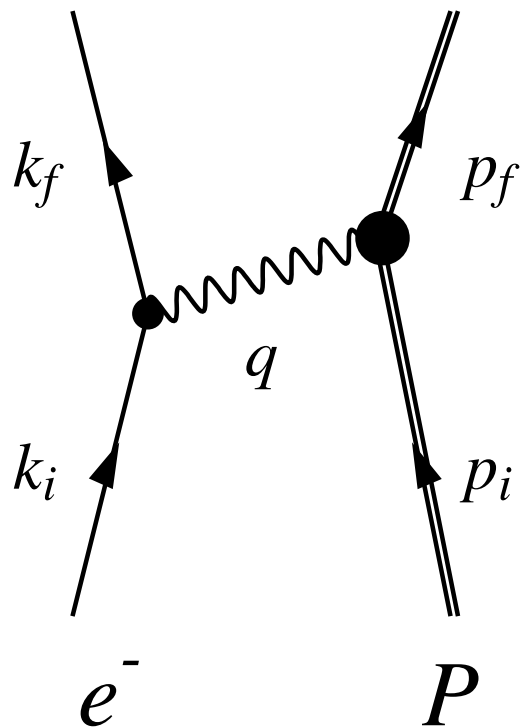


## Lezione 16

# Diffusione profondamente inelastica di leptoni da protoni



$$k \equiv (\epsilon, \mathbf{k}) \quad m_e \ll \epsilon \quad |\mathbf{k}| \simeq \epsilon$$

In laboratorio

$$p_i \equiv (M, 0) \quad p_f \equiv (E_f, \mathbf{p}_f)$$

$$q = k_i - k_f = p_f - p_i \quad \omega = \epsilon_i - \epsilon_f \quad \mathbf{q} = \mathbf{k}_i - \mathbf{k}_f$$

$$\begin{aligned}
k_i + p_i &= k_f + p_f \\
(k_i + p_i)^2 &= (k_f + p_f)^2 \\
k_i^2 + p_i^2 + 2k_i \cdot p_i &= k_f^2 + p_f^2 + 2k_f \cdot p_f \\
m_e^2 + M^2 + 2k_i \cdot p_i &= m_e^2 + M^2 + 2k_f \cdot p_f \\
k_i \cdot p_i &= k_f \cdot p_f \\
p_f &= k_i + p_i - k_f
\end{aligned}$$

$$k_i \cdot p_i = k_f \cdot (k_i + p_i - k_f) = k_f \cdot k_i + k_f \cdot p_i - k_f^2$$

lab

$$k_i \equiv (\epsilon_i, \mathbf{k}_i) \quad ; \quad p_i \equiv (M, 0) \quad ; \quad k_f \equiv (\epsilon_f, \mathbf{k}_f)$$

$$\epsilon_i M = \epsilon_i \epsilon_f - \mathbf{k}_i \cdot \mathbf{k}_f + \epsilon_f M - m_e^2$$

$$m_e \ll \epsilon \quad |\mathbf{k}| = \epsilon$$

$$\epsilon_i M = \epsilon_i \epsilon_f - \epsilon_i \epsilon_f \cos \theta + \epsilon_f M = \epsilon_f M \left[ 1 + \frac{\epsilon_i}{M} (1 - \cos \theta) \right]$$

$$\epsilon_f = \epsilon_i \left[ 1 + \frac{\epsilon_i}{M} (1 - \cos \theta) \right]^{-1}$$

## Massa invariante

$$W^2 = (p_i + q)^2 = p_f^2 = M^2 + 2p_i \cdot q + q^2 = M^2 + 2M\nu - Q^2$$

$$p_i \equiv (M, 0) \quad ; \quad q \equiv (\epsilon_i - \epsilon_f, \mathbf{q})$$

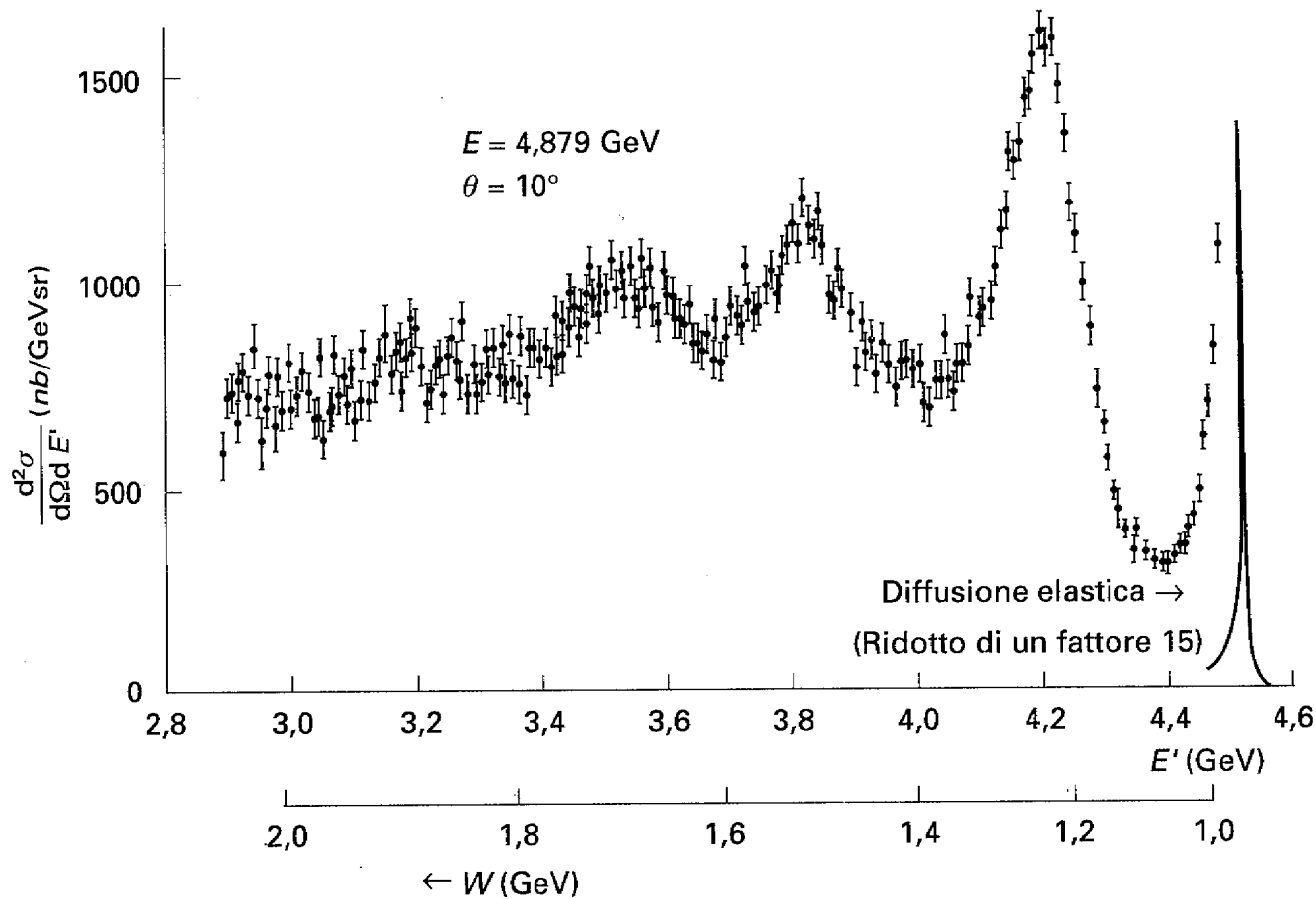
$$\nu = \frac{p_i \cdot q}{M} = \frac{1}{M} [M(\epsilon_i - \epsilon_f)] = \epsilon_i - \epsilon_f$$

