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It has become clear in recent years that electroweak radiative corrections at the Teraelettronvolt (TeV) scale, which is the energy scale relevant for present or near future accelerators, are much bigger than one could naïvely expect [1]. In fact, while at LEP energies ($\sim 100 \text{ GeV}$) electroweak corrections are parametrized by the weak coupling constant, $\frac{\alpha_W}{\pi} \lesssim 1\%$, a steady growth with energy makes them reach the $20 \div 30\%$ level at the TeV scale. At higher energies, electroweak corrections keep on growing and become as big as the tree level values for the cross sections, thus making a perturbative treatment problematic. As one tries to understand the asymptotic behavior, i.e. how the cross sections depend on the typical energy of the process when the energy itself becomes much bigger than all particle's masses, two striking features emerge:

- the behavior of cross sections for energies much higher than the weak scale M ~ 100 GeV is related to the *infrared*, rather than the ultraviolet, structure of the Standard Model; M plays the role of infrared cutoff [2].
- No "infrared safe" observable exists: even at the highest energies, all observables depend crucially on the low energy infrared cutoff M.

In order to clarify these two points, I will examine a simple example. Let us a consider a scattering process in which the center of mass energy E is much bigger than any scale of the theory: particle's masses, symmetry breaking scale and so on. At first sight, the asymptotic behavior of the cross section $\sigma(E)$ should depend on how the theory behaves for very high energies, that is on its ultraviolet properties, encoded into the Renormalization Group Equations (RGEs). In many cases, this amounts to the following recipe: calculate the tree level value for $\sigma = \sigma(\alpha_i, ...)$, then substitute the "fixed coupling constants" α_i with the "running coupling constants $\alpha_i(\mu)$ evaluated at a scale $\mu = E$. However, by doing this and comparing with, say a numerical calculation, one obtains a completely wrong result! What went wrong?

The crucial point is that all cross sections have an unavoidable dependence also on the weak symmetry breaking scale M, and not only on the process scale E. It has been clarified that this dependence is related to the infrared properties of the theory, and that M acts as an infrared cutoff: all quantities would be divergent in the limit $M \rightarrow 0$. Moreover, while RGEs equations produce asymptotic behaviors growing like single logs, the infrared behavior generates double logs: at one loop the correction to the cross section is proportional to $\alpha_w \log^2 \frac{E}{M}$. So, infrared originated weak corrections dominate the ultraviolet behavior.

To my knowledge, this is the only case in which studying RGEs is of little use in order to understand asymptotic behaviors. In fact, due to very general theorems, in the case of QED and QCD the dependence on the infrared cutoff is usually canceled between different contributions and one can measure "infrared safe" observables. This leads me to the second point sketched previously.

For very high energies, besides photons (γ) and gluons (g), also weak gauge bosons (W, Z) are radiated copiously when two initial particles undergo a hard scattering into a given final state. To be definite, let us consider $e^+e^- \rightarrow 2$ jets +X, a process relevant for future e^+e^- linear colliders. Here the experimental trigger is defined to be 2 "hard" jets (i.e. two hadronic jets forming a large invariant mass), while any type of radiation (γ, g, W, Z) is included in "X". The hope is that, by defining such an inclusive quantity, there is a better theoretical control over the perturbative predictions: this is what happens in QED and QCD, where a cancellation between virtual corrections and real γ, g emissions so that the final result is independent of the infrared cutoff and thus free of large logarithms. However, no such a cancellation happens when considering weak bosons emission, and the in the fully inclusive cross sections cross large double $\log \sim \alpha_w \log^2 \frac{E}{M}$ are still present. The theoretical reason for this is now well understood [3] and is related to the weak sector of the Standard Model being simultaneously non abelian and (spontaneously) broken. This noncancellation, baptised "Bloch-Nordsieck violation", is generically unavoidable since it is present whenever two or more weak isospin charges (belonging to the initial and/or final states) are detected experimentally. Since initial states in an accelerator (protons, electrons,..) do carry weak isospin charges, uncanceled double logs at high energies are truly ubiquitous.

The theoretical behavior of weak corrections at very high energies is by now well understood: techniques partly mutuated from QCD allow to resum these contributions at all orders, allowing a good theoretical control over the asymptotic behavior of cross sections. Then, a very relevant question arises: are these corrections phenomenologically relevant? The answer is not obvious, since next generation of hadronic (LHC) and electron-positron (ILC) colliders will probe the TeV scale, which is not much bigger than the weak scale of about 100 GeV. Is the double logarithmic enhancement enough to produce relevant effects? The answer is yes: the "effective coupling constant" $\alpha_w \log^2 \frac{E}{M}$ is of the order of 10 % at the TeV scale, and one loop weak corrections can reach the 50 % (!) level [4]. At a linear e^+e^- collider, with its clean experimental environment allowing for precision measurements, it is also clear that the one loop approximation is not enough: one needs to calculate also higher orders and/or all-order resummed corrections. The outcome is not so clearcut in an hadronic collider like the LHC, where strong interactions provide a dominant background and a lower experimental precision. Let me discuss this issue by taking a particular example, that we studied in [6].

The scattering of longitudinally polarized gauge bosons, $V_L V_L \rightarrow V_L V_L$ where V = W, Z, probes the physics responsible for the symmetry breaking mechanism. In fact the longitudinal degrees of freedom, that the gauge bosons acquire when the electroweak $SU(2) \otimes U(1)$ symmetry breaks, interact with each other through that very same sector of the Lagrangian (the so called "Higgs sector") which is also responsible for symmetry breaking¹. It turns out that even at the highest energies radiative corrections depend crucially on the low energy infrared cutoff M, featuring the infamous double logs. This holds true even for a fully inclusive quantity, i.e. a quantity where W, Z radiation is included in the final state. These kind of corrections can be resummed at all orders and a give well behaved result (see fig. 1, continuous line). In this figure, the dependence of the radiative correction to the cross section is drawn as a function of the center of mass energy $E = \sqrt{s}$. For a comparison, the "exclusive" case, where W, Z radiation is forbidden, is also depicted in fig. 1 (dashed line).

An important comment is in order. Whether gauge bosons emission is measured or not strongly depends on the experimental setup. This means that, depending on what are the "experimental cutoffs" on the final state, i.e. what kind of particles are detected and with what properties (mass, momenta and so on), the theoretical prediction can be anywhere between the dashed and continuous lines in fig. 1. A more detailed study on this subject is therefore needed; to this we will soon dedicate our efforts in order to get theoretical predictions nearer to actual exeriments.



Figure 1. Radiative electroweak cross sections relative to the Born value, in the case of W^+Z scattering. Dashed green line: additional gauge bosons are excluded from the observable definition. Continuous blue line: additional gauge boson radiation is included.

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¹a formal proof of this statement is given by the so called "equivalence theorem" [5]