Il comportamento asintotico delle due soluzioni e'

$$R_\ell \xrightarrow[r\to\infty]{} \sin\left(kr - \frac{\ell}{2}\pi + \delta_\ell\right) \tag{11a}$$

$$\tilde{R}_\ell \xrightarrow[r\to\infty]{} \sin\left(kr - \frac{\ell}{2}\pi + \tilde{\delta}_\ell\right) \tag{11b}$$

Il Wronskiano e'

$$\lim_{r\to\infty} W(R_\ell, \tilde{R}_\ell) = \lim_{r\to\infty}\left(\sin\left(kr - \frac{\ell}{2}\pi + \delta_\ell\right) k \cos\left(kr - \frac{\ell\pi}{2} + \tilde{\delta}_\ell\right) - \sin\left(kr - \frac{\ell}{2}\pi + \tilde{\delta}_\ell\right) k \cos\left(kr - \frac{\ell}{2}\pi + \delta_\ell\right)\right)$$

$$= k \sin\left(\delta_\ell - \tilde{\delta}_\ell\right) \tag{12}$$

Usando il teorema del Wronskiano

$$\lim_{r\to\infty} W(R_\ell, \tilde{R}_\ell) = k \sin\left(\delta_\ell - \tilde{\delta}_\ell\right) = -\int_0^\infty \left(U_\ell - \tilde{U}_\ell\right) R_\ell \tilde{R}_\ell \, dr \tag{13}$$

questo perche' $\lim_{r\to 0} W(R_\ell, \tilde{R}_\ell) = 0 \quad \left(R_\ell(0) = 0\right)$

da cui

$$\sin\left(\delta_\ell - \tilde{\delta}_\ell\right) = -\frac{2m}{\hbar^2}\int_0^\infty \left(V - \tilde{V}\right) R_\ell \tilde{R}_\ell \, dr \tag{14}$$

se $\tilde{V} = 0$

$$\sin \delta_\ell = -\frac{2m}{\hbar^2}\int_0^\infty V R_\ell \tilde{R}_\ell \, dr \tag{15}$$

Supponiamo $\Delta V = V - \tilde{V}$ molto piccolo quindi $R_\ell \simeq \tilde{R}_\ell$ si ha

$$\Delta \delta_\ell = -\frac{2m}{\hbar^2 k}\int_0^\infty \Delta V \, R_\ell^2 \, dr \tag{16}$$