Nei processi elastici  $W = M$

$$W^2 = M^2 = M^2 + 2M\nu - Q^2 \quad ; \quad 2M\nu - Q^2 = 0$$

In processi anelastici  $W > M$

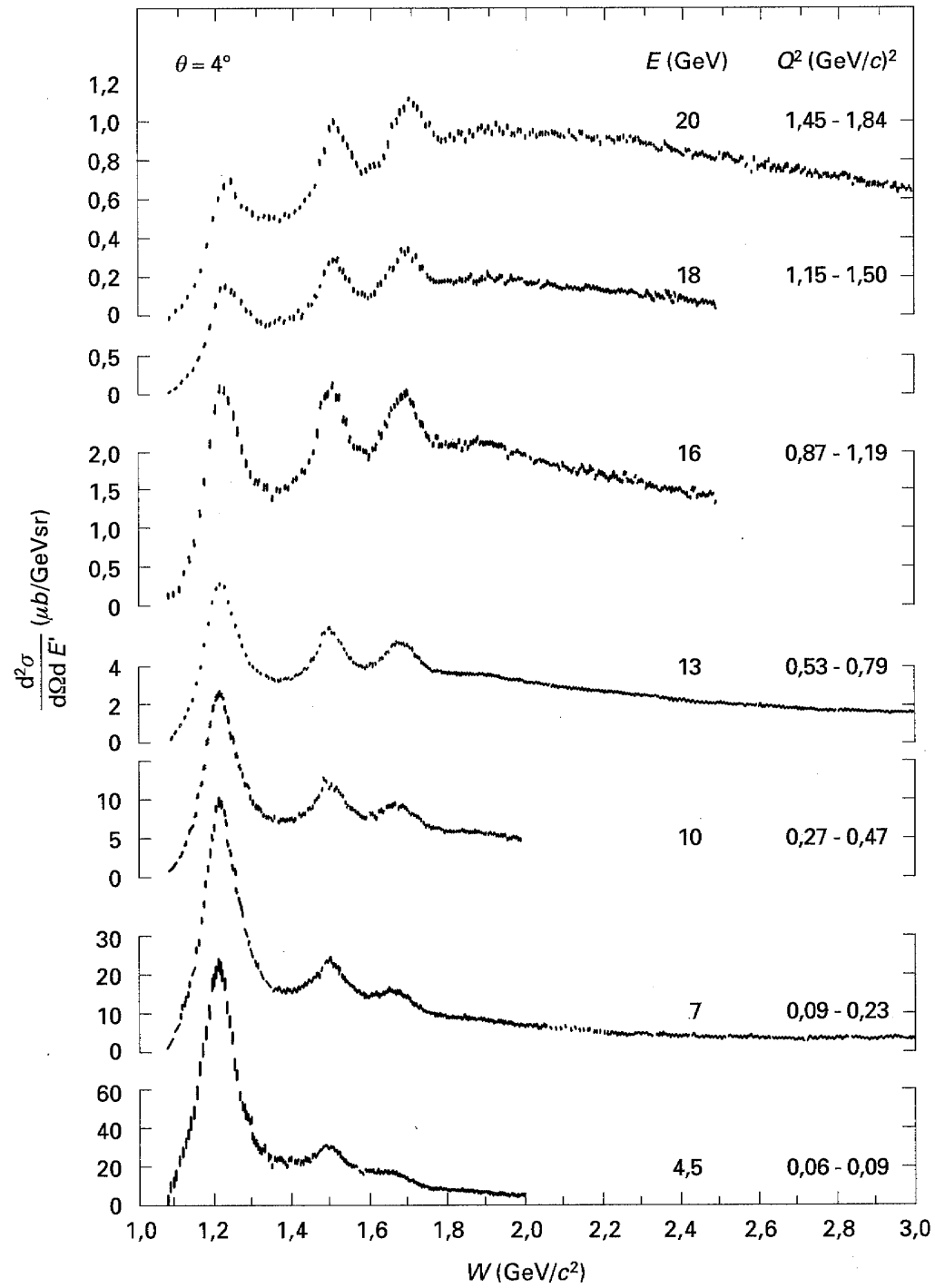
$$W^2 = p_f^2 = M^2 + \Delta_w^2 = M^2 + 2M\nu - Q^2 \quad ; \quad 2M\nu - Q^2 > 0$$



$$e + p \rightarrow X + e' \quad \epsilon_f = \epsilon_i - \omega$$

Risonanza  $\Delta$  a 1232 MeV.  $\Delta^+$  per conservazione della carica. Ampiezza (non legata all'esperimento)  $\Gamma = 120 \text{ MeV}$ .  $\hbar = 6.6 \cdot 10^{-22} \text{ MeV s}$ .

$$\tau = \frac{\hbar}{\Gamma} \simeq 5.5 \cdot 10^{-24} \text{ s} \quad \Delta^+ \rightarrow p + \pi^0 \quad \Delta^+ \rightarrow n + \pi^+$$



