

il comportamento asintotico delle due soluzioni e'

$$R_b \xrightarrow{r \rightarrow \infty} \sin(kr - \frac{l}{2}\tilde{u} + \delta_b) \quad (11a)$$

$$\tilde{R}_b \xrightarrow{r \rightarrow \infty} \sin(kr - \frac{l}{2}\tilde{u} + \tilde{\delta}_b) \quad (11b)$$

il Wronskiano e'

$$\begin{aligned} \lim_{r \rightarrow \infty} W(R_b, \tilde{R}_b) &= \lim_{r \rightarrow \infty} \left(\sin(kr - \frac{l}{2}\tilde{u} + \delta_b) k \cos(kr - \frac{l}{2}\tilde{u} + \tilde{\delta}_b) \right. \\ &\quad \left. - \sin(kr - \frac{l}{2}\tilde{u} + \tilde{\delta}_b) k \cos(kr - \frac{l}{2}\tilde{u} + \delta_b) \right) \\ &= k \sin(\delta_b - \tilde{\delta}_b) \end{aligned} \quad (12)$$

Usando il teorema del Wronskiano

$$\lim_{r \rightarrow \infty} W(R_b, \tilde{R}_b) = k \sin(\delta_b - \tilde{\delta}_b) = - \int_0^{\infty} (\tilde{u}_b - \tilde{\tilde{u}}_b) R_b \tilde{R}_b dr \quad (13)$$

questo perche' $\lim_{r \rightarrow 0} W(R_b, \tilde{R}_b) = 0$ ($R_b(0) = 0$)

da cui

$$\sin(\delta_b - \tilde{\delta}_b) = - \frac{2m}{\hbar^2} \int_0^{\infty} (V - \tilde{V}) R_b \tilde{R}_b dr \quad (14)$$

se $\tilde{V} = 0$

$$\sin \delta_b = - \frac{2m}{\hbar^2} \int_0^{\infty} V R_b \tilde{R}_b dr \quad (15)$$

Supponiamo $\Delta V = V - \tilde{V}$ molto piccolo quindi $R_b \approx \tilde{R}_b$ si ha

$$\Delta \delta_b = - \frac{2m}{\hbar^2 k} \int_0^{\infty} \Delta V R_b^2 dr \quad (16)$$