DE

Per esemplificare applico il tensore allo stato di singoletto.

$$\langle 00 | S_{12} | 00 \rangle = 3 \langle 00 | (\vec{\sigma}(1) \cdot \hat{r})(\vec{\sigma}(2) \cdot \hat{r}) | 00 \rangle - \langle 00 | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | 00 \rangle \qquad 23)$$

Definendo $z \parallel \hat{r}$ il primo termine risulta

$$\langle 00 | \sigma_z(1) \sigma_z(2) | 00 \rangle =$$

$$= \frac{1}{2} \left(\chi_{-\frac{1}{2}}(1) \chi_{\frac{1}{2}}(2) - \chi_{\frac{1}{2}}(1) \chi_{-\frac{1}{2}}(2) \,\middle|\, \sigma_z(1) \sigma_z(2) \,\middle|\, \chi_{-\frac{1}{2}}(1) \chi_{\frac{1}{2}}(2) - \chi_{\frac{1}{2}}(1) \chi_{-\frac{1}{2}}(2) \right) =$$

$$= \frac{1}{2} \left[-\left(\chi_{-\frac{1}{2}}(1) \chi_{\frac{1}{2}}(2) \,\middle|\, \chi_{-\frac{1}{2}}(1) \chi_{\frac{1}{2}}(2) \right) - \left(\chi_{\frac{1}{2}}(1) \chi_{-\frac{1}{2}}(2) \,\middle|\, \chi_{\frac{1}{2}}(1) \chi_{-\frac{1}{2}}(2) \right) \right] = -1 \qquad 24)$$

Dato che: $S = \vec{\sigma}_1 + \vec{\sigma}_2$ [illegible]

$$\vec{\sigma}(1) \cdot \vec{\sigma}(2) = 2 \left(\sigma_+(1) \sigma_-(2) + \sigma_-(1) \sigma_+(2) \right) + \sigma_z(1) \sigma_z(2) \qquad 25)$$

• Esercizio •

Dimostrare la 25).

$$\sigma_x(1) \sigma_x(2) + \sigma_y(1) \sigma_y(2) + \sigma_z(1) \sigma_z(2) =$$

$$= \left(\sigma_+(1) + \sigma_-(1) \right) \left(\sigma_+(2) + \sigma_-(2) \right) + i \left(\sigma_-(1) - \sigma_+(1) \right) i \left(\sigma_-(2) - \sigma_+(2) \right) + \sigma_z(1) \sigma_z(2) =$$

$$= \left(\sigma_+(1) \sigma_+(2) + \sigma_+(1) \sigma_-(2) + \sigma_-(1) \sigma_+(2) + \sigma_-(1) \sigma_-(2) \right)$$

$$- \left(\sigma_-(1) \sigma_-(2) - \sigma_-(1) \sigma_+(2) - \sigma_+(1) \sigma_-(2) + \sigma_+(1) \sigma_+(2) \right) + \sigma_z(1) \sigma_z(2) =$$

$$= 2 \left(\sigma_+(1) \sigma_-(2) + \sigma_-(1) \sigma_+(2) \right) + \sigma_z(1) \sigma_z(2) .$$

Da cui:

$$\langle 00 | \vec{\sigma}(1) \cdot \vec{\sigma}(2) | 00 \rangle = 2 \langle 00 | \sigma_+(1) \sigma_-(2) | 00 \rangle$$
$$+ 2 \langle 00 | \sigma_-(1) \sigma_+(2) | 00 \rangle$$
$$+ \langle 00 | \sigma_z(1) \sigma_z(2) | 00 \rangle \qquad 26)$$