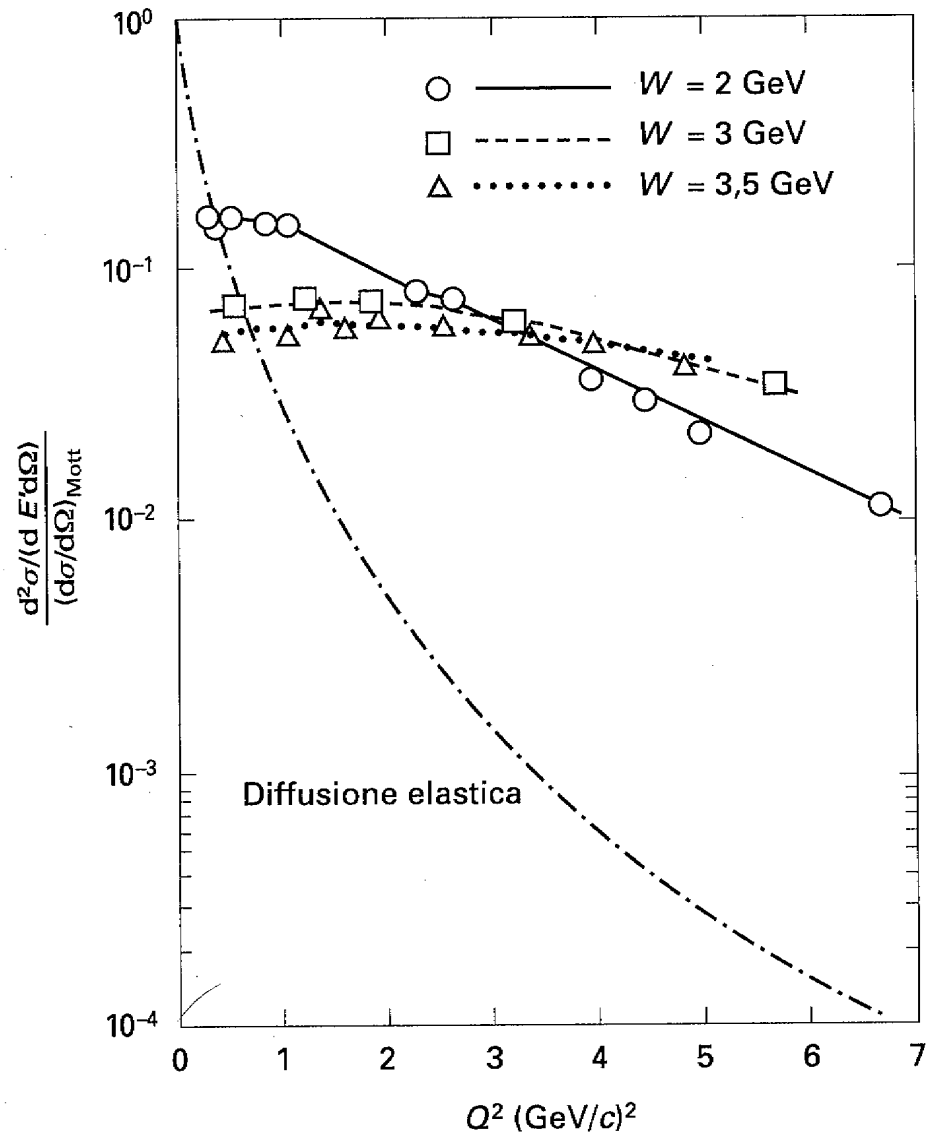
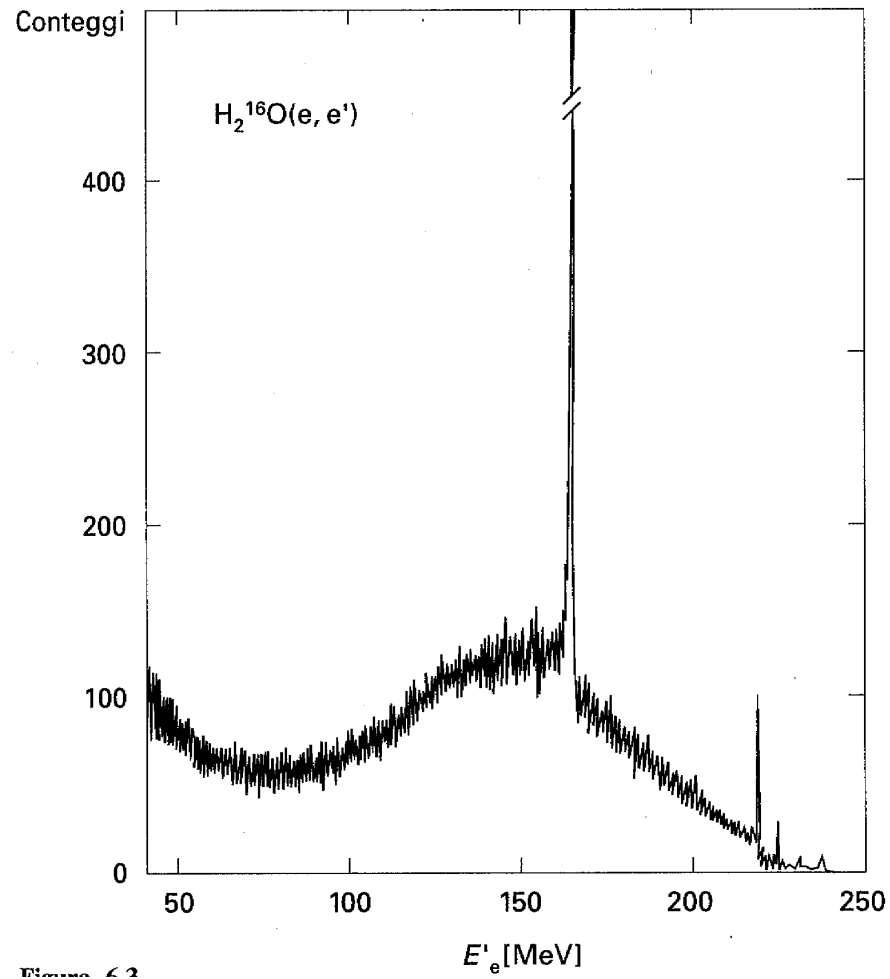


