

$$\begin{aligned}
 [H, a_n^\dagger a_j] &= \sum_{\alpha \beta} h_{\alpha \beta} (\delta_{n\beta} a_\alpha^\dagger a_j - \delta_{j\alpha} a_n^\dagger a_\beta) \\
 &+ \frac{1}{2} \sum_{ij} V_{ijij} [a_n^\dagger a_j - a_n^\dagger a_j] \rightarrow 0 \\
 &+ \frac{1}{4} \sum_{\alpha \beta \alpha' \beta'} \bar{V}_{\alpha \beta \alpha' \beta'} [N[a_\alpha^\dagger a_\alpha a_\beta^\dagger a_\beta], a_n^\dagger a_j]
 \end{aligned}
 \quad \begin{array}{l} \text{E' una costante che} \\ \text{si somma sia} \\ \text{a } E_0 \text{ che a } E_0 \end{array}
 \quad (1)$$

Consideriamo la seconda commutazione:

$$[a_i^\dagger a_m, [H, a_n^\dagger a_j]] \text{ e consideriamone il 1° termine.}$$

$$\begin{aligned}
 &h_{\alpha \beta} [a_i^\dagger a_m, (a_\alpha^\dagger a_j \delta_{n\beta} - a_n^\dagger a_\beta \delta_{j\alpha})] \\
 &= h_{\alpha \beta} (a_i^\dagger a_m a_\alpha^\dagger a_j \delta_{n\beta} - a_i^\dagger a_m a_n^\dagger a_\beta \delta_{j\alpha})
 \end{aligned}
 \quad \begin{array}{l} \text{Il secondo} \\ \text{termine è} \\ \text{nullo perché} \\ a_m |\bar{\Phi}_0\rangle = 0 \end{array}$$

il valore medio tra gli stati HF da'

$$h_{\alpha \beta} \langle \bar{\Phi}_0 | a_i^\dagger a_m a_\alpha^\dagger a_j | \bar{\Phi}_0 \rangle \delta_{n\beta}$$

$$- h_{\alpha \beta} \langle \bar{\Phi}_0 | a_i^\dagger a_m a_n^\dagger a_\beta | \bar{\Phi}_0 \rangle \delta_{j\alpha} = \text{usando le contrazioni} \neq 0$$

$$= h_{\alpha \beta} \delta_{ij} \delta_{mn} \delta_{n\beta} - h_{\alpha \beta} \delta_{ip} \delta_{mn} \delta_{j\alpha} =$$

Nella rappresentazione diagonale di $h_{\alpha \beta}$ abbiamo

$$= \epsilon_m \delta_{ij} \delta_{mn} - \epsilon_i \delta_{ij} \delta_{mn}$$

(2)