In questo limite

$$\frac{j_{J+1}(kr)}{j_{J-1}(kr)} \xrightarrow[kr\to 0]{} \frac{(kr)^{J+1}}{(2J+3)!!}\,\frac{(2J-1)!!}{(kr)^{J-1}} = \frac{(kr)^2}{(2J+3)(2J+1)} \qquad 7$$

questo per affermare che il termine $J-1$ è dominante in questo limite.

La parte longitudinale ha una struttura analoga

$$\vec{A}_{JM}(\vec{r},l) = \left(\frac{J}{2J+1}\right)^{1/2} j_{J-1}(kr)\,\vec{Y}^{M}_{J,J-1,1}(\hat{r}) + \left(\frac{J+1}{2J+1}\right)^{1/2} j_{J+1}(kr)\,\vec{Y}^{M}_{J,J+1,1}(\hat{r}) = \frac{1}{k}\vec{\nabla} j_J(kr)\,Y_{JM}(\hat{r})$$

escludendo i termini $J+1$ si ha che

$$\vec{A}_{JM}(\vec{r},e) \xrightarrow[kr\ll 1]{} \left(\frac{J+1}{2J+1}\right)^{1/2} j_{J-1}(kr)\,\vec{Y}^{M}_{J,J-1,1}(\hat{r}) \longrightarrow \left(\frac{J+1}{J}\right)^{1/2}\vec{A}_{JM}(\vec{r},l)$$

$$\longrightarrow \left(\frac{J+1}{J}\right)^{1/2}\frac{1}{k}\vec{\nabla} j_J(kr)\,Y_{JM}(\hat{r}) \qquad 7($$

$$M_{\beta\alpha}(EJ) = \sqrt{2\pi}\,[J]\,(i)^{J+1}\int \langle\beta|\vec{j}\,|\alpha\rangle\cdot\vec{A}(\vec{r},e)\,d^3r \Rightarrow kr\ll 1$$

$$= \sqrt{2\pi}\,[J]\,(i)^{J+1}\int \langle\beta|\vec{j}\,|\alpha\rangle \left(\frac{J+1}{J}\right)^{1/2}\frac{1}{k}\vec{\nabla} j_J(kr)\,Y_{JM}(\hat{r})\,d^3r = \text{ per parti}$$

$$= \sqrt{2\pi}\,[J]\,(i)^{J+1}\left(\frac{J+1}{J}\right)^{1/2}\frac{1}{k}\left\{\Big[\langle\beta|\vec{j}\,|\alpha\rangle\, j_J(kr)\,Y_{JM}(\hat{r})\Big]_{-\infty}^{\infty} - \int \vec{\nabla}\cdot\langle\beta|\vec{j}\,|\alpha\rangle\, j_J(kr)\,Y_{JM}(\hat{r})\,d^3r\right\}$$

= il primo termine è nullo e nel secondo uso l'eq. di continuità 65)

$$= \sqrt{2\pi}\,[J]\,(i)^{J+1}\left(\frac{J+1}{J}\right)^{1/2}\frac{1}{k}\,i\,\frac{E_\beta - E_\alpha}{\hbar}\int \langle\beta|\rho|\alpha\rangle\, j_J(kr)\,Y_{JM}(\hat{r})\,d^3r$$

$$= \sqrt{2\pi}\,(i)^{J}\left[\frac{(2J+1)(J+1)}{J}\right]^{1/2}\frac{i^2}{k}\,kc\int \langle\beta|\rho|\alpha\rangle\, j_J(kr)\,Y_{JM}(\hat{r})\,d^3r$$

$$= -c\sqrt{2\pi}\,(i)^{J}\left[\frac{(2J+1)(J+1)}{J}\right]^{1/2}\int j_J(kr)\,Y_{JM}(\hat{r})\,\langle\beta|\rho|\alpha\rangle\,d^3r \qquad 7$$

eq. 73) è nota come <u>teorema di Siegert</u>.