pulses which can be given on the input of an oscilloscope.
- The scope is triggered on an arbitrary pulse of the sequence such that this pulse and the subsequent pulses are displayed on the scope’s screen (*self-triggering mode*).
- The time between the triggering edge and the first visible edge of a succeeding pulse is the dead time of the detector.

Dead time estimation by use of an oscilloscope.



Kolanoski, Wermes 2015

- This method to measure the dead time is valid for the different scenarios discussed before *provided the event rate is high enough to allow clear recognition of the dead-time gap on the oscilloscope*.
- On the other hand, the rate must not be so high that the measurement becomes dead-time saturated, that is, no subsequent pulses are visible on the oscilloscope.
- The dead time determined this way is that of the specific system, in this example for the combination of counter and discriminator.

Dead time determination from decay curve.

- In the decay-curve method a short-lived radioactive isotope (e.g. long-lived isotopes like Indium $^{116m}\mathrm{In}$, a β^- emitter with a half-life of 54.3 min) is used having a decay constant λ and decaying with the rate:

$$n(t) = n_0 e^{-\lambda t}$$

- where n_0 is the true rate at the beginning of the measurement.
- Inserting $n(t)$ in the formulas obtained for the various cases of the yields:

$$n = \frac{m}{1 - m\tau} \longrightarrow m(t)\,\mathrm{e}^{\lambda t} = n_0 - n_0 \tau\, m(t) \qquad \text{(fixed dead-time interval)},$$

$$m = n\,\mathrm{e}^{-n\tau} \longrightarrow \ln \mathrm{m(t)} + \lambda \mathrm{t} = \ln \mathrm{n_0} - \underbrace{\mathrm{n_0}\tau\,\mathrm{e}^{-\lambda \mathrm{t}}}_{n\tau} \qquad \text{(updating dead-time interval)}$$

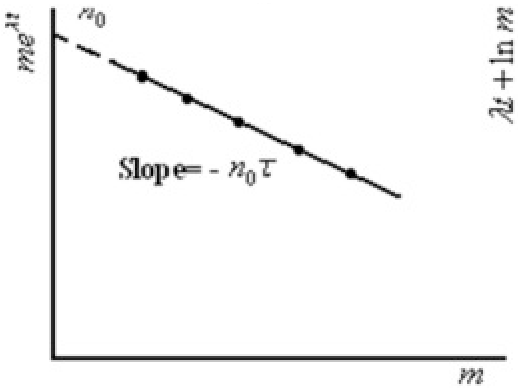
Dead time determination from decay curve.

- In the case of a fixed dead time, plotting $m(t)e^{\lambda t}$ versus $m(t)$ one obtains a straight line with slope $-n_0\tau$ and offset n_0;
- For updating dead time, plotting $\lambda t + \ln m(t)$ versus $e^{-\lambda t}$ results in the same slope $-n_0\tau$ with $\ln n_0$ as offset.
- From the straight-line fit results one can obtain both the dead time τ and the true rate at the beginning of the measurement (n_0)

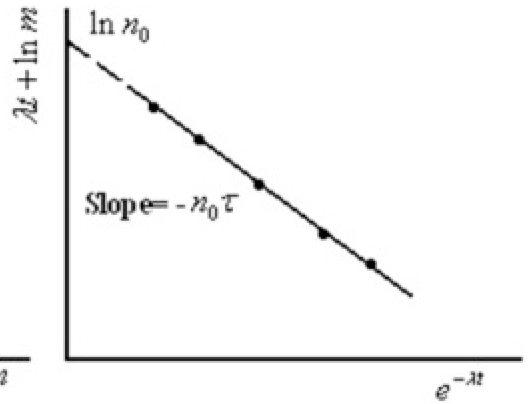
https://doi.org/10.1016/j.net.2018.06.014

See also:
https://doi.org/10.1038/s41598-020-75310-3

Fixed dead time



Updating dead time



Dead time determination from decay curve.

- The decaying source method has the advantage of not only measuring the value of deadtime, but also testing the validity of the idealized assumption of paralyzable and nonparalyzable models.
- However, care must be taken in selecting a suitable isotope.
- The isotope used for this technique must be pure with a single half-life, which is not too long or too short such that the entire counting rate range can be measured in a reasonable time.
- Moreover, the half-life of the decaying isotope must be known with good accurately.
- A disadvantage of decaying source method is that it takes a long time for deadtime determination.

https://doi.org/10.1016/j.net.2018.06.014

Dead time determination from time difference distributions

- If time stamps of statistically distributed individual events are measured, the dead time of a detector system can be determined from analysis of the distribution of time differences $\Delta t = t_{i+1} - t_i$ between two successive events.
- The principle is the same as that using an oscilloscope for observation of successively appearing events, but this method can be applied more generally, in particular also for low event rates.
- Without detector dead time these time differences are exponentially distributed:

$$\frac{dN}{d(\Delta t)} = N_0\, n\, \mathrm{e}^{-n\,\Delta t}$$

- where N_0 is the total number of events (the integral) and *n* the event rate.

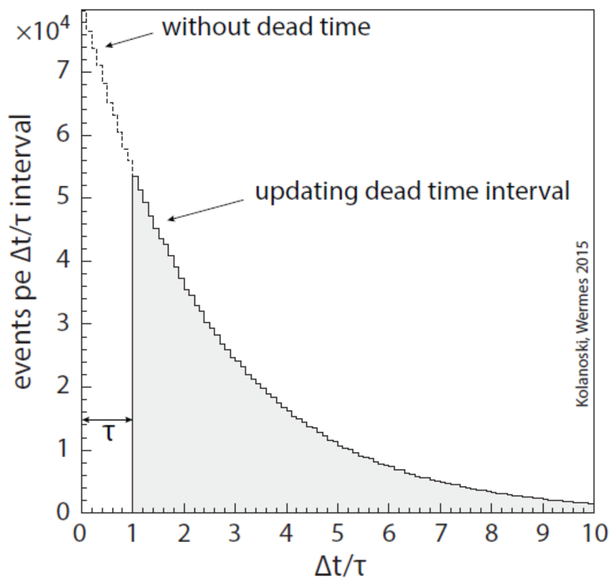
Dead time determination from time difference distributions

- Due to dead times after registration of an event, smaller time differences are suppressed, and the distribution deviates from a pure exponential shape.
- Assuming that the length of the system dead time is constant we analyze the two types of dead time:

a) fixed dead-time interval of length τ after a registered event: the observed event rate respecting the dead time is $m = \frac{n}{1+n\tau}$.

b) fixed dead time τ after every event (i.e. not only after registered events): This means that after any event the remaining dead time is extended and is always at least (updating of the dead-time interval). The observed rate for dead-time updating is $m = n\, e^{-n\tau}$.

