

Lezione 15

Struttura del protone

Momento magnetico anomalo

$$\mu = \frac{e\hbar}{2Mc} = g \frac{1}{2} \frac{e\hbar}{2Mc}$$

Analogo semi-classico. Elettrone che orbita. Corrente generata.

$$I = e\nu = e \frac{\omega}{2\pi}$$

Momento di dipolo magnetico

$$|\mathbf{M}| = A \frac{I}{c} = \pi r^2 \frac{1}{c} e \frac{\omega}{2\pi}$$

A area del circuito

Momento angolare

$$|\mathbf{L}| = m_e r^2 \omega = \hbar |l|$$

Sostituendo gli autovalori di l $\omega = \hbar l / m_e r^2$ quindi

$$|\mathbf{M}| = \frac{e r^2}{c} \frac{\hbar l}{2 m_e r^2} = \frac{e\hbar}{2m_e c} l$$

Unità minima ($l = 1$) è il magnetone di Bohr

$$\mu_B = \frac{e\hbar}{2m_e c}$$

Momento magnetico dell'elettrone

$$\mu_e = g \frac{1}{2} \mu_B$$

In QED $g = 2$ fino a 10^{-8}

Per i nucleoni

$$\mu_N = \frac{e\hbar}{2m_N c} \quad m_N = \frac{1}{2}(m_p + m_n)$$

protone $\mu_p = 2.79\mu_N$

neutrone $\mu_n = -1.91\mu_N$

Diffusione elastica di elettroni da protoni

1) Cinematica

$$k_i + p_i = k_f + p_f \quad ; \quad \epsilon_i + M = \epsilon_f + E_f \quad ; \quad \mathbf{k}_i + 0 = \mathbf{k}_f + \mathbf{p}_f$$

$$E_f - M = \epsilon_i - \epsilon_f \equiv \omega \quad ; \quad \mathbf{p}_f = \mathbf{k}_i - \mathbf{k}_f \equiv \mathbf{q}$$

$$E_f^2 = M^2 + \mathbf{p}_f^2 \quad ; \quad (\omega + M)^2 = M^2 + \mathbf{q}^2$$

$$\omega^2 + 2M\omega + M^2 = M^2 + \mathbf{q}^2 \quad ; \quad \omega^2 + 2M\omega - \mathbf{q}^2 = 0$$

$$\omega = -M \pm \sqrt{M^2 + \mathbf{q}^2} = -M \pm M \left(1 + \frac{1}{2} \frac{\mathbf{q}^2}{M^2} + \dots \right)$$

Selezionando la soluzione +.

$$\omega = -M + M + \frac{1}{2} \frac{\mathbf{q}^2}{M} = \frac{\mathbf{q}^2}{2M}$$

Relazione univoca tra q e ω .

Sezione d'urto

Rinculo del bersaglio non più trascurabile

$$\frac{d\sigma}{d\Omega} = \sigma_M \frac{1}{1 + \frac{2\epsilon_i}{M} \sin^2 \theta/2} = \sigma_M \frac{\epsilon_f}{\epsilon_i} = \sigma_M f_{rec}$$

Sezione d'urto di Mott

$$\sigma_M = \frac{\alpha^2 \cos^2 \theta/2}{4\epsilon_i^2 \sin^4 \theta/2}$$

$$|\mathbf{k}| = \epsilon \quad ; \quad m_e \ll \epsilon \quad ; \quad q^2 = \epsilon_i^2 + \epsilon_f^2 - 2\epsilon_i\epsilon_f \cos \theta$$

$$\epsilon_i^2 \simeq \epsilon_f^2 \quad ; \quad q^2 \simeq 2\epsilon_i\epsilon_f(1 - \cos \theta) = 4\epsilon_i\epsilon_f \sin^2 \theta/2$$

$$1 + \frac{2\epsilon_i}{M} \sin^2 \theta/2 = 1 + \frac{q^2}{2M\epsilon_f} = \frac{1}{\epsilon_f} \left(\epsilon_f + \frac{q^2}{2M} \right) = \frac{1}{\epsilon_f} (\epsilon_f + \omega) = \frac{\epsilon_i}{\epsilon_f}$$

Sezione d'urto

Momento magnetico del bersaglio.

