$$u_L(r) \xrightarrow[r\to\infty]{} (2L+1)\, i^L e^{i\sigma_L} \frac{A_L}{k} \sin\left(kr - \frac{\pi}{2}L - \eta \ln(kr) + \sigma_L + \delta_L\right)$$

$$u^c_L(r) \xrightarrow[r\to\infty]{} (2L+1)\, i^L e^{i\sigma_L} \frac{1}{k} \sin\left(kr - \frac{\pi}{2}L - \eta \ln(kr) + \sigma_L\right)$$

$$u^+_L(r) \xrightarrow[r\to\infty]{} e^{i\left(kr - \frac{\pi}{2}L - \eta \ln(kr)\right)}$$

$$u^-_L(r) \xrightarrow[r\to\infty]{} e^{-i\left(kr - \frac{\pi}{2}L - \eta \ln(kr)\right)}$$

$$u_L(r) - u^c_L(r) = (2L+1)\, i^L \frac{1}{k}$$

$$e^{i\sigma_L}\left\{\frac{A_L}{2i}\left[u^+_L(r)\, e^{i\sigma_L} e^{+i\delta_L} - u^-_L(r)\, e^{-i\sigma_L} e^{-i\delta_L}\right] - \left[u^+_L(r)\, e^{i\sigma_L} e^{+i\delta_L} - u^-_L(r)\, e^{-i\sigma_L} e^{-i\delta_L}\right]\right.$$

$$= (2L+1)\, i^L \frac{1}{k} \frac{e^{i\sigma_L}}{2i}\left\{u^+_L(r)\, e^{i\sigma_L}\left(A_L e^{i\delta_L} - 1\right) - u^-_L(r)\, e^{-i\sigma_L}\left(A_L e^{-i\delta_L} - 1\right)\right\}$$

$$A_L = e^{i\delta_L}$$

$$u_L(r) - u^c_L(r) = (2L+1)\, i^L \frac{1}{k} \frac{1}{2i}\, u^+_L(r)\, e^{2i\sigma_L}\left(e^{2i\delta_L} - 1\right)$$

$$\psi(\vec{r}) - \psi^c(\vec{r}) = \frac{1}{2i} \frac{1}{kr} \sum_{L=0}^{\infty} (2L+1)\, i^L e^{2i\sigma_L}\left(e^{2i\delta_L} - 1\right) u^+_L(r)\, P_L(\cos\vartheta)$$

Dal confronto con la 19) si ha che:

$$f(\vartheta) = \frac{1}{2i} \frac{1}{k} \sum_{L=0}^{\infty} (2L+1)\, i^L e^{2i\sigma_L}\left(e^{2i\delta_L} - 1\right) e^{-i\frac{\pi}{2}L}\, P_L(\cos\vartheta)$$

$$= \frac{1}{2i} \frac{1}{k} \sum_{L=0}^{\infty} (2L+1)\, e^{2i\sigma_L}\left(e^{2i\delta_L} - 1\right) P_L(\cos\vartheta) \qquad 19a$$