Quindi nell'equazione precedente i termini incrociati $l+1$ e $l-1$ danno contributo nullo.

$$\sum_M \left(\vec{\nabla}\phi Y^*_{l\mu}\right)\left(\vec{\nabla}\phi Y_{l\mu}\right) =$$

$$\frac{l+1}{2l+1}\left[\left(\frac{d}{dr}-\frac{l}{r}\right)\phi\right]^2\frac{2l+1}{4\pi} + \frac{l}{2l+1}\left[\left(\frac{d}{dr}+\frac{l+1}{r}\right)\phi\right]^2\frac{2l+1}{4\pi} =$$

$$= \frac{l+1}{4\pi}\left[\left(\frac{d}{dr}\phi\right)^2 + \frac{l^2}{r^2}\phi^2 - 2\frac{l}{r}\phi\left(\frac{d}{dr}\phi\right)\right] + \frac{l}{4\pi}\left[\left(\frac{d}{dr}\phi\right)^2 + \frac{(l+1)^2}{r^2}\phi^2 + 2\frac{l+1}{r}\phi\frac{d}{dr}\phi\right]$$

$$= \frac{1}{4\pi}\left[(l+1)\left(\frac{d}{dr}\phi\right)^2 + \frac{l^2(l+1)}{r^2}\phi^2 - \cancel{2\frac{(l+1)l}{r}\phi\frac{d}{dr}\phi}\right.$$
$$\left. + l\left(\frac{d}{dr}\phi\right)^2 + \frac{l(l+1)^2}{r^2}\phi^2 + \cancel{2\frac{(l+1)l}{r}\phi\frac{d}{dr}\phi}\right] =$$

$$= \frac{1}{4\pi}\left[(2l+1)\left(\frac{d}{dr}\phi\right)^2 + \frac{1}{r^2}\left(l^3+l^2+l^3+2l^2+l\right)\phi^2\right] =$$

$$= \frac{2l+1}{4\pi}\left[\left(\frac{d}{dr}\phi\right)^2 + \frac{l(l+1)}{r^2}\phi^2\right]$$

• Sostituiamo la 11 nella 2

$$S_J = \cancel{8}^{\,2}\pi^{\cancel{3}\,2}\frac{J+1}{\cancel{J}}\frac{1}{[(2J+1)!!]^2}\frac{1}{2J_i+1}\frac{1}{\hbar c}\frac{\hbar^2c^2}{\cancel{2}mc^2}\frac{\cancel{J}(2J+1)^2}{\cancel{4\pi}}\sum_a\langle i|r_a^{2J-2}|i\rangle e_a^2$$

$$= \frac{1}{2J_i+1}\left(\frac{1}{\hbar c}\right)\left(\frac{\hbar c}{mc^2}\right)^2\pi^2\frac{J+1}{[(2J-1)!!]^2}\sum_a\langle i|r_a^{2J-2}|i\rangle e_a^2$$