The matrix elements involved in eq. 6 are:

Direct

$$D_p = \int \varphi^{*\beta}_k(\vec{r})\, U_p(\vec{r})\, \varphi^{\alpha}_k(\vec{r})\, d^3r \qquad 7)$$

with $p = 1, \dots, 6$

$$\varphi^{n}_k(\vec{r}) = h_{n,\ell}(r)\, \mathcal{Y}^{m}_{\ell j}\, \chi_t \qquad k \equiv \{\ell, j, m, t\},\ n = 1, 2 \qquad 8)$$

$$h_{n,\ell}(r) = A_{n\ell} \cdot (\alpha r)^{\ell} \cdot \mathrm{L}^{\ell+1/2}_{n-1}(\alpha^2 r^2)\, e^{-\frac{1}{2}(\alpha^2 r^2)} \qquad 9a)$$

$$A_{n\ell} = \left(\frac{2\,(n-1)!}{\Gamma(n+\ell+1/2)} \right)^{1/2} \alpha^{3/2} \qquad 9b)$$

$$\mathrm{L}^{\ell+1/2}_{n-1}(x) = \sum_{r=0}^{n-1} \frac{\Gamma(n-1+\ell+1/2+1)}{\Gamma(r+\ell+1/2+1)} \frac{(-x)^r}{r!\,(n-1-r)!} \qquad 9c)$$

α - oscillator constant

- D_1 - Central term, eq. HE 6)

$$D_1 = \int dr_1 r_1^2\, h_{n,\ell_1}(r_1)\, h_{n',\ell_1}(r_1) \cdot 4\pi (2\pi)^{1/2} \int dr_2 r_2^2\, \rho(r_2) \int dq\, q^2 j_0(qr_1) j_0(qr_2) V_1(q)$$

$$= 4\pi (2\pi)^{1/2} \sum_{\substack{n_2 \ell_2 j_2 t_2 \\ n_2'}} a^{t_2}_{n_2,\ell_2 j_2}\, a^{t_2}_{n_2',\ell_2 j_2}\, \frac{2j_2+1}{4\pi}$$

$$\int dr_1 r_1^2\, h_{n,\ell_1}(r_1)\, h_{n',\ell_1}(r_1) \int dr_2 r_2^2\, h_{n_2',\ell_2}(r_2)\, h_{n_2,\ell_2}(r_2) \int dq\, q^2 j_0(qr_1) j_0(qr_2) V_1(q) \qquad 10)$$