- Sviluppo in multipoli della soluzione in onde piane dell'equazione di Helmoltz

$$\vec{A}(\vec{r}) = \vec{\mathcal{E}} \cdot e^{i\vec{k}\cdot\vec{r}} \qquad \text{con} \quad \vec{\mathcal{E}}\cdot\vec{k} = 0 \tag{57}$$

$$A_\mu(\vec{r}) = \mathcal{E}_\mu e^{i\vec{k}\cdot\vec{r}} = 4\pi \sum_{LM} (i)^L j_L(kr) \, Y^*_{LM}(\hat{k}) \, Y_{LM}(\hat{r}) \, \mathcal{E}_\mu = \text{inserisco } \delta_{\mu\mu'} \delta_{MM'}$$

$$= 4\pi \sum_{LM} (+i)^L j_L(kr) \, Y^*_{LM}(\hat{k}) \sum_{\substack{M'\mu' \\ JN}} \langle LM \, 1\mu | JN \rangle \langle LM' \, 1\mu' | JN \rangle \, Y_{LM'}(\hat{r}) \, \mathcal{E}_{\mu'}$$

$$= 4\pi \sum_{LM} (+i)^L j_L(kr) \, Y^*_{LM}(\hat{k}) \sum_{JN} \langle LM \, 1\mu | JN \rangle \, \vec{Y}^N_{J,L,1}(\hat{r}) = \text{scegliendo } k /\!/ z$$

$$= 4\pi \sum_{L} (i)^L j_L(kr) \frac{[L]}{\sqrt{4\pi}} \delta_{M,0} \sum_{J} \langle L0 \, 1\mu | J\mu \rangle \, \vec{Y}^\mu_{J,L,1}(\hat{r})$$

$$= \sqrt{4\pi} \sum_{LJ} (i)^L j_L(kr) \, [L] \, \langle L0 \, 1\mu | J\mu \rangle \, \vec{Y}^\mu_{J,L,1}(\hat{r}) \, . \tag{58}$$

Per il C-G L può essere solo $L = J \pm 1$ o $L = J$ quindi

$$A_\mu(\vec{r}) = \sqrt{4\pi} \sum_J \Big[(i)^{J-1} j_{J-1}(kr) \, [J-1] \, \langle J-1 \, 0 \, 1\mu | J\mu \rangle \, \vec{Y}^\mu_{J,J-1,1}(\hat{r})$$

$$+ (i)^J j_J(kr) \, [J] \, \langle J0 \, 1\mu | J\mu \rangle \, \vec{Y}^\mu_{J,J,1}(\hat{r})$$

$$+ (i)^{J+1} j_{J+1}(kr) \, [J+1] \, \langle J+1 \, 0 \, 1\mu | J\mu \rangle \, \vec{Y}^\mu_{J,J+1,1}(\hat{r}) \Big]$$

solo per $\mu = \pm 1$

$$= \sqrt{4\pi} \sum_J \Big[(i)^{J-1} j_{J-1}(kr) \, [J-1] \frac{\sqrt{J+1}}{\sqrt{2}\,[J-1]} \, \vec{Y}^\mu_{J,J-1,1}(\hat{r})$$

$$+ (i)^J j_J(kr) \, [J] \frac{-\mu}{\sqrt{2}} \, \vec{Y}^\mu_{J,J,1}(\hat{r})$$

$$+ (i)^{J+1} j_{J+1}(kr) \, [J+1] \frac{\sqrt{J}}{\sqrt{2}\,[J+1]} \, \vec{Y}^\mu_{J,J+1,1}(\hat{r}) \Big] =$$