- Alcune osservazioni su come si comporta il tensore:

Ricordo le matrici di Pauli:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{18a}$$

$$\sigma_+ = \tfrac{1}{2}(\sigma_x + i\sigma_y) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \sigma_x = \sigma_+ + \sigma_- \tag{18b}$$

$$\sigma_- = \tfrac{1}{2}(\sigma_x - i\sigma_y) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = i(\sigma_- - \sigma_+) \tag{18c}$$

$$\chi^{1/2}_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi^{1/2}_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad |1/2, +1/2\rangle \quad |1/2, -1/2\rangle \tag{19a}$$

$$\sigma_+ \chi^{1/2}_{1/2} = 0 \qquad \sigma_+ \chi^{1/2}_{-1/2} = \chi^{1/2}_{1/2} \tag{19b}$$

$$\sigma_- \chi^{1/2}_{1/2} = \chi^{1/2}_{-1/2} \qquad \sigma_- \chi^{1/2}_{-1/2} = 0 \tag{19c}$$

La funzione d'onda che descrive l'accoppiamento dei due spin è

$$\chi^1_\Sigma = \sum_{s_1, s_2} \langle \tfrac{1}{2} s_1 \tfrac{1}{2} s_2 | 1\Sigma \rangle \, \chi^{1/2}_{s_1}(1) \, \chi^{1/2}_{s_2}(2) \equiv |1, \Sigma\rangle \tag{20}$$

nel caso di spin 1, e

$$\chi^0_0 = \sum_{s_1, s_2} \langle \tfrac{1}{2} s_1 \tfrac{1}{2} s_2 | 00 \rangle \, \chi^{1/2}_{s_1}(1) \, \chi^{1/2}_{s_2}(2) \equiv |0, 0\rangle \tag{21}$$

nel caso di spin 0.

- Esercizio:

Verificare che le equazioni 20 e 21 corrispondono a:

$$\chi^0_0 = \frac{1}{\sqrt{2}} \left(\chi^{1/2}_{-1/2}(1) \chi^{1/2}_{1/2}(2) - \chi^{1/2}_{1/2}(1) \chi^{1/2}_{-1/2}(2) \right) \tag{22a}$$

$$\chi^1_1 = \chi^{1/2}_{1/2}(1) \chi^{1/2}_{1/2}(2) \qquad \chi^1_{-1} = \chi^{1/2}_{-1/2}(1) \chi^{1/2}_{-1/2}(2) \tag{22b}$$

$$\chi^1_0 = \frac{1}{\sqrt{2}} \left(\chi^{1/2}_{-1/2}(1) \chi^{1/2}_{1/2}(2) + \chi^{1/2}_{1/2}(2) \chi^{1/2}_{-1/2}(1) \right) \tag{22c}$$