Dead time determination from time difference distributions



- For the distribution of time differences including dead times, *case (b)* is the simpler case: no time difference smaller than τ occurs.
- All other time differences are not affected.
- The number *M* of still registered events is given by the integral of:

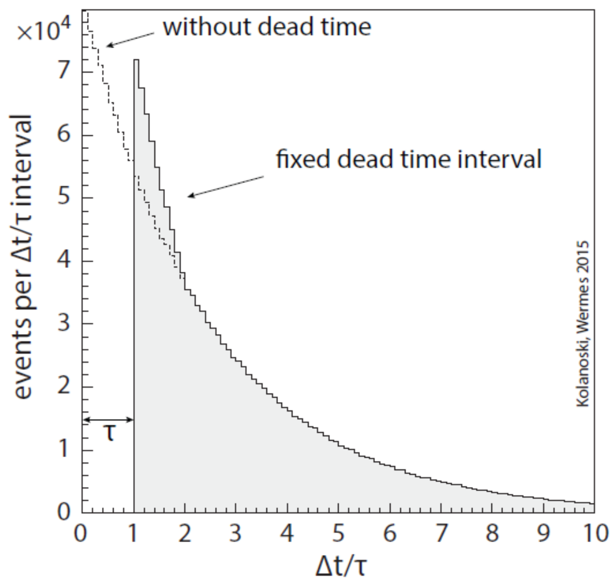
$$\frac{dN}{d(\Delta t)} = N_0\, n\, \mathrm{e}^{-n\,\Delta t}$$

- From τ to ∞:

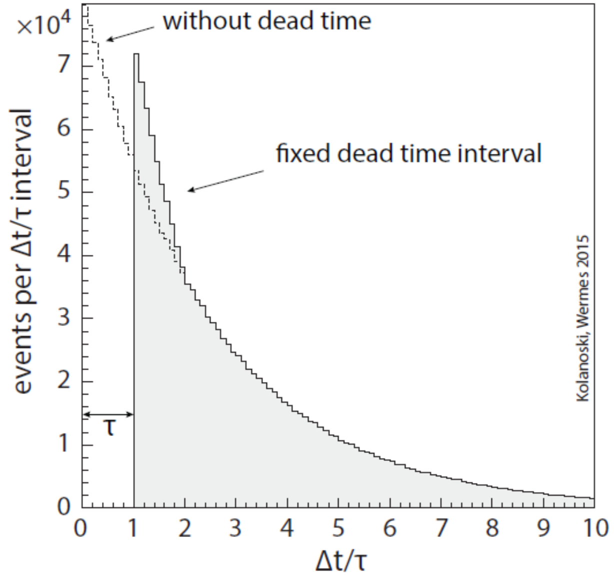
$$M = N_0\, n \int_{\tau}^{\infty} \mathrm{e}^{-n\,\Delta t} d(\Delta t) = N_0\, \mathrm{e}^{-n\,\tau}$$

Dead time determination from time difference distributions

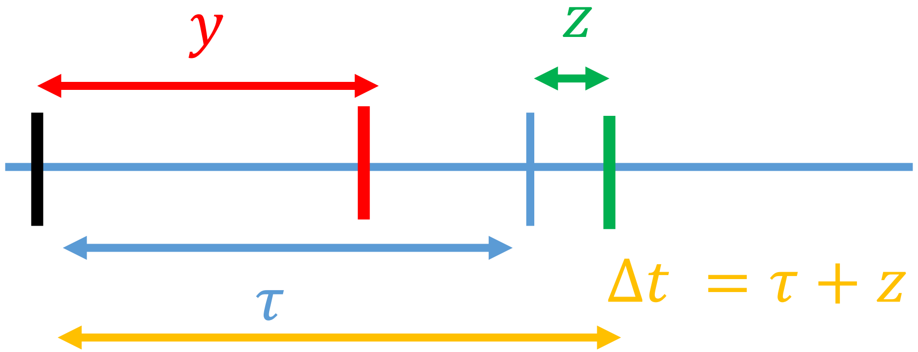
- *Case (a)*, that is, a fixed dead-time interval only after a *registered* event, is more complex than (b).
- In every dead-time interval there are on average $n\tau$ events that are not registered.
- The time difference between the last of these not registered events to the first event occurring right after the interval can be shorter than τ.
- Since the latter event (appearing after τ) is registered it contributes to the counting rate m.
- Measured, however, is the time difference to the last registered event, which is τ plus the time of the event after the end of the dead-time interval.



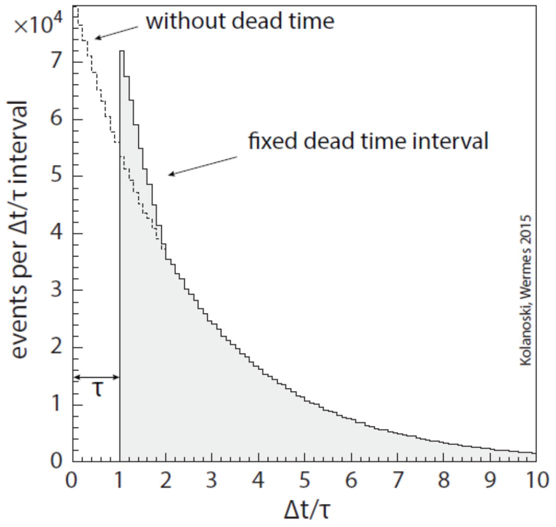
Dead time determination from time difference distributions



- By contrast the same event would not be registered in case (b) when having a time distance smaller than τ to the previous event, since this previous event would have extended the dead-time interval.
- To find the distribution of these additional time differences we consider the distribution of time distances *y* of last events inside the dead-time interval convoluted with the distribution of the distance *z* of the first event after the dead-time interval, with the condition $\Delta t' = y + z < 2\tau$.



Dead time determination from time difference distributions



- The distribution of y, and z, each in an interval τ, is given by:

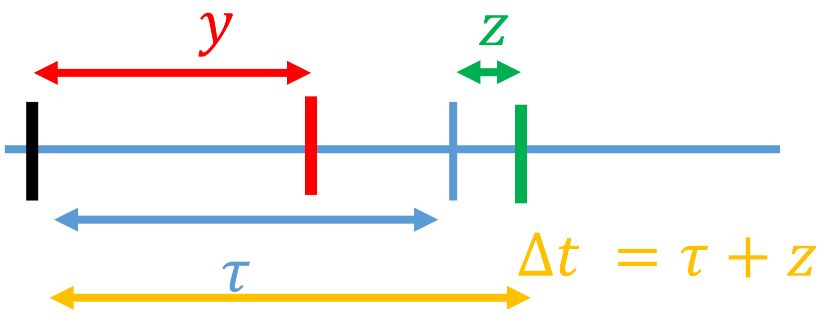
$$f(y)dy = ne^{-ny}dy$$

- The distribution of the (additionally) registered time differences $\Delta t = \tau + z$ arises from the product of the probability distributions for y and z with integration over the unobserved variable y:

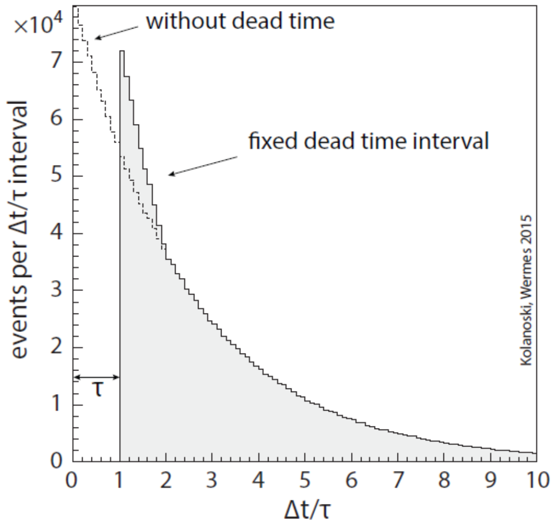
$$2\tau - \Delta t = 2\tau - (\tau + z) = \tau + z$$

$$f(\Delta t)d(\Delta t) = \int_0^{2\tau-\Delta t} \left(n\,\mathrm{e}^{-ny}\right)\left(n\,\mathrm{e}^{-n(\Delta t-\tau)}\right)\,dy\;d(\Delta t)$$

$$= \begin{cases} n\left(\mathrm{e}^{-n(\Delta t-\tau)} - \mathrm{e}^{-n\tau}\right)d(\Delta t) & \text{for } \tau < \Delta t < 2\tau\,, \\ 0 & \text{else}\,. \end{cases}$$



