$$\sum_{m=1}^{M}\left\{-\frac{\hbar^2}{2m_p}[\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d}{dr})-\frac{l(l+1)}{r^2}]+\frac{1}{2}m_p\omega^2 r^2-E_{ml}^{HO}\right\}R_{ml}^{HO}(r)a_m(nlj)=0 \tag{A.19}$$

Sottraendo la (A.19) alla (A.18) si ottiene

$$\sum_{m=1}^{M}(V_{cent.}(r)+V_{S.O.}(r)+V_C(r)-\frac{1}{2}m_p\omega^2 r^2+E_{nlj}^{HO})a_m(nlj)R_{ml}^{HO}(r)=E_{nlj}\sum_{m=1}^{M}a_m(nlj)R_{ml}^{HO}(r) \tag{A.20}$$

Si moltiplica per $R_{m'l}^{*HO}(r)$ e si integra

$$\int drr^2 R_{m'l}^{*HO}\sum_{m=1}^{M}(V_{cent.}(r)+V_{S.O.}(r)+V_C(r)-\frac{1}{2}m_p\omega^2 r^2+E_{ml}^{HO})a_m(nlj)R_{ml}^{HO}(r)=$$

$$=E_{nlj}\int drr^2 R_{m'l}^{*HO}(r)\sum_{m=1}^{M}a_m(nlj)R_{ml}^{HO}(r) \tag{A.21}$$

$$\sum_{m=1}^{M}a_m(nlj)\left\{E_{ml}^{HO}\delta_{mm'}+\int drr^2 R_{m'l}^{*HO}(r)R_{ml}^{HO}(r)[V_{cent.}(r)+V_{S.O.}(r)+V_C(r)-\frac{1}{2}m_p\omega^2 r^2]\right\}=$$

$$=E_{nlj}\sum_{m=1}^{M}a_m(nlj)\delta_{mm'}=E_{nlj}a_{m'}(nlj) \tag{A.22}$$

Il problema è stato ridotto quindi alla risoluzione della seguente equazione agli autovalori

$$H_{nlj}a(nlj)=E_{nlj}a(nlj) \tag{A.23}$$

con

$$H_{mm'}^{nlj}=E_{ml}^{HO}\delta_{mm'}+\int drr^2 R_{m'l}^{*HO}(r)R_{ml}^{HO}(r)[V_{cent.}(r)+V_{S.O.}(r)+V_C(r)-\frac{1}{2}\omega^2 r^2] \tag{A.24}$$

Normalizzazione degli autovettori

$$\int drr^2 R_{nlj}^{*}(r)R_{nlj}(r)=1 \tag{A.25}$$

$$\int drr^2\sum_{m'=1}^{M}a_{m'}(nlj)R_{m'l}^{HO}(r)\sum_{m=1}^{M}a_m(nlj)R_{ml}^{HO}(r)=\sum_{m'=1}^{M}a_{m'}(nlj)\sum_{m=1}^{M}a_m(nlj)\int drr^2 R_{m'l}^{HO}(r)R_{ml}^{HO}(r)=$$

$$=\sum_{m'=1}^{M}a_{m'}(nlj)\sum_{m=1}^{M}a_m(nlj)\delta_{mm'}=\sum_{m=1}^{M}(a_m(nlj))^2=1 \tag{A.26}$$