FIGURA 7.2

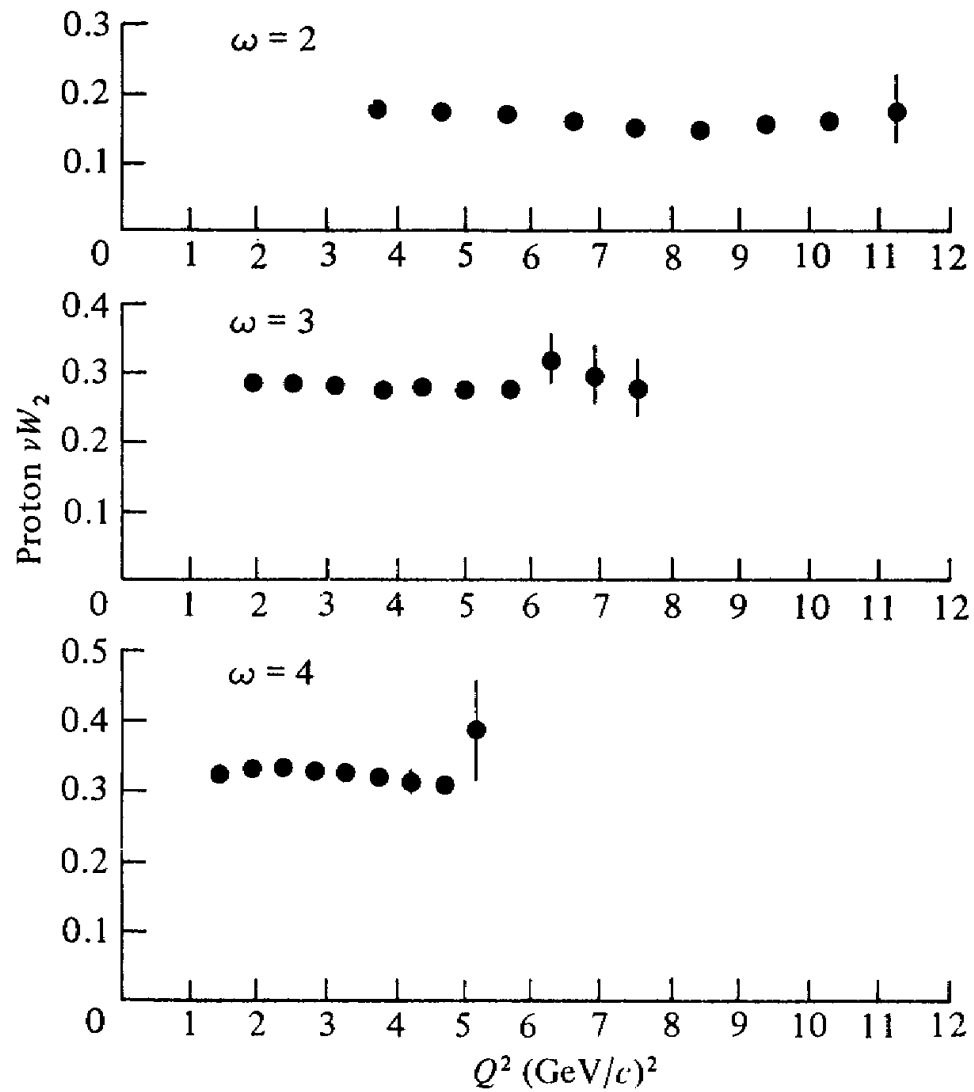
$$\frac{d^2\sigma}{d\Omega d\epsilon_f} = \sigma_M \left[ W_2(Q^2, \nu) + 2W_1(Q^2, \nu) \tan^2 \frac{\theta}{2} \right]$$

# Diffusione quasi-elastica da nuclei



$$\omega_{picco} \simeq \frac{q^2}{2m} - E_B$$





$$\omega = 1/x$$

$$x = \frac{Q^2}{2p_i \cdot q} = \frac{Q^2}{2M\nu}$$

$$x = 1 ; 2M\nu - Q^2 = 0$$

$$x < 1 ; 2M\nu - Q^2 > 0$$

## Bjorken scaling

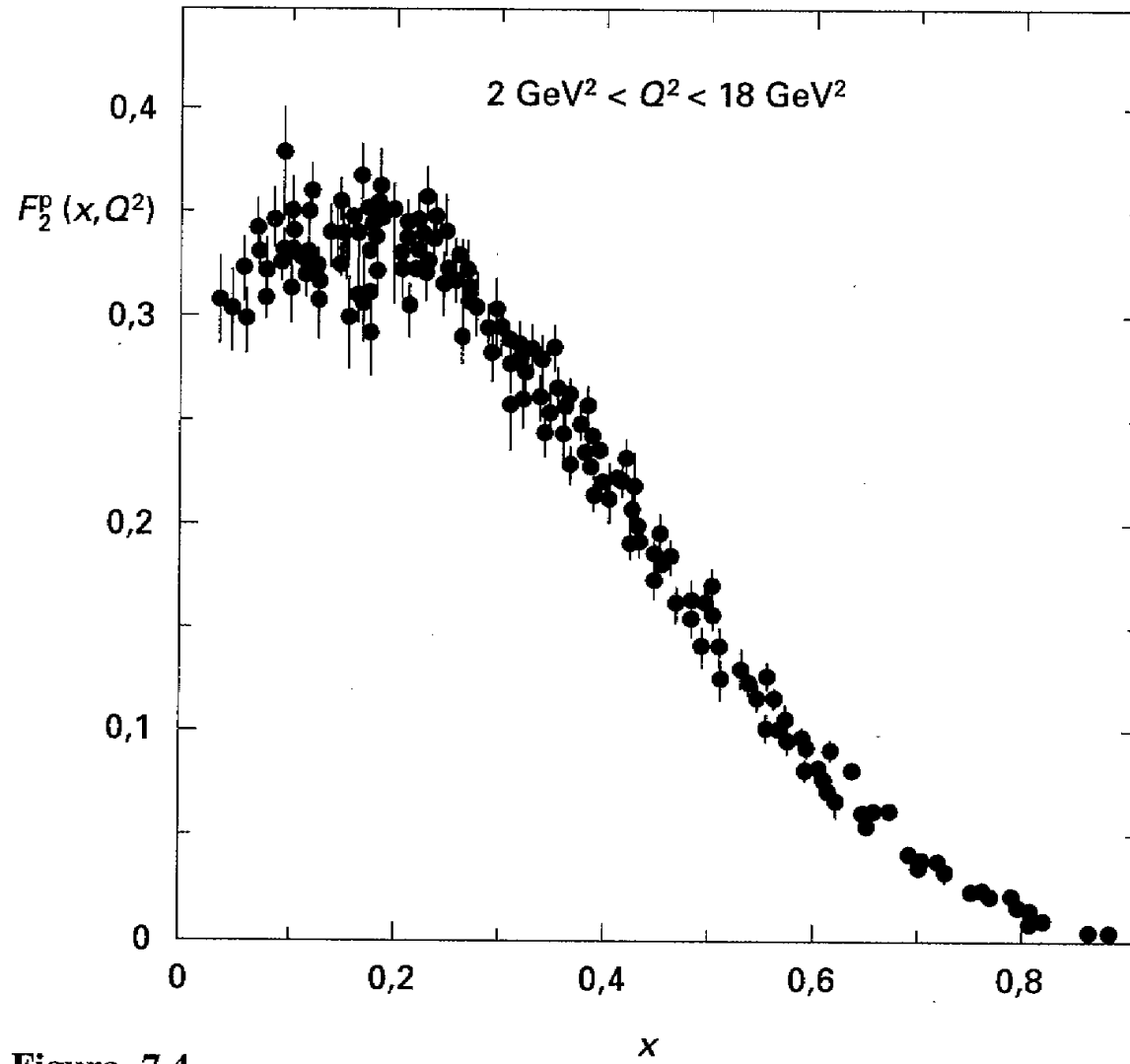
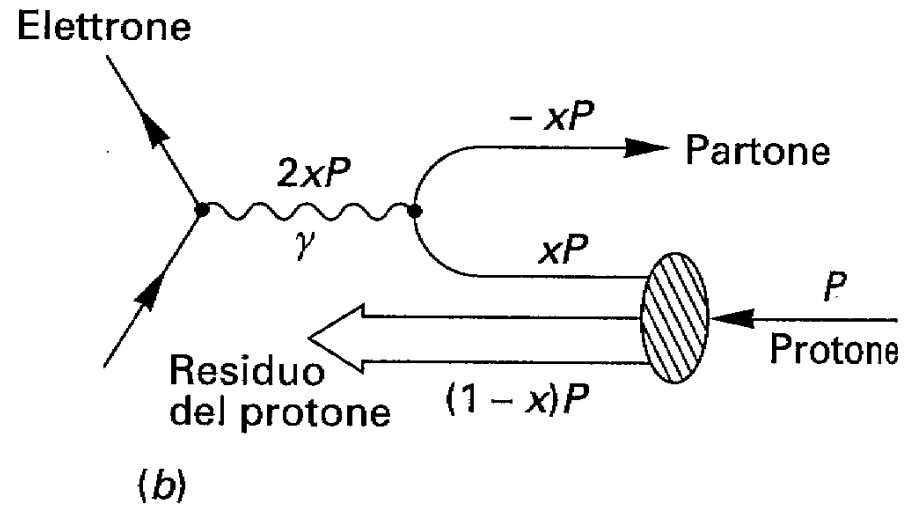
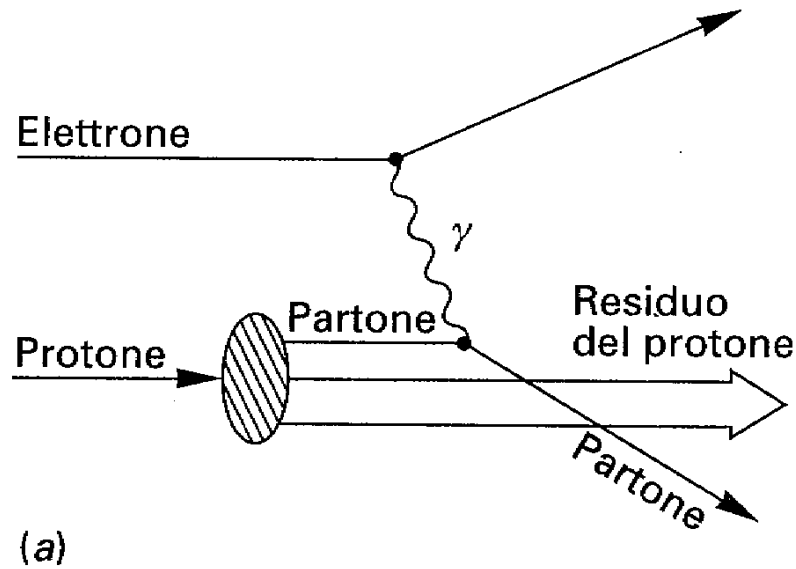