Equivalent to $0 < z < \tau$

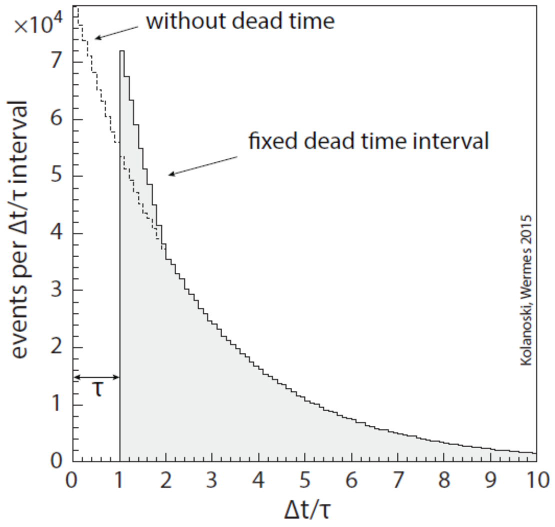


Dead time determination from time difference distributions

- Besides the gap at $0 \leq \Delta t \leq \tau$, an enhancement of events occurs for $\tau < \Delta t < 2\tau$
- The size (normalisation) of this enhancement can be derived from the measured rate as a function of the true rate ($m = \frac{n}{1+n\tau}$) and the value of $M = N_0 e^{-n\tau}$ obtained for the updating dead time interval (number of events with temporal distance $> \tau$).
- The measured number of events with dead time:

$$\widetilde{M} = \frac{N_0}{1+n\tau} = N_0\, \mathrm{e}^{-n\,\tau} + \Delta M \quad \Longrightarrow \quad \Delta M = N_0 \left(\frac{1}{1+n\tau} - \mathrm{e}^{-n\,\tau} \right)$$

Dead time determination from time difference distributions



- Here $\widetilde{M}$ is the measured number of events composed of the part from the integral from τ to ∞ (as for the updating dead time interval) plus the enhancement contribution ΔM.
- Hence the distribution of the measured time differences in case (a) arises by adding the original distribution above τ:

$$\frac{dN}{d(\Delta t)} = N_0\, n\, \mathrm{e}^{-n\,\Delta t}$$

- and the enhancement proportional to $f(t)$ calculated before:

$$\begin{aligned} \frac{dN}{d(\Delta t)} &= N_0 \left(n\, \mathrm{e}^{-n\,\Delta t} + \frac{1}{1+n\tau} f(\Delta t) \right) && \text{for } \Delta t > \tau \\ &= 0 && \text{else}\,. \end{aligned}$$

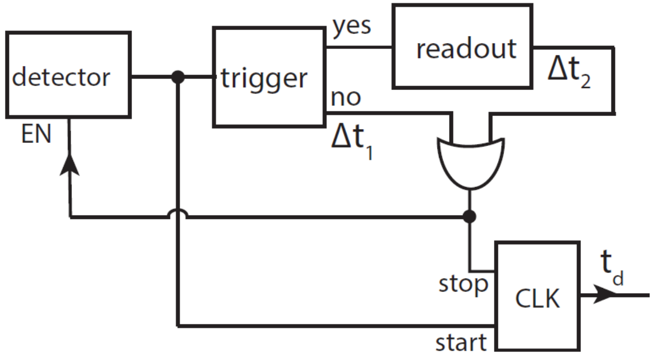
Dead time determination from time difference distributions

- The coefficient $f(t)$ has been determined such that the integral over this term yields ΔM in the previous equation

Electronic determination of the inactive time of a large detector system

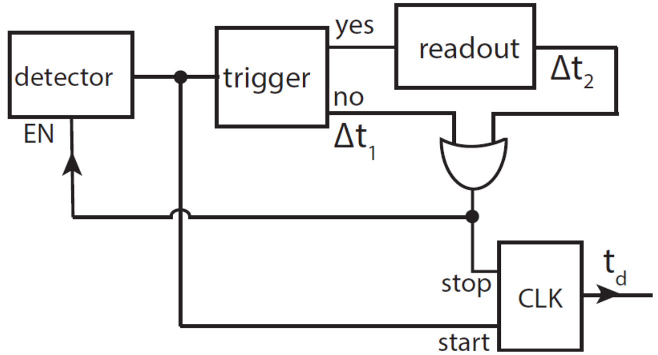
- If not only the dead time of an individual detector is concerned, but that of a larger system consisting of many detector components usually being hooked to one common computer, then the dead time is determined by an ‘electronic stopwatch’.
- Schematically, they consist of the detector, a triggering system deciding if an event should be read out or not, and the data taking system.

Inactive time



- The detector signals (or a selected subset of signals) are input to a trigger system at a certain time t_0.
- At the same time a clock is started measuring the time until the trigger decision.
- During this time the detector is deactivated for data taking.
- If the trigger decision is negative the detector is reactivated, the clock is stopped, and the measured time is stored as dead time (t_d).
- If the trigger decision is positive the dead time is extended by the time needed to read out the detector to the computer (readout time).
- A ‘ready’ signal ends the process.

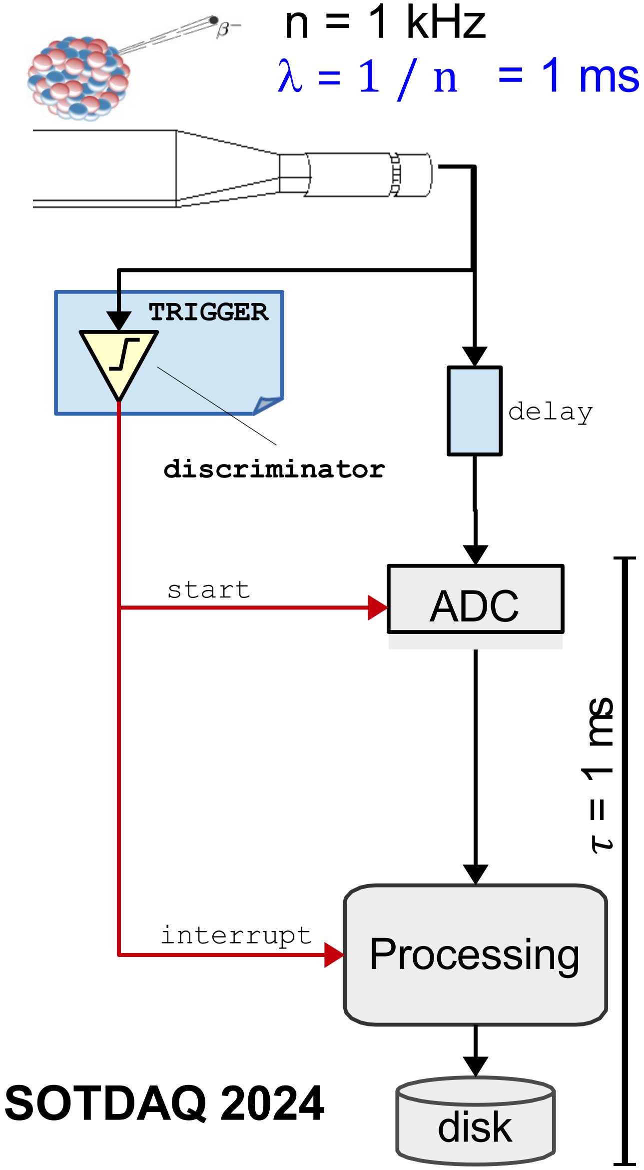
Inactive time



- It should be noted that in this scheme the individual channel dead times do not explicitly enter the consideration.
- They enter as inefficiencies of the individual channels in the analysis, either by determining them explicitly or by exploiting redundant information.
- For example, the inefficiency (containing the dead time) of a particular channel in a tracking detector can be determined using information from other channels concerned with the track that crosses many detector layers.
- In cosmic ray experiments, with events uniformly distributed in time; to measure for example the cosmic ray flux, the integrated dead time must be subtracted from the total time the detector is recording data.

