Three-body force in nuclear many-body systems

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Microscopic nuclear structure calculations require the presence of three-body forces in addition to the nucleon-nucleon interaction built to reproduce the properties of two-body systems [1]. The simplest three-body forces given in the literature contain a term which involves the exchange of two-pions with an intermediate excitation of a virtual Δ. This term, represented by the diagram A in Fig. 1, is attractive, therefore, in addition to this term, a new phenomenological repulsive term is added. This term is represented by the diagram B of Fig. 1, and its parameters are chosen to reproduce the binding energy of the ^{3}H. Since the use of only two-nucleon interactions underbinds the triton, the three-nucleon interaction produces a globally *attractive* contribution.

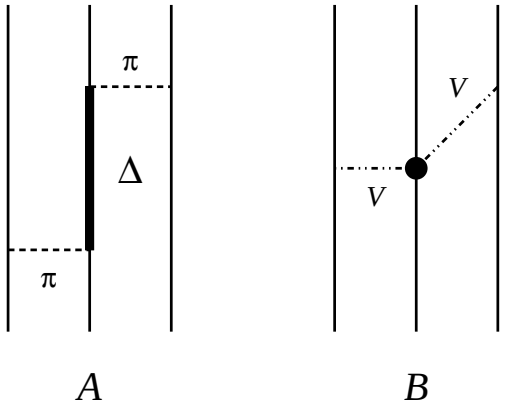

Figure 1. Diagrams representing the two terms which compose the three-body force.

The microscopic two- plus three-body interactions have been used with success to calculate binding energies of light nuclei up to ^{12}C [2]. The contribution of the three-body term is *attractive* in all the cases considered.

These interactions constructed to reproduce the properties of the two and three-body systems have been also used to calculate the binding energy per nucleus of nuclear matter as a function of the density of the systems [3]. In these calculations the Schrödinger equations is solved by making some approximations. In these calculations the Correlated Basis Function (CBF) theory has been used. The theory is based on the Ritz's variational principle, and the energy functional is defined as

$$\delta[E(\Phi)] = \delta\left[\frac{<\Phi|F^\dagger H F|\Phi>}{<\Phi|F^\dagger F|\Phi>}\right] = 0 \ , \qquad (1)$$

where $|\Phi>$ is a Slater determinant, H is the nuclear hamiltonian containing kinetic energy operator, two- and three-body interactions, and the many-body correlation function F is defined as product of two-body state-dependent correlation functions

$$F(1,2,.....,A) = \mathcal{S}\left(\prod_{i<j}^{A} F(i,j)\right) \ , \qquad (2)$$

where $\mathcal{S}$ is the symmetry operator,

$$F(i,j) = \sum_{p=1}^{6} f_p(r_{ij}) O_{ij}^p \ , \qquad (3)$$

and O_{ij}^p corresponds to the first six channels in the two-body interaction.

In nuclear matter calculations the Slater determinant is constructed by using plane wave single particle wave functions, therefore only the correlation function contains variational parameters. The many-body integral of Eq. (1) is calculated by making an expansion based on the number of the two-body correlation functions, f_p. From a topological analysis it turns out that all the diagrams of a certain type can be summed in a closed form by solving a set of integral equations. This method is called Fermi Hypernetted Chain (FHNC) resummation techniques.

In nuclear matter calculations the contribution of the three-body interaction is *repulsive* [3]. At present there is no explanation of this discrepancy with the few-body nuclei results. The explanation commonly advocated claims that, since in the few-body systems the density is relatively lower than that of nuclear matter, the attractive term dominates. The situation is reversed

in nuclear matter. If this explanation is correct, it should be possible, by analizing the binding energy of various nuclei as a function of the nucleon number, to identify the change of sign of the three-body force.

In these years we have extended the CBF-FHNC theory to describe finite nuclear sustems [4], and we have reached an accuracy comparable to that obtained by nuclear matter calculations. We have calculated the binding energies of various doubly magic nuclei. Our calculations have been done with the Argonne V8' two-body interaction plus the Urbana IX (UIX) three-body interaction.

	^{16}O	^{40}Ca	^{48}Ca	^{208}Pb
E_2	-5.66	-6.83	-6.24	-5.80
$E_{2,3}$	-4.80	-5.05	-4.62	-3.78

Table 1
Binding energies per nucleon in MeV.

In Tab. 1 we show the binding energies per nucleon for some doubly-magic nuclei all over the periodic table [4]. In the upper line of the table we show the energies obtained by considering only the two-body nucleon interaction. In the second row of the table we show the results obtained by considering also the three-body force. We observe that the contribution of this interaction is always repulsive, therefore we do not find the claimed change of sign as a function of the nucleon number.

This change of sign could be present in nuclei lighter than ^{16}O, therefore we have calculated the ^{4}He binding energy. In this nucleus we can directly compare our results with those obtained in Variational Monte Carlo approach (VMC). In this approach the many-body integrals of Eq. (2) are calculated with Monte Carlo techniques, a method more accurate than the FHNC expansion. At present, the Monte Carlo techniques are computationally too expansive to be used in nuclei heavier than those with A=12 [2].

	FHNC	VMC
E_2	-23.78	-23.72
$E_{2,3}$	-20.49	-27.78

Table 2
Binding energies per nucleon of ^{4}He expressed in MeV. E_2 indicates the results obtained with the two-body interaction only, E_3 those where also the tree-body interactions have been used.

The FHNC and VMC results are compared in Tab. 2. The agreement between the energies obtained with the two-body interaction only is excellent. Clearly the inclusion of the tree-body interaction produces wrong results in the FHNC calculations. The work is still in progress to understand which approximations done in the FHNC calculations are responsible for the disagreement. We think that the difference of behavior is caused by the number of state-dependent correlation functions included in the calculations. In the FHNC ones, the diagrams calculated include only two state-dependent correlation functions [3,4] respect to the six ones that may appear in the external points. This makes that it necessary to calculate several hundreds of spin and isospin traces.

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