$$\left(\mathcal{Y}^{M}_{011}\middle|\mathcal{Y}_{20}\middle|\mathcal{Y}^{M}_{211}\right) = \langle 00\,11|11\rangle\,\langle 20\,11|11\rangle\,\langle 00|\mathcal{Y}_{20}|20\rangle \qquad 57$$

$$\langle 00\,11|11\rangle = 1 \qquad \langle 20\,11|11\rangle = \left(\frac{1}{10}\right)^{1/2} \qquad 58$$

↳ questo e' 0 per $\delta_{MM'}$ in 55)

$$\langle 00|\mathcal{Y}_{20}|20\rangle = \frac{[0][2][2]}{\sqrt{4\pi}}\begin{pmatrix}0&2&2\\0&0&0\end{pmatrix}\begin{pmatrix}0&2&2\\0&0&0\end{pmatrix} = \frac{1}{\sqrt{4\pi}} \qquad 59$$

$$\begin{pmatrix}0&2&2\\0&0&0\end{pmatrix} = \frac{1}{[2]} \qquad \left(\mathcal{Y}^{M}_{011}\middle|\mathcal{Y}_{20}\middle|\mathcal{Y}^{M}_{211}\right) = \left(\frac{1}{10}\right)^{1/2}\frac{1}{\sqrt{4\pi}}$$

$$\left(\mathcal{Y}^{M}_{211}\middle|\mathcal{Y}_{20}\middle|\mathcal{Y}^{M}_{211}\right) = \langle 20\,11|11\rangle^2\,\langle 20|\mathcal{Y}_{20}|20\rangle +$$

$$+\langle 21\,10|11\rangle^2\,\langle 21|\mathcal{Y}_{20}|21\rangle + \langle 22\,1\,{-1}|11\rangle^2\,\langle 22|\mathcal{Y}_{20}|22\rangle \qquad 60$$

$$\langle 20\,11|11\rangle = \sqrt{\frac{1}{10}} \qquad \langle 21\,10|11\rangle = -\sqrt{\frac{3}{10}} \qquad \langle 22\,1\,{-1}|11\rangle = +\sqrt{\frac{3}{5}} \qquad 61$$

$$\langle 20|\mathcal{Y}_{20}|20\rangle = \frac{[2][2][2]}{\sqrt{4\pi}}\begin{pmatrix}2&2&2\\0&0&0\end{pmatrix}\begin{pmatrix}2&2&2\\0&0&0\end{pmatrix} \qquad 62a$$

$$\langle 21|\mathcal{Y}_{20}|21\rangle = \frac{[2][2][2]}{\sqrt{4\pi}}\begin{pmatrix}2&2&2\\0&0&0\end{pmatrix}\begin{pmatrix}2&2&2\\-1&0&1\end{pmatrix}(-1) \qquad 62b$$

$$\langle 22|\mathcal{Y}_{20}|22\rangle = \frac{[2][2][2]}{\sqrt{4\pi}}\begin{pmatrix}2&2&2\\0&0&0\end{pmatrix}\begin{pmatrix}2&2&2\\-2&0&2\end{pmatrix} \qquad 63b$$

• Esercizio •

Sapendo che

$$\begin{pmatrix}J&J&2\\M&-M&0\end{pmatrix} = (-)^{J-M}\frac{2\left[3M^2 - J(J+1)\right]}{\left[(2J+3)(2J+2)(2J+1)2J(2J-1)\right]^{1/2}} \qquad 64$$

Trovare il risultato di eq. 65)