$$|\vec{r}-\vec{r}'| = \left[r^2 + r'^2 - 2\vec{r}\cdot\vec{r}'\right]^{1/2} = r\left[1 - 2\frac{\vec{r}}{r^2}\cdot\vec{r}' + \frac{\vec{r}'^2}{r^2}\right]^{1/2} \xrightarrow[r \gg r']{}$$

$\rightarrow$ expansion $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$

$$\rightarrow r\left[1 + \frac{1}{2}\left(-2\frac{\vec{r}}{r^2}\right)\cdot\vec{r}' + \dots\right] = r - \frac{\vec{r}}{r}\cdot\vec{r}' + O\left(\frac{\vec{r}'}{r}\right)$$

$$\frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} = \frac{e^{ik\left(r - \frac{\vec{r}}{r}\cdot\vec{r}'\right)}}{r} + O\left(\frac{\vec{r}'}{r}\right)$$

nel numeratore il termine $\frac{\vec{r}}{r}\cdot\vec{r}' < r$ è necessario perché produce oscillazioni non trascurabili rispetto a quelle di r. Nel denominatore $\frac{\vec{r}}{r}\cdot\vec{r}' \ll r$.

$$\psi(\vec{r}) - e^{i\vec{k}\cdot\vec{r}} = \int -\frac{m}{2\pi\hbar^2}\frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}V(\vec{r}')\psi(\vec{r}')d^3r' \quad |\vec{r}| \gg |\vec{r}'|$$

$$\rightarrow -\frac{m}{2\pi\hbar^2}\int \frac{e^{ikr}}{r}e^{-ik\frac{\vec{r}}{r}\cdot\vec{r}'}V(\vec{r}')\psi(\vec{r}')d^3r'$$

$$= \frac{e^{ikr}}{r}\underbrace{\left[-\frac{m}{2\pi\hbar^2}\int e^{-i\vec{k}\cdot\vec{r}'}V(\vec{r}')\psi(\vec{r}')d^3r'\right]}_{f(\Omega)} \qquad \vec{k} = k\frac{\vec{r}}{r}$$

$$\frac{e^{ikr}}{r} \qquad f(\Omega)$$