$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 \qquad H_0|\phi_k\rangle = E|\phi_k\rangle \qquad E = \frac{\hbar^2 k^2}{2m}$$

$$\langle r|\phi_k\rangle = e^{i\vec{k}\cdot\vec{r}}$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r}) \qquad H|\psi^{+}_{k_a}\rangle = E|\psi^{+}_{k_a}\rangle$$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + \tilde{U}\right]\tilde{\psi}_b^{(-)} = E\tilde{\psi}_b^{(-)} \qquad \psi_b^{(-)} \xrightarrow[r\to\infty]{} e^{i\vec{k}_b\cdot\vec{r}} + \tilde{f}_b^{(-)}(\Omega)\frac{e^{-ikr}}{r}$$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + U\right]\psi_a^{(+)} = E\psi_a^{(+)} \qquad \psi_a^{(+)} \xrightarrow[r\to\infty]{} e^{i\vec{k}_a\cdot\vec{r}} + \tilde{f}_a^{(+)}(\Omega)\frac{e^{ikr}}{r}$$

Moltiplicando a sinistra la prima eq. per $\psi_a^{*(+)}$ e $\psi_b^{(-)}$ il complesso coniugato della seconda si può dimostrare che:

$$\langle\tilde{\psi}_b^{(-)}|U-\tilde{U}|\psi_a^{(+)}\rangle = -\frac{2\pi\hbar^2}{m}\left[f_a^{(+)}(\Omega_b) - \tilde{f}_b^{(-)*}(-\Omega_a)\right]$$

A) $U=V \quad \tilde{U}=0$ $\qquad \langle\phi_b|V|\psi_a^{+}\rangle = -\frac{2\pi\hbar^2}{m} f_a^{(+)}(\Omega_b)$

B) $U=\tilde{U}=V$ $\qquad f_a^{(+)}(\Omega_b) = f_b^{(-)*}(-\Omega_a)$

C) $U=0 \quad \tilde{U}=V$ $\qquad \langle\psi_b^{(-)}|V|\phi_a\rangle = -\frac{2\pi\hbar^2}{m} f_b^{(-)*}(-\Omega_a)$

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