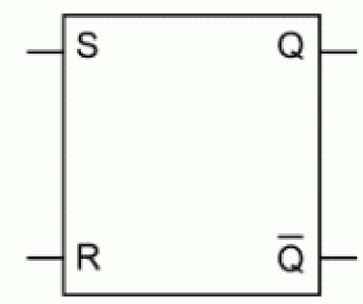
Basic DAQ: "real" trigger

- Events asynchronous and unpredictable
  - E.g.: beta decay studies
- Signal split in:
  - **Trigger path**: from dedicated detectors to trigger logic
  - **data paths**: from all the detectors to storage on positive trigger decision
- Discriminator: generates an output digital signal if amplitude of the input pulse is greater than a given threshold
- Delay needed to compensate for the trigger latency



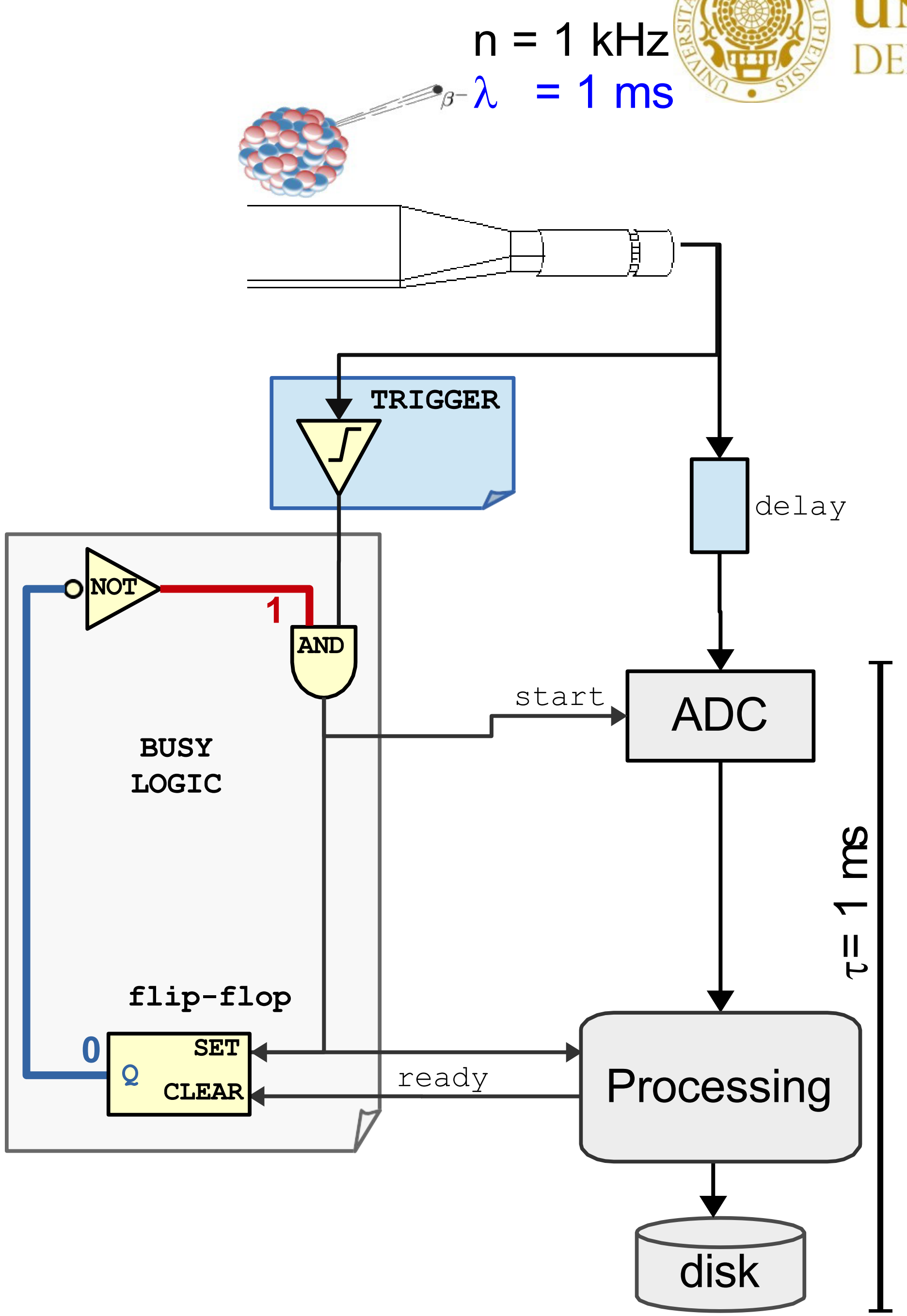
Busy logic

- The busy logic avoids triggers while the system is busy in processing
- A minimal busy logic can be implemented with
  - an AND gate
  - a NOT gate
  - a flip-flop (flip-flop)



