Calcoliamo 37) nel caso $\hat{r} // z$ e $M=1$ $(\vartheta = 0)$

$$S_{12}\,|01,11\rangle = \left[3\left(\sigma_z(1)\sigma_z(2)\right) - \left(\vec{\sigma}(1)\cdot\vec{\sigma}(2)\right)\right] \chi^{1/2}_{1/2}(1)\,\chi^{1/2}_{1/2}(2)\,Y_{00}(\hat{r}) =$$

visto in eq. 30 a)

$$= \frac{2}{\sqrt{4\pi}}\,\chi^{1/2}_{1/2}(1)\,\chi^{1/2}_{1/2}(2) = \frac{2}{\sqrt{4\pi}}\,\chi^1_1 \qquad 41$$

$$b\,|21,11\rangle = b\langle 2\,0\,1\,1\,|\,1\,1\rangle\, Y_{20}(\hat{r})\,\chi^{1/2}_{1/2}(1)\,\chi^{1/2}_{1/2}(2)$$

$$= b\left(\frac{1}{10}\right)^{1/2}\sqrt{\frac{2L+1}{4\pi}}\,\chi^1_1 = b\sqrt{\frac{1}{2}}\sqrt{\frac{1}{4\pi}}\,\chi^1_1 \qquad 42$$

uguagliando 41) e 42) si ha:

$$\frac{2}{\sqrt{4\pi}} = b\,\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{4\pi}} \qquad \text{quindi} \quad \underline{b=\sqrt{8}}\,. \qquad 43$$

Adesso calcolo:

$$S_{12}\,|21;1M\rangle = b'\,|01,1M\rangle + c\,|21;1M\rangle \qquad 44$$

Posso scrivere le eq. 44 e 45 in forma matriciale

$$S_{12}\begin{pmatrix}|01\rangle\\|21\rangle\end{pmatrix} = \begin{pmatrix}a & b\\ b' & c\end{pmatrix}\begin{pmatrix}|01\rangle\\|21\rangle\end{pmatrix} = \begin{matrix}a|01\rangle + b|21\rangle\\ b'|01\rangle + c\,|21\rangle\end{matrix}$$

S_{12} e' hermitiano quindi deve essere

$$S_{12}^{\dagger} = S_{12} \longrightarrow S_{12}^{\dagger} = \left(S_{12}^{T}\right)^{*} = S_{12}^{T} \qquad \text{cioe'}$$

$$\begin{pmatrix}a & b\\ b' & c\end{pmatrix} = \begin{pmatrix}a & b'\\ b & c\end{pmatrix} \quad \text{da cui} \quad \underline{b'=b} \qquad 45$$

La 45 puo' anche essere ridefinita dicendo che:

$$\langle 01|\,S_{12}\,|21\rangle = \langle 21|\,S_{12}\,|01\rangle$$