Figura 7.4

## Modello a partoni



Analogia con il quasi-elastico sul nucleo. Non c'è emissione di partone.

Sistema di riferimento comodo per i calcoli.

Il partone scambia solo impulso l'energia è la stessa prima e dopo l'urto.

$$|q| = 2|p|.$$

Componenti trasverse di  $p$  trascurabili.

Piccole distanze - partoni non interagenti.

Nuove variabili  $Q^2 = 4\epsilon_i\epsilon_f \sin^2 \theta/2$  e  $\nu = \epsilon_i - \epsilon_f$

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{\pi}{\epsilon_i\epsilon_f} \frac{d^2\sigma}{d\Omega d\epsilon_f}$$

Sezione d'urto **elastica** su bersaglio puntiforme di carica  $z_j$  (frazione della carica elementare  $|e|$ ) e massa  $m_j = Mx = MQ^2/2M\nu$ .

$$\left(\frac{d^2\sigma}{dQ^2 d\nu}\right)_{part} = \sigma_M \frac{\pi}{\epsilon_i\epsilon_f} \left[ z_j^2 + z_j^2 \frac{Q^2}{4m_j^2} 2 \tan^2 \frac{\theta}{2} \right] \delta\left(\nu - \frac{Q^2}{2m_j}\right)$$

La sezione d'urto totale si ottiene sommando su tutte le sezioni d'urto individuali. Per la  $W_2$ .

$$W_2(Q^2, \nu) = \sum_j \int_0^1 dx q_j(x) z_j^2 \delta\left(\nu - \frac{Q^2}{2m_j}\right)$$

$$\delta(g(x)) = \frac{\delta(x - x_0)}{\left|\frac{dg}{dx}\right|_{x=x_0}} \quad g(x_0) = 0$$

$$g(x) = \nu - \frac{Q^2}{2xM} \quad x_0 = \frac{Q^2}{2\nu M} \quad \text{per} \quad g(x_0) = 0$$

$$\frac{dg(x)}{dx} \Big|_{x=x_0} = -\frac{Q^2}{2M} \left( \frac{-1}{x^2} \right) \Big|_{x=x_0} = \frac{Q^2}{2M} \left( \frac{2M\nu}{Q^2} \frac{1}{x_0} \right) = \frac{\nu}{x_0}$$

$$W_2(Q^2, \nu) = \sum_j \int_0^1 dx q_j(x) z_j^2 \frac{x_0}{\nu} \delta(x - x_0) = \frac{1}{\nu} \sum_j z_j^2 q_j(x_0) x_0$$

$$\begin{aligned} W_1(Q^2, \nu) &= \sum_j \int_0^1 dx q_j(x) z_j^2 \frac{Q^2}{4x^2 M^2} \delta\left(\nu - \frac{Q^2}{2m_j}\right) \\ &= \sum_j \int_0^1 dx q_j(x) z_j^2 \frac{Q^2}{4x^2 M^2} \frac{x_0}{\nu} \delta(x - x_0) \\ &= \sum_j q_j(x_0) z_j^2 \frac{Q^2}{4M^2} \frac{x_0}{x_0^2 \nu} \\ &= \sum_j q_j(x_0) z_j^2 \frac{1}{2} \frac{Q^2}{2M^2 \nu} \frac{1}{x_0} = \frac{1}{M} \frac{1}{2} \sum_j q_j(x_0) z_j^2 \end{aligned}$$