$$\left(\frac{d\sigma}{d\omega}\right)_{punt} = f_{rec} \sigma_M \left[1 + 2 \frac{\mathbf{q}^2 - \omega^2}{4M^2} \tan^2 \theta/2 \right]$$

Cambio di variabili

$$Q^2 = -q_\mu q^\mu = \mathbf{q}^2 - \omega^2 \quad \tau = \frac{Q^2}{4M^2}$$

$$\left(\frac{d\sigma}{d\omega}\right)_{punt} = f_{rec} \sigma_M \left[1 + 2\tau \tan^2 \theta/2 \right]$$

Fattori di forma

Dirac

$$\left(\frac{d\sigma}{d\omega}\right) = f_{rec} \sigma_M \left[F_E(Q^2) + 2\tau F_M(Q^2) \tan^2 \theta/2 \right]$$

Sachs (più usati)

$$\left(\frac{d\sigma}{d\omega}\right) = f_{rec} \sigma_M \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \theta/2 \right]$$

$$\begin{aligned} G_E^p(0) &= 1 & ; & & G_E^n(0) &= 0 \\ G_M^p(0) &= 2.79 & ; & & G_M^n(0) &= -1.91 \end{aligned}$$

Separazione di Rosenbluth

$$\frac{\left(\frac{d\sigma}{d\omega}\right)}{f_{rec} \sigma_M} = \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \theta/2 \right]$$

