The density dependent term:

$$K_0 = \int dr_1 r_1^2 \, h_{n_1 l_1}(r_1) \, K(r_1) \, h_{n' l_1}(r_1) \qquad 16)$$

Remember:

$$\rho(r) = \sum_{nljt} \frac{2j+1}{4\pi} R^2_{nlj}(r) = \sum_{nljt} \frac{2j+1}{4\pi} \sum_{n_1} a^t_{n_1 l j} h_{n_1 l}(r) \sum_{n_2} a^t_{n_2 l j} h_{n_2 l}(r) \qquad 17)$$

$$f^D_\rho(r_1, r_2) = \frac{1}{4\pi} \frac{\partial f_\rho(\rho)}{\partial \rho(r_1)} \qquad 18)$$

Let's assume: $f_\rho(\rho) = \frac{1}{2}\left(\rho(r_1) + \rho(r_2)\right)^\alpha \qquad 19)$

$$\frac{\partial}{\partial \rho} f_\rho = \frac{\alpha}{2}\left(\rho(r_1) + \rho(r_2)\right)^{\alpha-1} \qquad 20)$$

$$= \frac{\alpha}{2}\left(\sum_{n_1 l_1 j_1 t_1} \frac{2j_1+1}{4\pi} R^2_{n_1 l_1 j_1}(r_1) + \sum_{n_2 l_2 j_2 t_2} \frac{2j_2+1}{4\pi} R^2_{n_2 l_2 j_2}(r_2)\right)^{\alpha-1}$$

$$= \frac{\alpha}{2}\left[\sum_{n_1 l_1 j_1 t_1} \frac{2j_1+1}{4\pi}\left(R^2_{n_1 l_1 j_1}(r_1) + R^2_{n_1 l_1 j_1}(r_2)\right)\right]^{\alpha-1}$$

$$R^2_{nlj}(r_1) = \sum_{n n'} a^t_{nlj} \, a^t_{n' l j} \, h_{nl}(r) \, h_{n'l}(r)$$

$$\frac{\partial}{\partial \rho} f_\rho = \frac{\alpha}{2}\left[\sum_{ljt} \frac{2j+1}{4\pi}\left(\sum_{nn'} a^t_{nlj} \, a^t_{n'lj} \, h_{nl}(r) h_{n'l}(r_1) + \sum_{nn'} a^t_{nlj} \, a^t_{n'lj} \, h_{nl}(r) h_{n'l}(r)\right)\right]^{\alpha-1}$$

$$= \frac{\alpha}{2}\left[\sum_{ljt} \frac{2j+1}{4\pi} \sum_{nn'} a^t_{nlj} \, a^t_{n'lj} \left(h_{nl}(r_1) h_{n'l}(r_1) + h_{nl}(r) h_{n'l}(r)\right)\right]^{\alpha-1} \qquad 21)$$