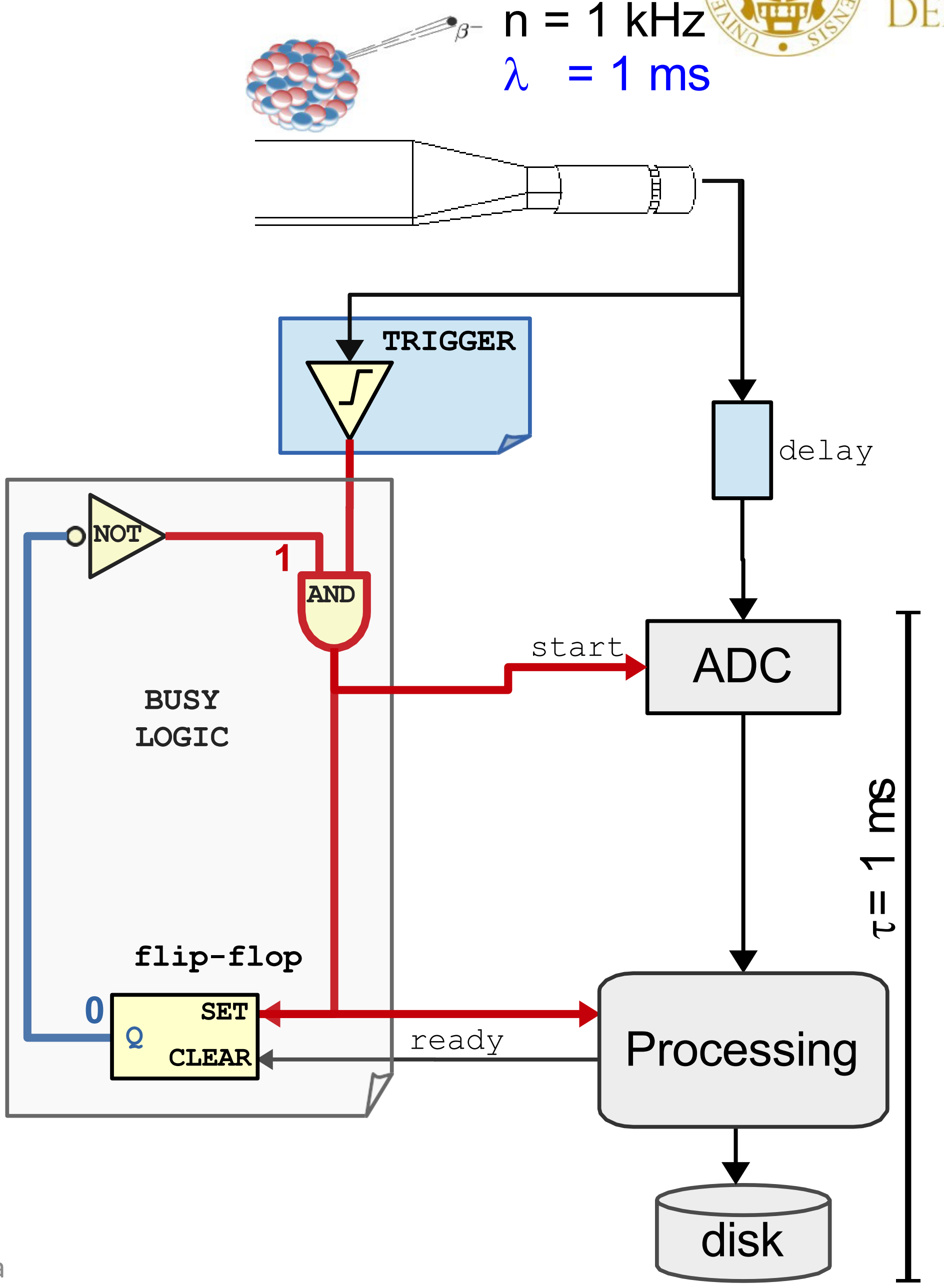
S	R	Q
0	0	Q_0
0	1	0
1	0	1
1	1	?

Memory!!



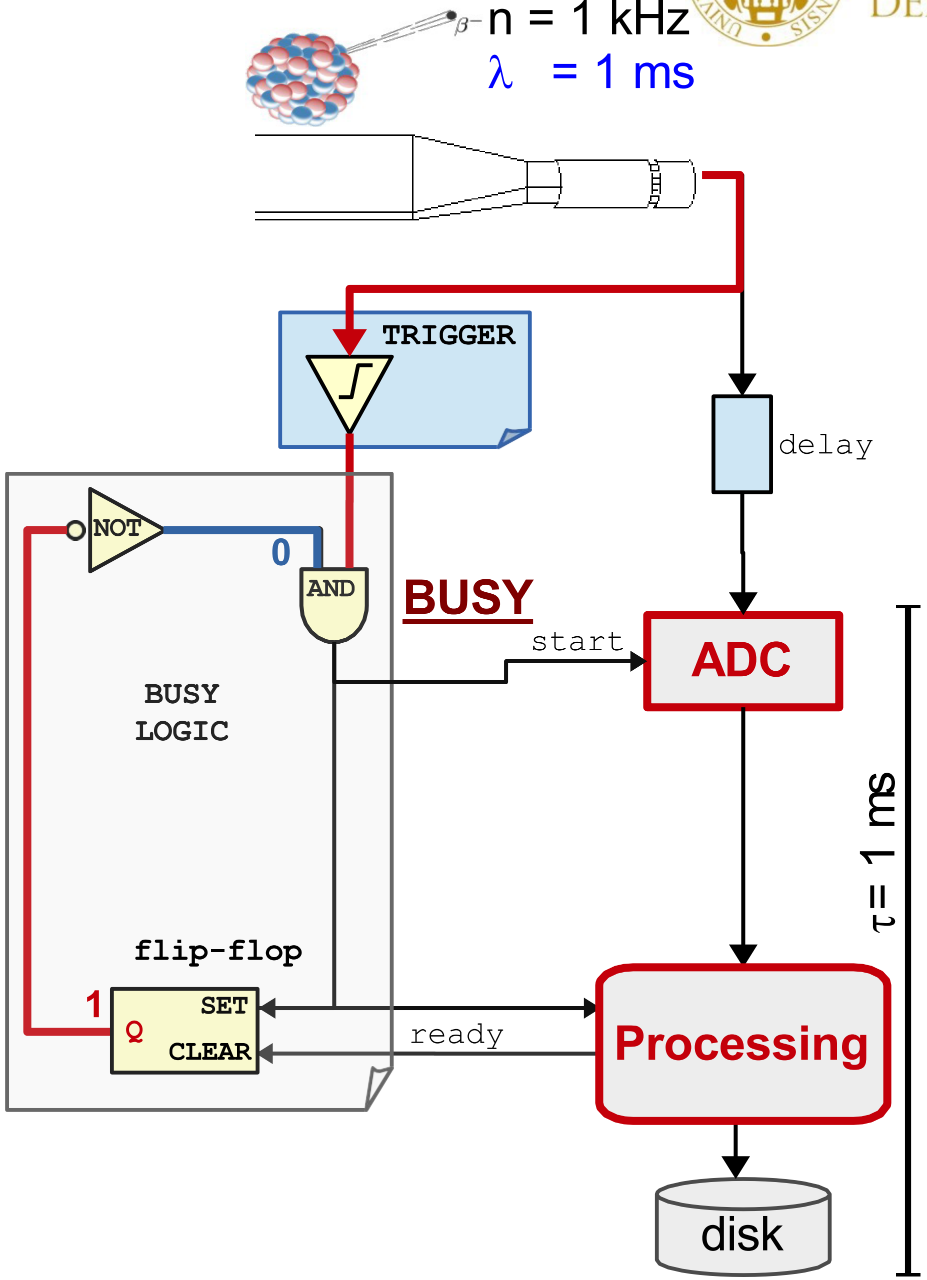
Busy logic

- Start of run, system ready for triggers:
  - the flip-flop output is down (ground state)
  - via the NOT, one of the port of the AND gate is set to up (opened)
- When a new trigger arrives, the signal finds the AND gate open, so:
  - The ADC is started
  - The processing is started
  - The flip-flop receives a signal to flip



