

Lezione 5

**Isospin**

# Momenti Angolari in Meccanica Quantistica

$$[J_i, J_j] = i\hbar \sum_{k=1,2,3} \epsilon_{ijk} J_k$$

$$\begin{aligned} \epsilon_{ijk} &= 1 && \text{per } i = 1, j = 2, k = 3 \text{ o permutazione ciclica} \\ \epsilon_{ijk} &= -1 && \text{per } i = 1, j = 3, k = 2 \text{ o permutazione ciclica} \\ \epsilon_{ijk} &= 0 && \text{per due indici uguali} \end{aligned}$$

$$J^2 |jm\rangle = j(j+1)\hbar^2 |jm\rangle \quad J_z |jm\rangle = m\hbar |jm\rangle$$

$j$  assume valori interi o semi-interi

$$-j \leq m \leq j$$

# Somma momenti angolari

$$\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2$$

Autostati di  $J_{1,2}^2$  e  $J_{z,1,2}$

$$|j_1 m_1\rangle \quad e \quad |j_2 m_2\rangle$$

Autostato di  $J^2$  e  $J_z$

$$|j_1 j_2; jm\rangle = \sum_{m_1, m_2} \langle j_1 m_1 j_2 m_2 | jm\rangle |j_1 m_1\rangle |j_2 m_2\rangle$$

$\langle j_1 m_1 j_2 m_2 | jm\rangle$  coefficiente di Clebsch-Gordan.

$$J^2 |j_1 j_2; jm\rangle = j(j+1)\hbar^2 |j_1 j_2; jm\rangle \quad J_z |j_1 j_2; jm\rangle = m\hbar |j_1 j_2; jm\rangle$$

$$|j_1 - j_2| \leq j \leq j_1 + j_2 \quad ; \quad m = m_1 + m_2$$

## Due fermioni con spin 1/2

$$|j_1 m_1\rangle = \left| \frac{1}{2} s_1 \right\rangle \quad e \quad |j_2 m_2\rangle = \left| \frac{1}{2} s_2 \right\rangle$$

Caso  $J=0$  Singoletto

$$\begin{aligned} \left| \frac{11}{22}; 00 \right\rangle &= \left\langle \frac{111}{222} - \frac{1}{2} | 00 \right\rangle \left| \frac{11}{22} \right\rangle \left| \frac{1}{2} - \frac{1}{2} \right\rangle \\ &+ \left\langle \frac{1}{2} - \frac{111}{222} | 00 \right\rangle \left| \frac{1}{2} - \frac{1}{2} \right\rangle \left| \frac{11}{22} \right\rangle \\ &= \frac{1}{\sqrt{2}} \left( \left| \frac{11}{22} \right\rangle \left| \frac{1}{2} - \frac{1}{2} \right\rangle - \left| \frac{1}{2} - \frac{1}{2} \right\rangle \left| \frac{11}{22} \right\rangle \right) \end{aligned}$$

## Caso J=1 Tripletto

$$\begin{aligned}
 \left| \frac{11}{22}; 11 \right\rangle &= \left\langle \frac{1111}{2222} \middle| 11 \right\rangle \left| \frac{11}{22} \right\rangle \left| \frac{11}{22} \right\rangle = \left| \frac{11}{22} \right\rangle \left| \frac{11}{22} \right\rangle \\
 \left| \frac{11}{22}; 10 \right\rangle &= \left\langle \frac{111}{222} - \frac{1}{2} \middle| 10 \right\rangle \left| \frac{11}{22} \right\rangle \left| \frac{1}{2} - \frac{1}{2} \right\rangle \\
 &+ \left\langle \frac{1}{2} - \frac{111}{222} \middle| 10 \right\rangle \left| \frac{1}{2} - \frac{1}{2} \right\rangle \left| \frac{11}{22} \right\rangle \\
 &= \frac{1}{\sqrt{2}} \left( \left| \frac{11}{22} \right\rangle \left| \frac{1}{2} - \frac{1}{2} \right\rangle + \left| \frac{1}{2} - \frac{1}{2} \right\rangle \left| \frac{11}{22} \right\rangle \right) \\
 \left| \frac{11}{22}; 1 - 1 \right\rangle &= \left\langle \frac{1}{2} - \frac{11}{22} - \frac{1}{2} \middle| 1 - 1 \right\rangle \left| \frac{1}{2} - \frac{1}{2} \right\rangle \left| \frac{1}{2} - \frac{1}{2} \right\rangle \\
 &= \left| \frac{1}{2} - \frac{1}{2} \right\rangle \left| \frac{1}{2} - \frac{1}{2} \right\rangle
 \end{aligned}$$

