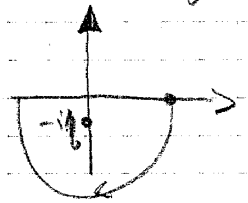
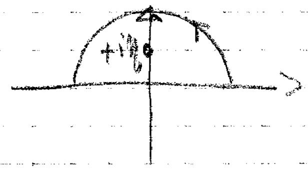
$$iG^0(\vec{x}t, \vec{x}'t') = (2\pi)^{-3} \int_{-\infty}^{\infty} d^3k \; e^{i\vec{k}(\vec{x}-\vec{x}')} \; e^{-i\omega_k(t-t')}$$

$$\left[-\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi i} \frac{e^{-i\omega'(t-t')}}{\omega' + i\eta} \vartheta(k-k_F) - \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi i} \frac{e^{-i\omega'(t-t')}}{\omega' - i\eta} \vartheta(k_F-k) \right]$$



segno −



segno +

Moltiplicando per $-i$

$$G^0(\vec{x}, t, \vec{x}'t') = (2\pi)^{-4} \int d^3k \, e^{i\vec{k}(\vec{x}-\vec{x}')}$$

$$\int_{-\infty}^{\infty} d\omega' \, e^{-i\omega'(t-t')} e^{-i\omega_k(t-t')} \left[\frac{\vartheta(k-k_F)}{\omega' + i\eta} + \frac{\vartheta(k_F-k)}{\omega' - i\eta} \right]$$

Definendo $\omega = \omega' + \omega_k \Rightarrow \omega' = \omega - \omega_k$

$$G^0(\vec{x}t, \vec{x}'t') = (2\pi)^{-4} \int d^3k \, e^{i\vec{k}(\vec{x}-\vec{x}')} \int_{-\infty}^{\infty} d\omega \, e^{-i\omega(t-t')}$$

$$\left[\frac{\vartheta(k-k_F)}{\omega - \omega_k + i\eta} + \frac{\vartheta(k_F-k)}{\omega - \omega_k - i\eta} \right] \quad 9$$

Da cui si ottiene:

$$G^0(\vec{k}, \omega) = \left[\frac{\vartheta(k-k_F)}{\omega - \omega_k + i\eta} + \frac{\vartheta(k_F-k)}{\omega - \omega_k - i\eta} \right] \quad 10$$

Somiglianza con la funzione di Green soluzione dell'equazione differenziale.