$$\{\alpha_k, \alpha_{-k}\} = \alpha_k \alpha_{-k} + \alpha_{-k}\alpha_k$$

Uso la prima delle eq. 2 con $k \to -k$

$$= \left(u_k a_k - v_k a^{+}_{-k}\right)\left(u_{-k} a_{-k} - v_{-k} a^{+}_{k}\right) + \left(u_{-k} a_{-k} - v_{-k} a^{+}_{k}\right)\left(u_k a_k - v_k a^{+}_{-k}\right)$$

$$= u_k u_{-k} a_k a_{-k} - u_k v_{-k} a_k a^{+}_k - v_k u_{-k} a^{+}_{-k} a_{-k} + v_k v_{-k} a^{+}_{-k} a^{+}_k$$

$$+ \left[u_k u_{-k} a_{-k} a_k - v_{-k} u_{+k} a^{+}_k a_k - u_{-k} v_{+k} a_{-k} a^{+}_{-k} + v_k v_{-k} a^{+}_k a^{+}_{-k}\right]$$

$$= u_k u_{-k} \underbrace{\{a_k, a_{-k}\}}_{0} - u_k v_{-k} \underbrace{\{a_k, a^{+}_k\}}_{1} - u_{-k} v_{+k} \underbrace{\{a^{+}_{-k}, a_{-k}\}}_{1} + v_k v_{-k} \underbrace{\{a^{+}_{-k}, a^{+}_k\}}_{0}$$

$$= -\left(u_k v_{-k} + v_k u_{-k}\right) \overset{!}{=} 0 \quad \text{solo se} \quad \frac{u_k}{v_k} = -\frac{u_{-k}}{v_{-k}} \qquad 5)$$

Si sceglie $v_{-k} = -v_k \quad u_{-k} = u_k$ (Belyaev)

da cui le definizioni nelle seconde colonne della 2)

Quindi si ha che:

$$\{\alpha_k, \alpha^{+}_{k'}\} = \delta_{kk'} \qquad \{\alpha_k, \alpha_{-k}\} = \{\alpha^{+}_k, \alpha^{+}_{-k}\} = 0 \qquad 6)$$