## Nuove variabili

$$F_1(x) = MW_1(Q^2, \nu) = \frac{1}{2} \sum_j q_j(x) z_j^2$$

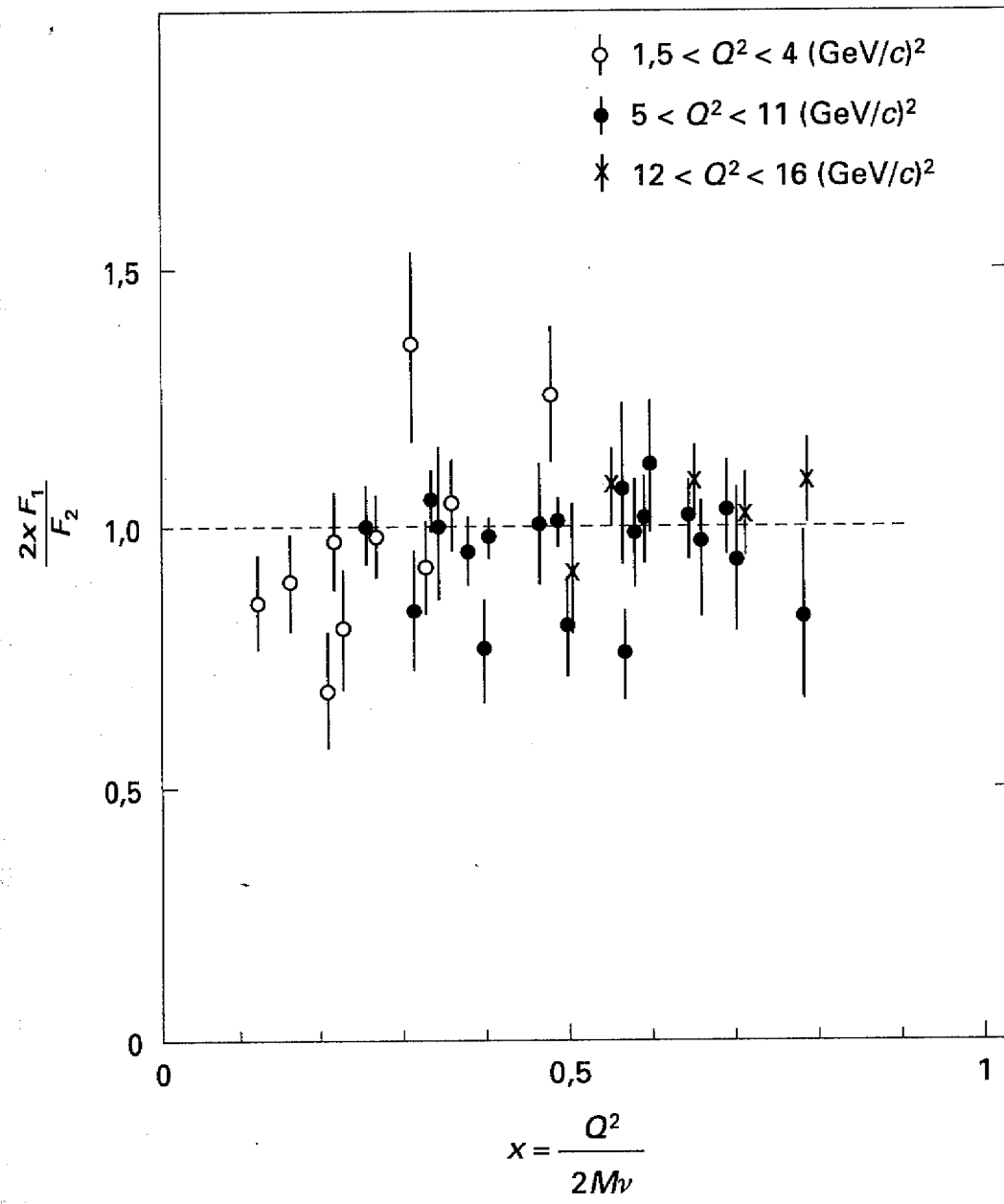
$$F_2(x) = \nu W_2(Q^2, \nu) = x \sum_j q_j(x) z_j^2$$

## Relazione di Callan - Gross

$$2xF_1(x) = F_2(x) = x \sum_j q_j(x) z_j^2$$

Tutti i centri diffusori hanno spin 1/2.

Approssimazione impulsiva, i partoni non interagiscono.



Probabilità di trovare il partone di tipo  $j$  con frazione  $x$  dell'impulso totale del protone moltiplicato per la frazione della carica.  $q_j(x)z_j^2$

$$\int_0^1 dx \sum_j q_j(x)z_j^2 = \int_0^1 dx \frac{F_2(x)}{x} \rightarrow \infty$$

$F_2(x)$  finito per  $x \rightarrow 0$   $N \rightarrow \infty$

## Identificazione dei partoni con i quarks

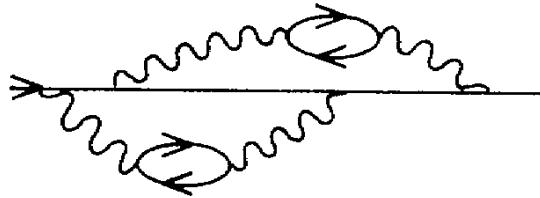
Dalla spettroscopia adronica

sapore	carica	$I$	$I_3$	spin
u	2/3	1/2	1/2	1/2
d	-1/3	1/2	-1/2	1/2

$$p \equiv (u, u, d) \quad n \equiv (d, d, u)$$



Dalla QED



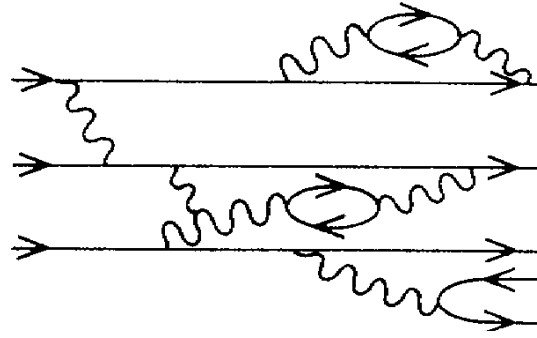
Fluttuazioni quantistiche del vuoto

Spettroscopia dell'H, Lamb shift

Magnetone di Bohr

$$\mu_B = g \frac{1}{2} \frac{e\hbar}{2M} \quad \text{Dirac} \quad g = 2$$

$$\begin{aligned} \frac{g - 2}{2} &= (1159657.7 \pm 3.5) 10^{-9} \text{ Exp} \\ &= (1159655.4 \pm 3.3) 10^{-9} \text{ Th} \end{aligned}$$



Il fotone può interagire sia con i quark di valenza sia con quelli del mare.  
Per il protone

$$\begin{aligned}
 \frac{1}{x} F_2^p &= \sum_j z_j^2 q_j(x) \\
 &= \left(\frac{2}{3}\right)^2 [u^p(x) + \bar{u}^p(x)] + \left(\frac{-1}{3}\right)^2 [d^p(x) + \bar{d}^p(x)] + \dots \\
 &= \left(\frac{2}{3}\right)^2 [u_V^p(x) + u_S^p(x) + \bar{u}_S^p(x)] + \left(\frac{-1}{3}\right)^2 [d_V^p(x) + d_S^p(x) + \bar{d}_S^p(x)] + \dots \\
 &= \frac{1}{9} [4u_V^p(x) + d_V^p(x)] + \frac{1}{9} [4u_S^p(x) + 4\bar{u}_S^p(x) + d_S^p(x) + \bar{d}_S^p(x)] + \dots
 \end{aligned}$$

Protone e neutrone fanno parte di un doppietto di isospin. Rotazione dell'isospin. La distribuzione degli  $u$  quark nel protone corrisponde a quella dei  $d$  del neutrone.

$$u^p(x) = d^n(x) \equiv u(x) \quad d^p(x) = u^n(x) \equiv d(x)$$

Tutte le distribuzioni di quark del mare sono identiche.

