$$i\,G^A(\nu_1\nu_2\nu_3\nu_4;E) = \langle\Psi_0|a_{\nu_3}a^\dagger_{\nu_4}\left[\int_{-\infty}^{0} e^{+\frac{i}{\hbar}(H-E_0)\tau}\, e^{\frac{i}{\hbar}E\tau}\, d\tau\right] a_{\nu_1}a^\dagger_{\nu_2}|\Psi_0\rangle$$

$$\lim_{\eta\to 0}\int_{-\infty}^{0} e^{\frac{i}{\hbar}(E+H-E_0-i\eta)\tau}\, d\tau = \left.\frac{e^{\frac{i}{\hbar}(E+H-E_0)\tau}\, e^{\frac{1}{\hbar}\eta}}{\frac{i}{\hbar}(E+H-E_0-i\eta)}\right|_{-\infty}^{0}$$

$$= +\left[\frac{i}{\hbar}\left(E+H-E_0-i\eta\right)\right]^{-1} = -i\hbar\,\frac{1}{E+H-E_0-i\eta}$$

$$G^A(\nu_1\nu_2\nu_3\nu_4;E) = \langle\Psi_0|a_{\nu_3}a^\dagger_{4}\,\frac{\hbar}{E+H-E_0-i\eta}\,a_{\nu_1}a^\dagger_{\nu_2}|\Psi_0\rangle(-1)$$

$$G(\nu_1\nu_2\nu_3\nu_4;E) = G^A + G^R =$$

$$= -\hbar\left[\langle\Psi_0|a_{\nu_1}a^\dagger_{\nu_2}\frac{1}{E-(H-E_0)+i\eta}a_{\nu_3}a^\dagger_{\nu_4}|\Psi_0\rangle - \langle\Psi_0|a_{\nu_3}a^\dagger_{\nu_4}\frac{1}{E+(H-E_0)-i\eta}a_{\nu_1}a^\dagger_{\nu_2}|\Psi_0\rangle\right]$$

Inseriamo $1=\sum_n |\Psi_n\rangle\langle\Psi_n|$ con $H|\Psi_n\rangle = E_n|\Psi_n\rangle$.
Nota $|\Psi_n\rangle$ ha lo stesso numero di particelle di $|\Psi_0\rangle$

$$G(\nu_1\nu_2\nu_3\nu_4;E) = -\hbar\sum_n\left[\frac{\langle\Psi_0|a_{\nu_1}a^\dagger_{\nu_2}|\Psi_n\rangle\langle\Psi_n|a_{\nu_3}a^\dagger_{\nu_4}|\Psi_0\rangle}{E-(E_n-E_0)+i\eta} - \frac{\langle\Psi_0|a_{\nu_3}a^\dagger_{\nu_4}|\Psi_n\rangle\langle\Psi_n|a_{\nu_1}a^\dagger_{\nu_2}|\Psi_0\rangle}{E+(E_n-E_0)-i\eta}\right] \quad 6)$$