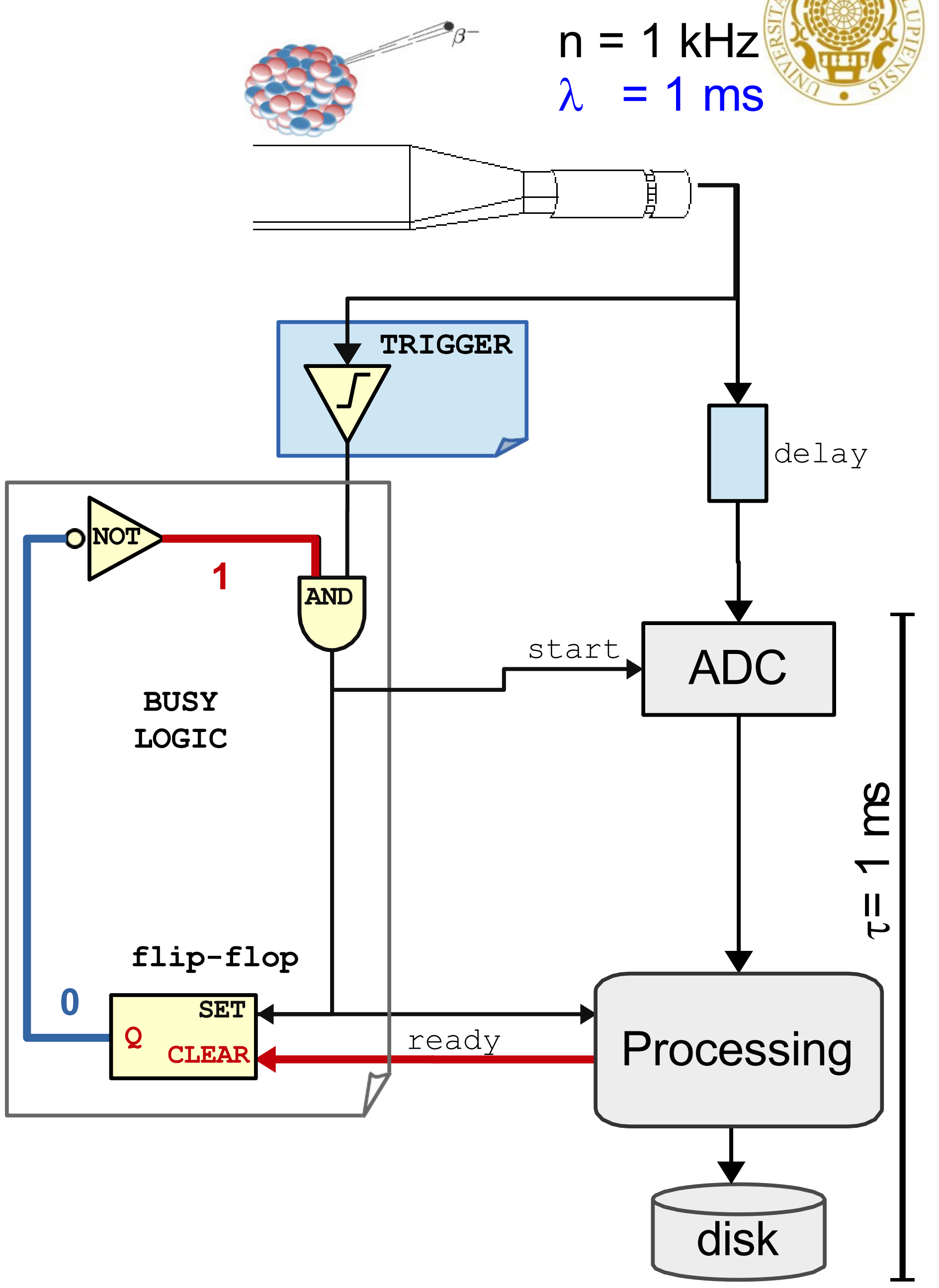
Busy logic

- One of the AND inputs is now steadily down (closed)
- Any new trigger is inhibited by the AND gate (busy)



Busy logic

- At the end of processing a ready signal is sent to the flip-flop
  - The flip-flop flips again
  - The gate is now opened
  - The system is ready to accept a new trigger
- busy logic avoids triggers while DAQ is busy in processing
  - New triggers do not interfere w/ previous data



DAQ deadtime and system efficiency

- Given the busy logic design, the system is *non-paralysable* and it as a fixed dead-time interval.
- Given the true rate n and their stochastic fluctuations, the DAQ rate is:

$$m = \frac{n}{1 + n\tau} \leq n$$

- We can define the efficiency as:

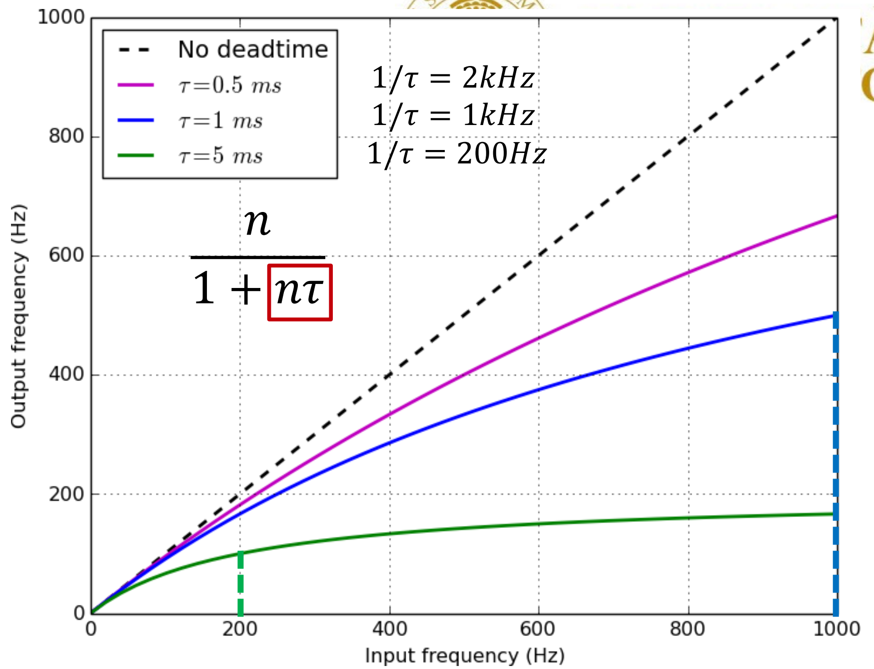
$$\epsilon = \frac{M}{N} = \frac{m}{n} = \frac{1}{1 + n\tau} \leq 100\%$$

- With a physic rate of 1kHz and a deadtime of 1ms we get:

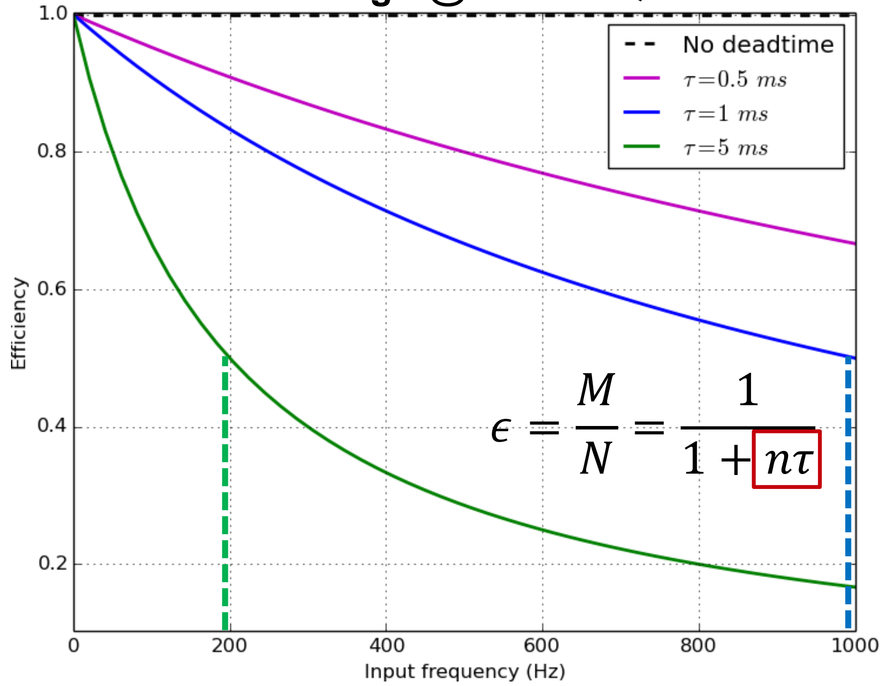
$$m = \frac{1kHz}{1+1} = 500Hz\,, \qquad \epsilon = \frac{m}{n} = 0.5 = 50\%$$

Deadtime and efficiency

- To obtain $\epsilon \sim 100\%$ (i.e.: $m \sim n$) $\rightarrow \; n\tau \ll \; 1$
- E.g.: $\epsilon \sim \; 99\%$ for $n = \; 1 \, kHz \; \rightarrow \; \tau < \; 0.01 \, ms \; \rightarrow \; 1/\tau > \; 100 \, kHz$
- *To cope with the input signal fluctuations, we have to over-design our DAQ system by a factor 100!*
- How can we mitigate this effect?