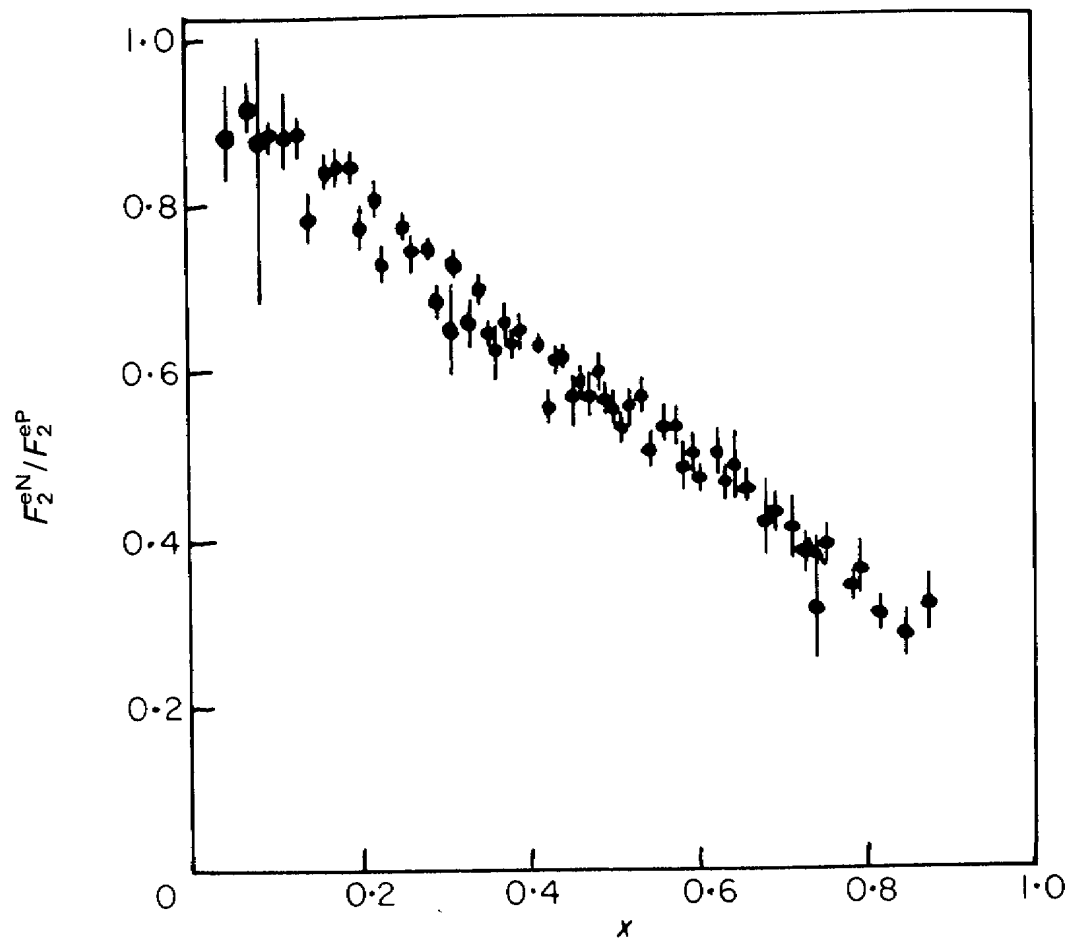
$$u_S^p(x) = \bar{u}_S^p(x) = d_S^p(x) = \bar{d}_S^p(x)$$

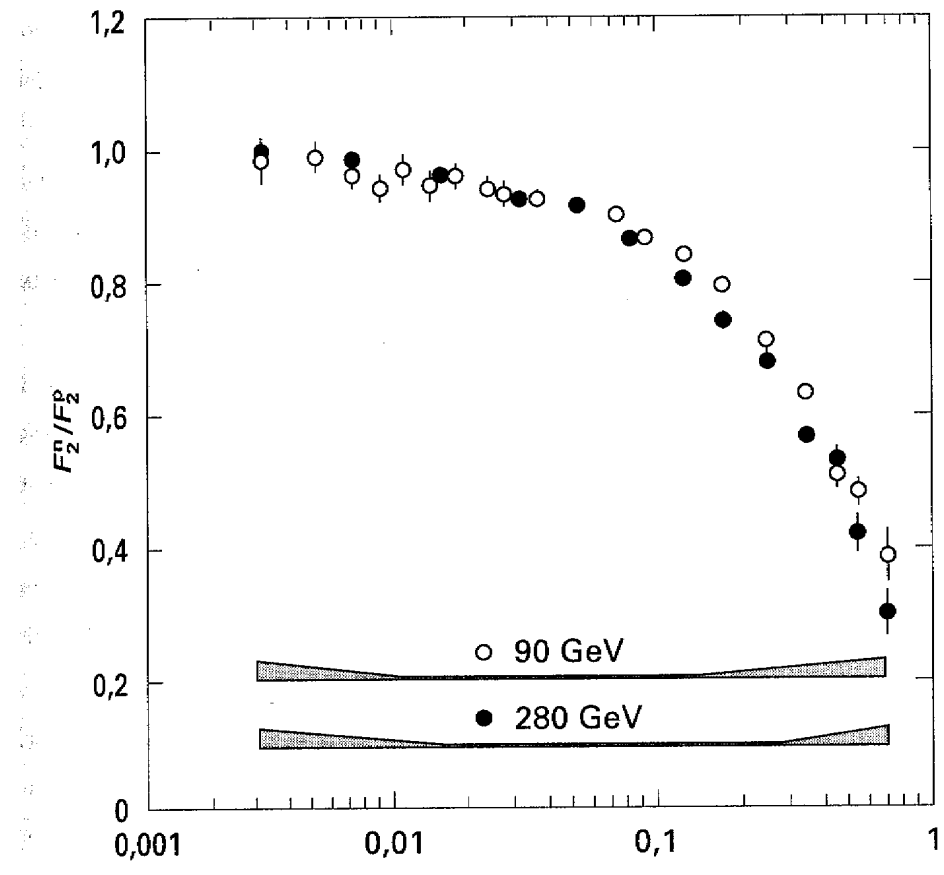
$$\frac{1}{x} F_2^p = \frac{1}{9} [4u_V(x) + d_V(x)] + S(x)$$

