$$G(x,y) = \frac{1}{(2\pi)^4}\int d^4k\, e^{ik(x-y)}\, G^0(k)$$

$$+ \frac{1}{(2\pi)^4}\int d^4k_1 d^4k_2 d^4k_3\, e^{ik_1x}e^{-ik_3y}\, G^0(k_1)\Sigma(k_2)G(k_3)\frac{1}{(2\pi)^8}(2\pi)^8\delta(k_1-k_2)\,\delta(k_2-k_3)$$

$$= \frac{1}{(2\pi)^4}\int d^4k\, e^{ik(x-y)}\left[G^0(k) + G^0(k)\Sigma(k)G(k)\right]$$

Quindi $\quad G(k) = G^0(k) + G^0(k)\Sigma(k)G(k)$

da cui $\quad G(k) - G^0(k)\Sigma(k)G(k) = G^0(k)$

$$\left(1 - G^0(k)\Sigma(k)\right)G(k) = G^0(k)$$

$$G(k) = \frac{G^0(k)}{1 - G^0(k)\Sigma(k)} = \frac{1}{\left[G^0(k)\right]^{-1} - \Sigma(k)}$$

Poichè:

$$G^0(\vec{k},\omega) = \left[\frac{\theta(k-k_F)}{\omega-\omega_k+i\eta} + \frac{\theta(k_F-k)}{\omega-\omega_k-i\eta}\right]$$

$$G^0(\vec{k},\omega)^{-1} = \omega - \omega_k$$

quindi $\quad G(k) = \dfrac{1}{\omega-\omega_k - \Sigma(\vec{k},\omega)}$