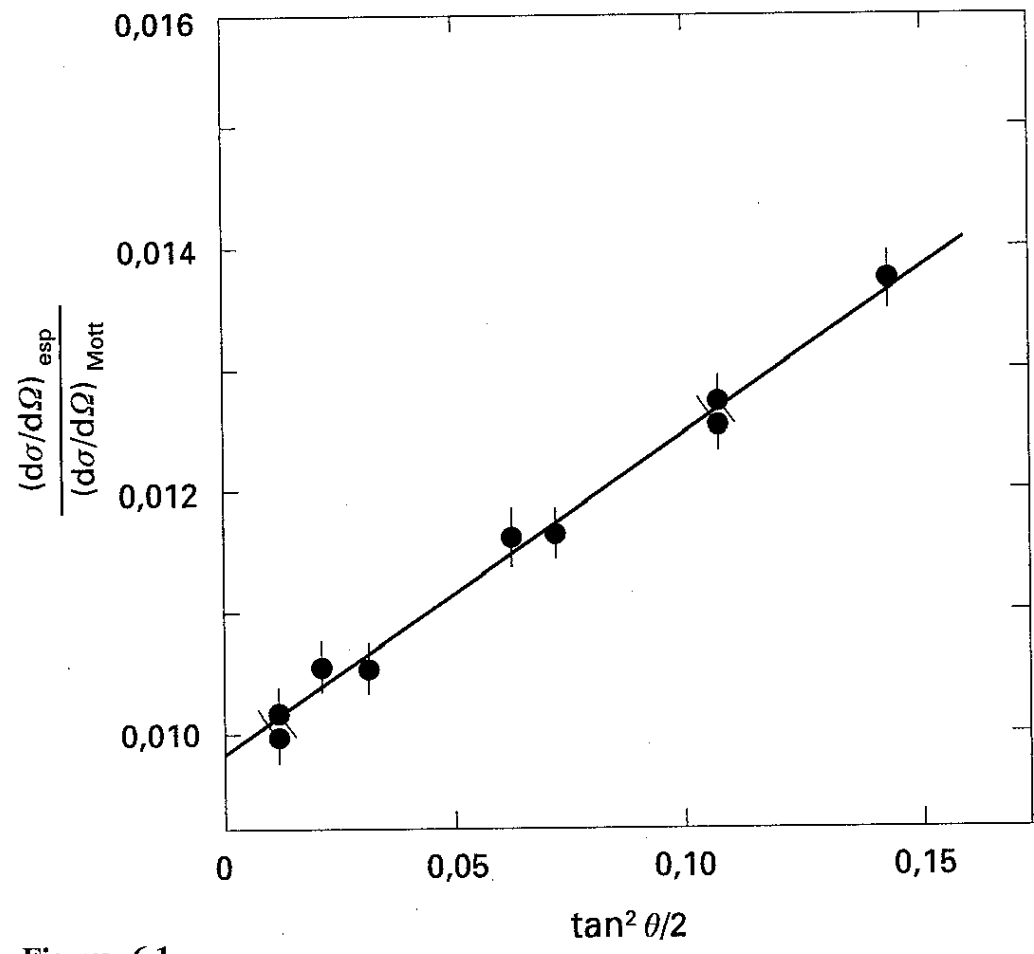


Figura 6.1

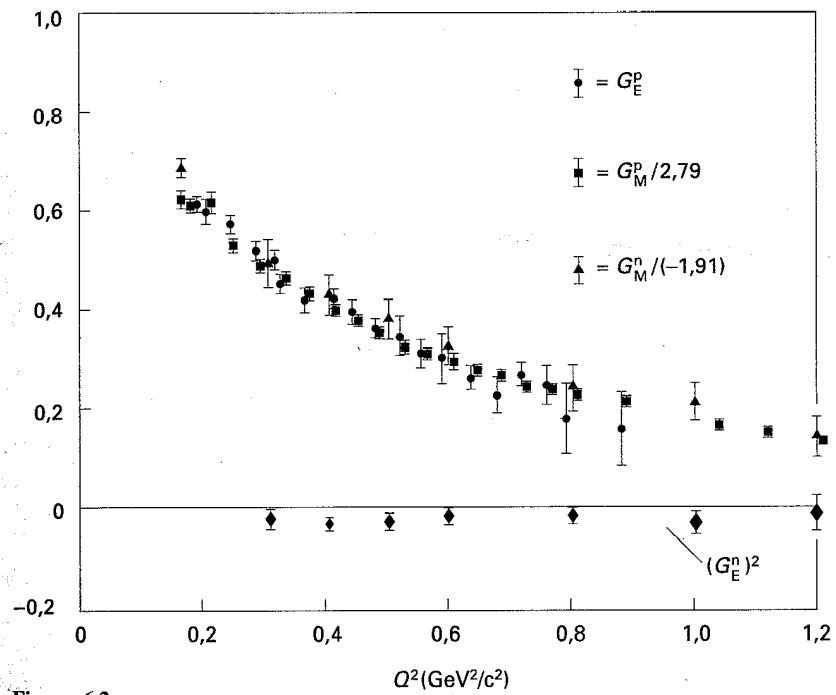


Figura 6.2

$$G_E^p(Q^2) \simeq \frac{G_M^p(Q^2)}{2.79} \simeq \frac{G_M^n(Q^2)}{-1.91} \simeq G^{\text{dipolo}}(Q^2) = \left(1 + \frac{Q^2}{0.71[\text{GeV}^2]}\right)^{-2}$$

$$\omega \ll M$$

$$\begin{aligned} F(|\mathbf{q}|^2) &= \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} \rho(r) = \int d^3r \left[\sum_{n=0}^{\infty} \frac{1}{n!} (i\mathbf{q}\cdot\mathbf{r})^n \right] \rho(r) \\ &= \int d\Omega \int_0^{\infty} dr r^2 \left[1 - \frac{1}{2} (|\mathbf{q}||\mathbf{r}|)^2 \cos^2 \theta + \dots \right] \rho(r) \\ &= 4\pi \int_0^{\infty} dr r^2 \rho(r) - \frac{1}{2} \int_0^{2\pi} d\phi \int_{-1}^1 d(\cos \theta) \cos^2 \theta \int_0^{\infty} dr r^2 (|\mathbf{q}||\mathbf{r}|)^2 \rho(r) + \dots \\ &= 1 - \frac{1}{6} |\mathbf{q}|^2 4\pi \int_0^{\infty} dr r^4 \rho(r) + \dots = 1 - \frac{1}{6} |\mathbf{q}|^2 \langle r^2 \rangle \end{aligned}$$

$$\frac{dF(|\mathbf{q}|^2)}{d|\mathbf{q}|^2} = -\frac{1}{6} \langle r^2 \rangle \quad ; \quad \langle r^2 \rangle = -6 \frac{dF(|\mathbf{q}|^2)}{d|\mathbf{q}|^2} \Big|_{q^2=0}$$

$$\sqrt{\langle r^2 \rangle_p} \simeq 0.862 \text{ fm}$$

Domande

[P2-2] [P2-3] [P4-2] [P4-3] [P4-4]