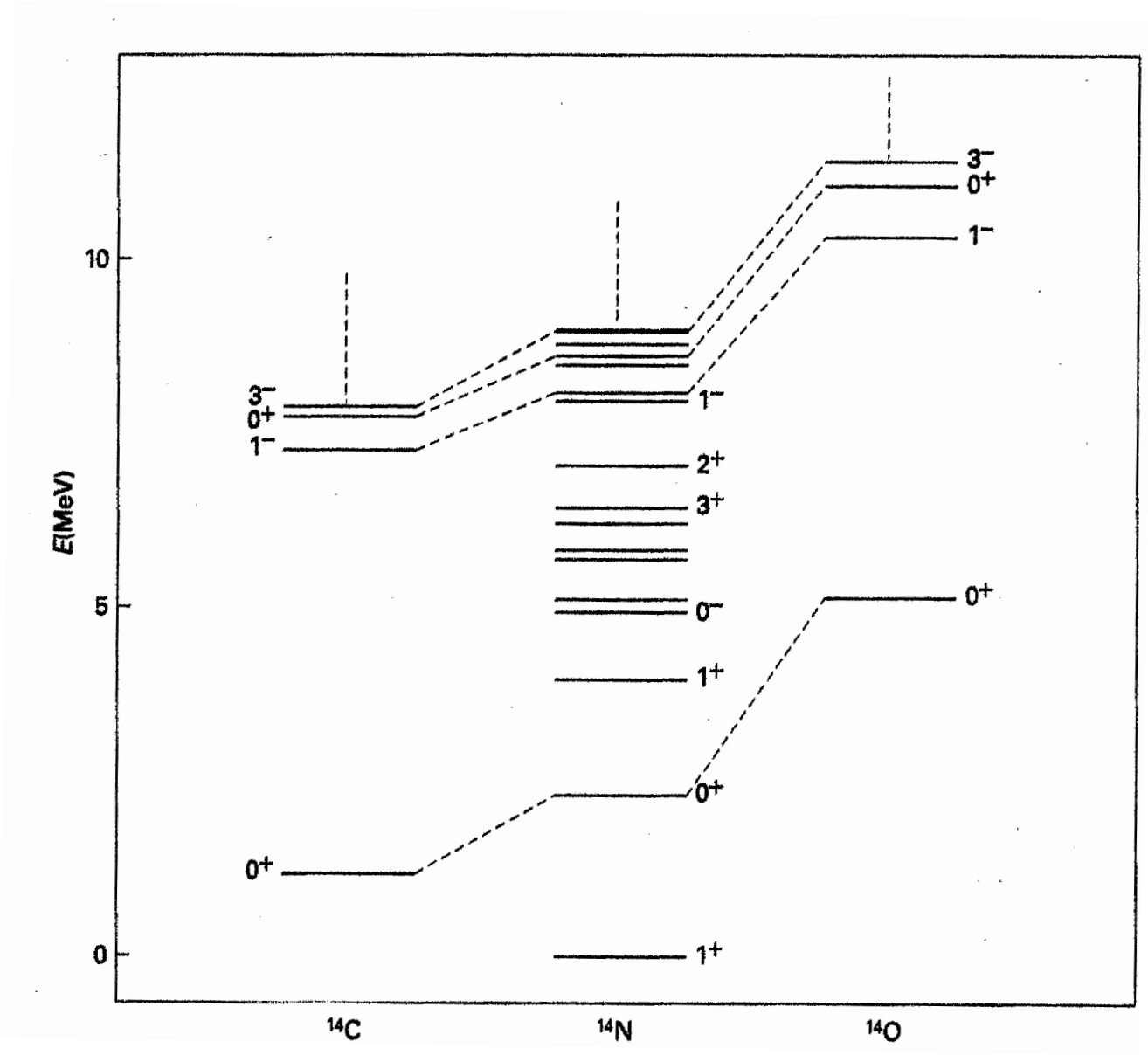
# Isospin

- $m_p \simeq m_n$
- Interazione p-p, n-n e p-n identica ogni volta che è possibile il confronto (senza Coulomb)

Protone e neutrone due casi della stessa particella.

Si assegna un numero quantico detto isospin, o spin isotopico, che si comporta come un momento angolare, e nella sua terza proiezione assume due valori

$$\frac{1}{2} \text{ protone} \quad -\frac{1}{2} \text{ neutrone}$$



Spettri di eccitazione dei nuclei  $^{14}\text{C}$ ,  $^{14}\text{N}$  e  $^{14}\text{O}$ .

# Domande

[N4-1]