Andrea Negri @ ISOTDAQ 2024



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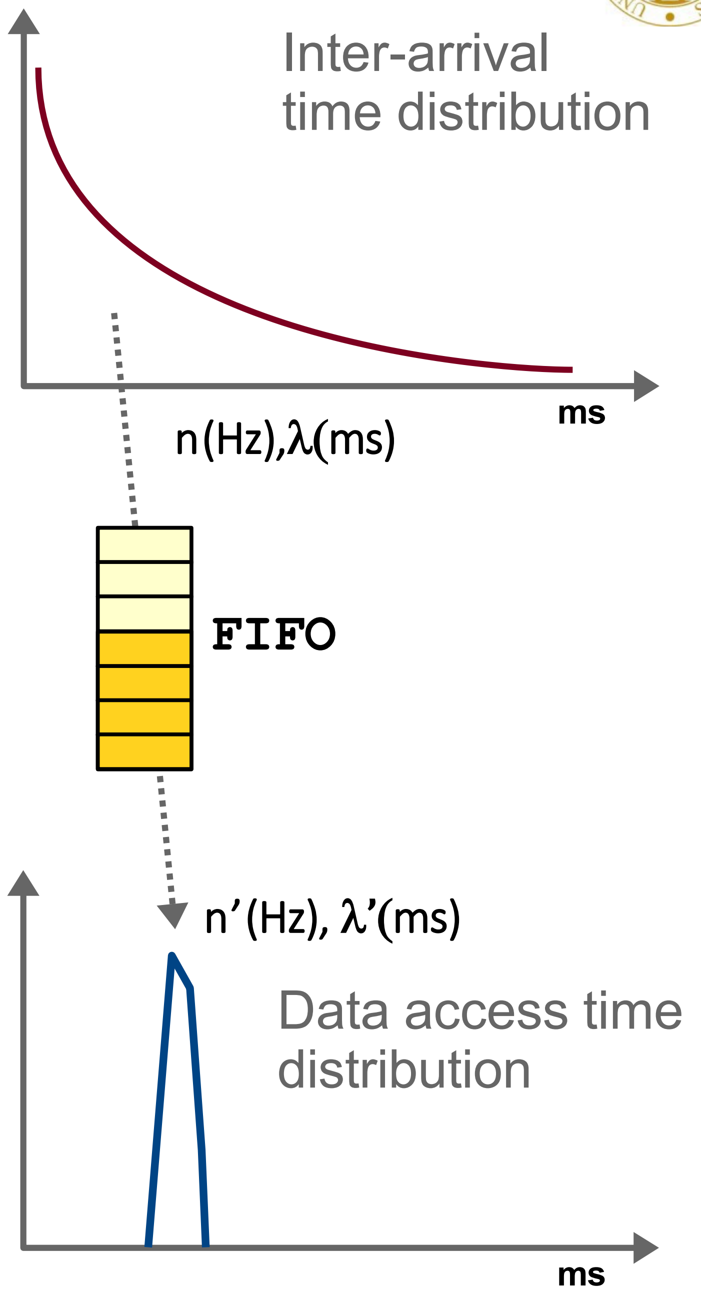
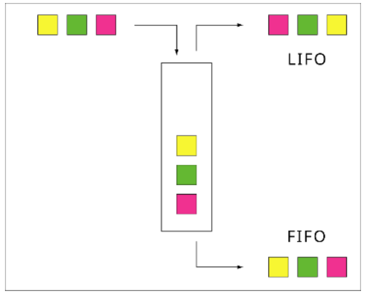
De-randomization

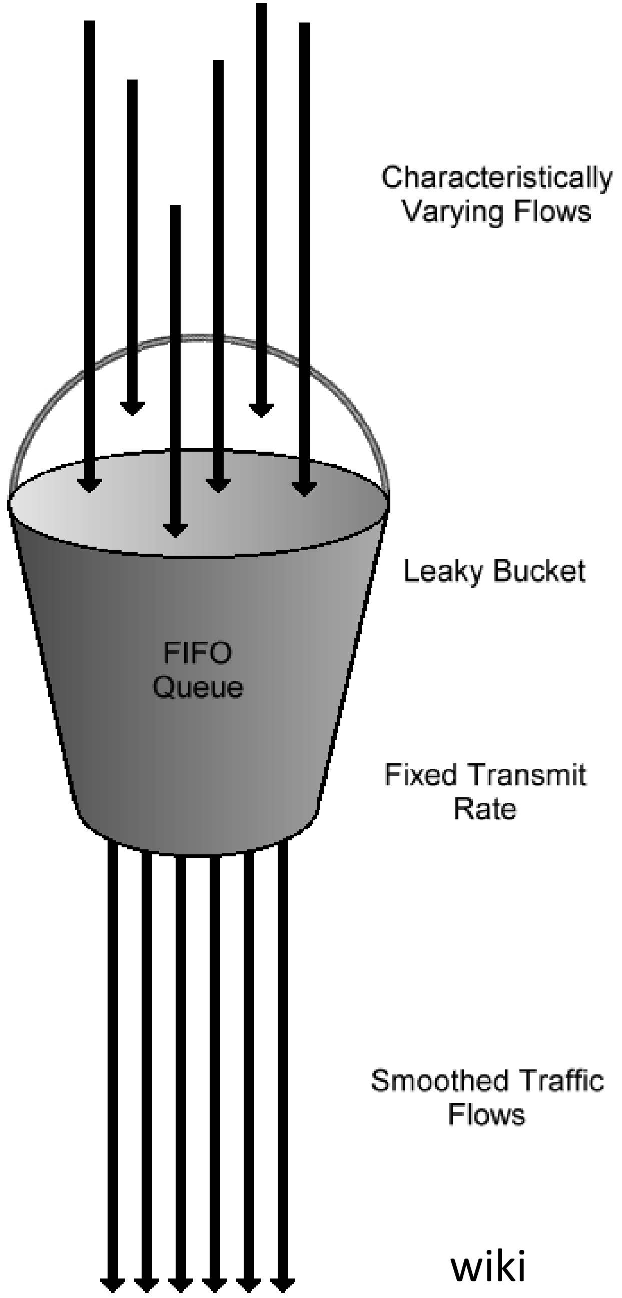
- What if we were able to make the system more deterministic and less dependent on the stochastic arrival time of our signals?
- *Then we could ensure that events don't arrive when the system is busy*
- This is called **de-randomization**
- How it can be achieved?
  - by buffering the data (having a holding queue where we can slot it up to be processed)