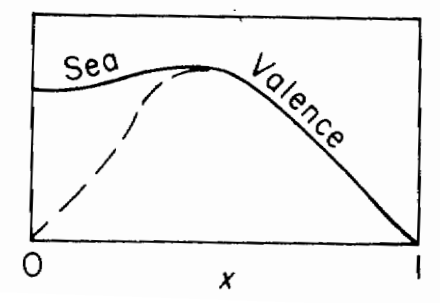
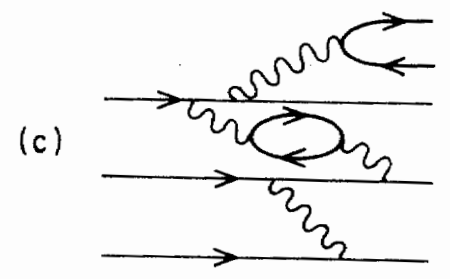
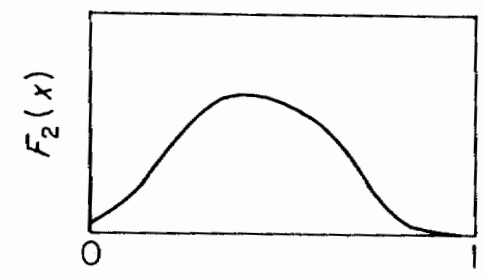
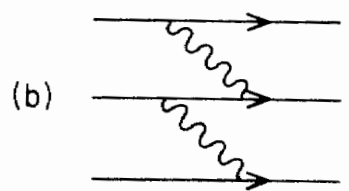
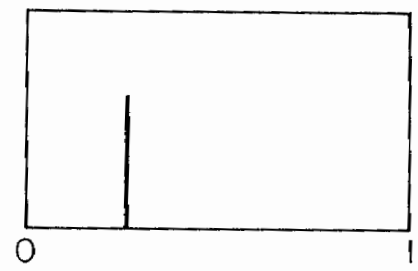
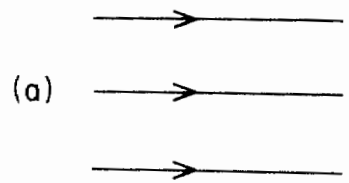
$$\frac{1}{x} F_2^n = \frac{1}{9} [4d_V(x) + u_V(x)] + S(x)$$

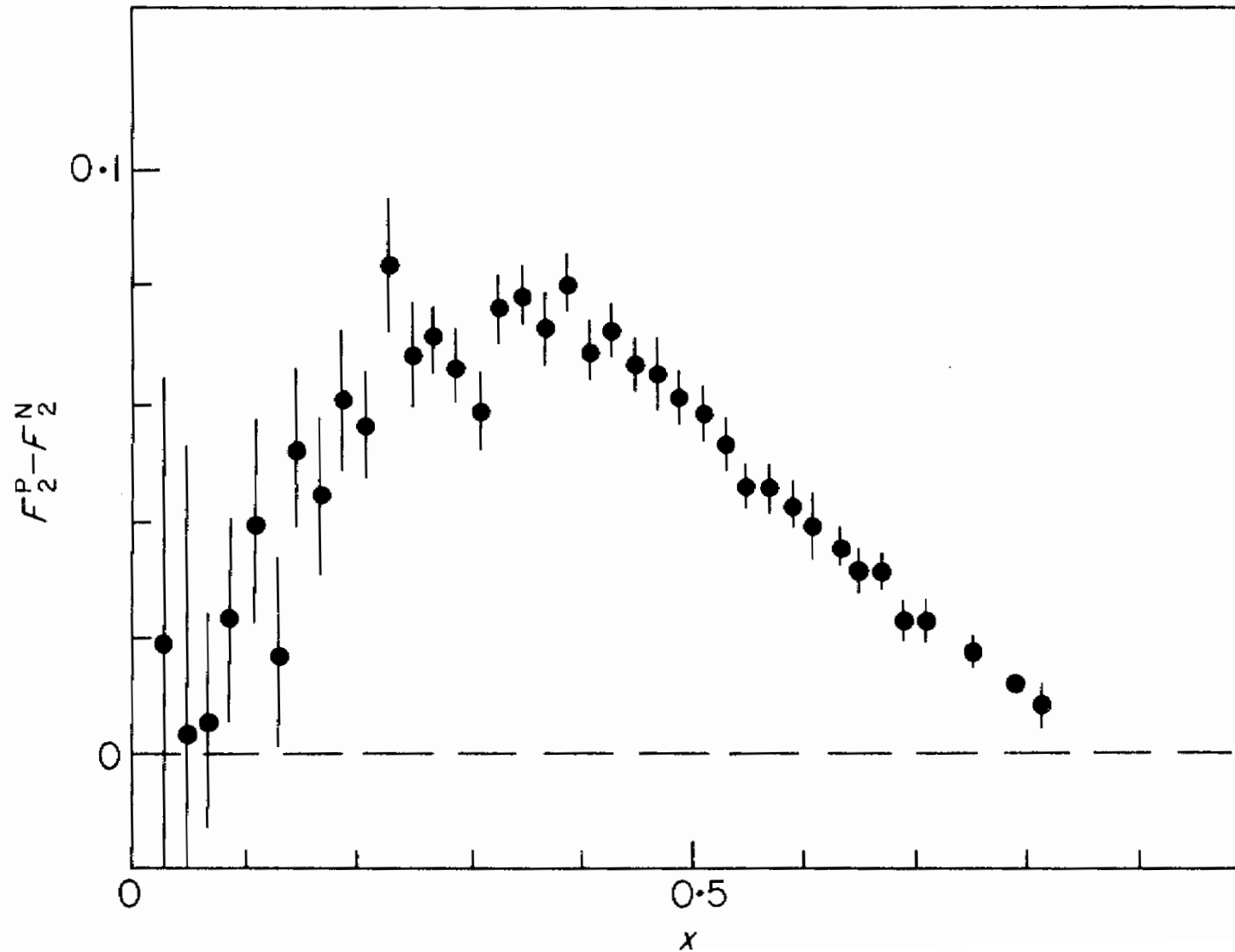
$$\lim_{x \rightarrow 0} \frac{F_2^n(x)}{F_2^p(x)} \rightarrow 1$$

$$\lim_{x \rightarrow 1} \frac{F_2^n(x)}{F_2^p(x)} \rightarrow \frac{4d_V(x) + u_V(x)}{4u_V(x) + d_V(x)}$$









Sottratto il contributo del mare

$$\frac{1}{x} [F_2^p(x) - F_2^n(x)] = \frac{1}{3} [u_V(x) - d_V(x)]$$

Per il nucleone

$$\begin{aligned} F_2^N(x) &= \frac{1}{2} (F_2^p(x) + F_2^n(x)) \\ &= \frac{x}{2} \left[ \left( \frac{4}{9} + \frac{1}{9} \right) (u_V^p(x) + u_S^p(x) + \bar{u}_S^p(x)) \right. \\ &\quad \left. + \left( \frac{4}{9} + \frac{1}{9} \right) (d_V^p(x) + d_S^p(x) + \bar{d}_S^p(x)) \right] \\ &= \frac{5}{18} x \sum_{j=u,d} q_j(x) \end{aligned}$$

Momento totale del protone

$$\int_0^1 dx x \sum_{j=u,d} q_j(x) z_j^2 = \frac{18}{5} \int_0^1 F_2^N(x) dx = 1$$

Sperimentalmente  $\simeq 0.5$

**Gluoni**



## Approfondimenti.

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Ian J. R. Aitchison & Anthony J. Hey

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# Domande

[N4-9] [P1-1] [P1-2] [P2-1] [P3-1] [P4-1] [P4-4] [P4-5] [P4-6]