De-randomization Leacky bucket

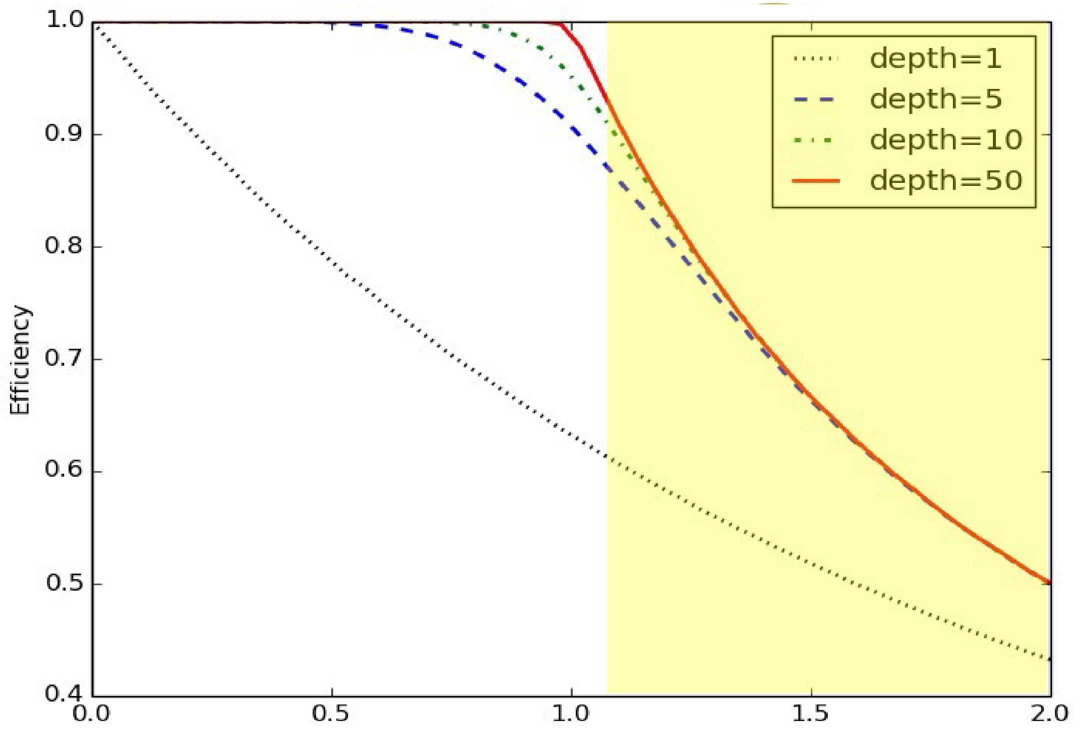
- "The leaky bucket consists of a finite queue. When a packet arrives, if there is room on the queue it is appended to the queue; otherwise, it is discarded.
- At every clock tick one packet is transmitted (unless the queue is empty)", so the packets are only ever transmitted at a fixed rate.
- "*The leaky bucket algorithm enforces a rigid output pattern at the average rate, no matter how bursty the [input] traffic is*". (From A. S. Tanenbaum)
- This is only strictly true if the queue does not become empty (for a small arrival rate or gaps in it)



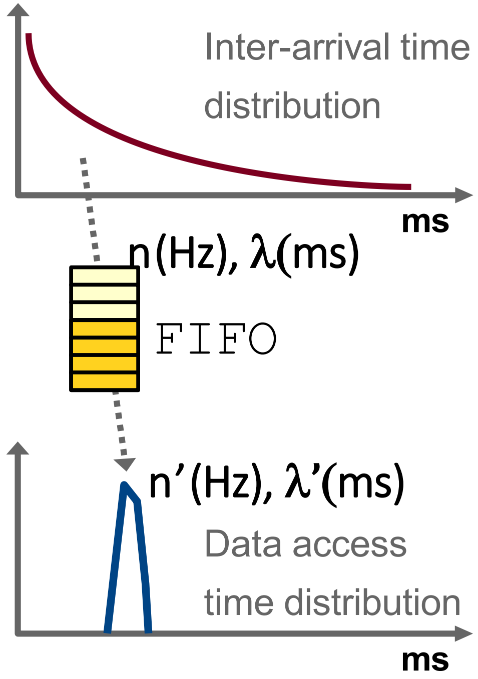
wiki

De-randomization

- Queue depth is the number of values that can be stored in the FIFO
- Efficiency vs traffic intensity ($\rho = \frac{\lambda'}{\lambda} = n/n'$) for different queue depths is:
  - $\rho > 1$, the system is overloaded, the FIFO output time is larger than the input time($\lambda' > \lambda$). The output rate will be smaller than the input one.



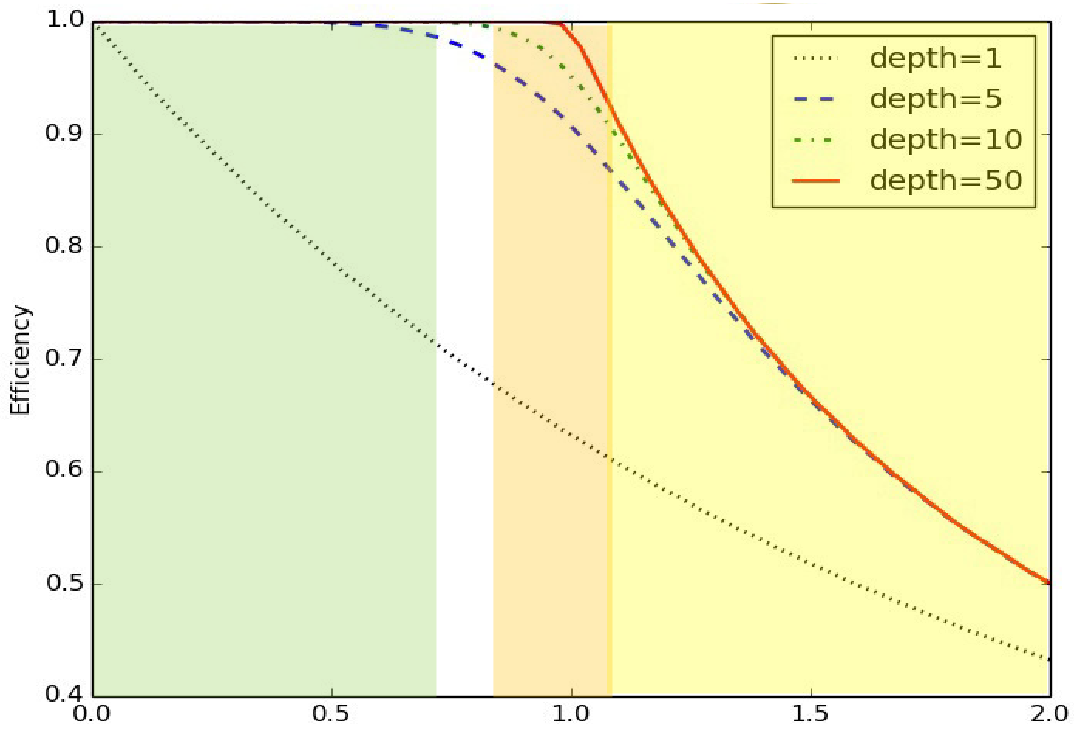
$\rho = \lambda'/\lambda$



De-randomization

- $\rho \ll 1$, the system is over-designed, the FIFO output time is shorter than the input time ($\lambda' \ll \lambda$). The output rate will be saturated.
- $\rho \sim 1$, the FIFO output time is similar than the input time ($\lambda' \sim \lambda$). *Using a queue, we have a high efficiency even with moderate depth.*

- *Analytic calculation possible for very simple systems only, otherwise MonteCarlo simulation is required*



$\rho = \lambda'/\lambda$