Lezione 13

Equazione di Dirac

Equazione di Schrödinger col principio di equivalenza

$$E = \frac{p^2}{2m} \quad ; \quad E \to i\hbar\frac{\partial}{\partial t} \quad ; \quad \mathbf{p} \to -i\hbar\nabla$$

$$i\hbar\frac{\partial}{\partial t}\psi = \frac{(-i\hbar\nabla)^2}{2m}\psi = \frac{-\hbar^2}{2m}\nabla^2\psi$$

Conservazione della corrente

$$\frac{\partial}{\partial t}\rho + \nabla\cdot\mathbf{J} = 0$$

Moltiplichiamo l'equazione di Schrödinger per ψ^* e poi ne sottraiamo il complesso coniugato.

$$\psi^* \left[i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \right] \psi = 0 \quad ; \quad \psi \left[-i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \right] \psi^* = 0$$

$$i\hbar \left[\psi^* \frac{\partial}{\partial t} \psi + \psi \frac{\partial}{\partial t} \psi^* \right] \; + \; \frac{\hbar^2}{2m} \left[\psi^* \nabla^2 \psi - \psi \nabla^2 \psi^* \right] = 0$$

Dividendo per $i\hbar$

$$\frac{\partial}{\partial t} \left(\psi^* \psi \right) - \frac{i\hbar}{2m} \nabla \cdot \left[\psi^* \nabla \psi - \psi \nabla \psi^* \right] = 0$$

$$\rho = |\psi|^2 \quad ; \quad \mathbf{J} = -\frac{i\hbar}{2m} \left[\psi^* \nabla \psi - \psi \nabla \psi^* \right]$$

Soluzione

$$\psi(\mathbf{r}, t) = N e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

$$\rho = |\psi|^2 = |N|^2 \;\; ; \;\; \mathbf{J} = -\frac{i\hbar}{2m} [i\mathbf{k} - (-i\mathbf{k})] |N|^2 = \frac{\hbar \mathbf{k}}{m} |N|^2$$

Equazione di Klein-Gordon

$$E^2 = p^2c^2 + m^2c^4$$

$$\left(i\hbar\frac{\partial}{\partial t}\right)^2 \psi - (-i\hbar\nabla)^2 c^2 \psi = m^2c^4\psi$$

$$\left[-\hbar^2\frac{\partial^2}{\partial t^2} + \hbar^2\nabla^2c^2\right]\psi = m^2c^4\psi \;\; ; \;\; \left[-\frac{\partial^2}{\partial t^2} + \nabla^2c^2\right]\psi = \frac{m^2}{\hbar^2}c^4\psi$$

Moltiplichiamo per $-i\psi^*$ e consideriamo la complessa coniugata

$$-i\psi^*\left[-\frac{\partial^2}{\partial t^2} + \nabla^2c^2\right]\psi = \frac{m^2}{\hbar^2}c^4\psi\psi^*(-i)$$

$$i\psi\left[-\frac{\partial^2}{\partial t^2} + \nabla^2c^2\right]\psi^* = \frac{m^2}{\hbar^2}c^4\psi\psi^*(i)$$

Sottraendo

$$i\left[\psi^*\frac{\partial^2}{\partial t^2}\psi - \psi\frac{\partial^2}{\partial t^2}\psi^*\right] - i\psi^*\nabla^2c^2\psi + i\psi\nabla^2c^2\psi^* = 0$$

$$\frac{\partial}{\partial t}\left[i\psi^*\frac{\partial}{\partial t}\psi - i\psi\frac{\partial}{\partial t}\psi^*\right] + \nabla\cdot\left[-i\left(\psi^*\nabla c^2\psi - \psi\nabla c^2\psi^*\right)\right]$$

$$\rho = i\left[\psi^*\frac{\partial}{\partial t}\psi - \psi\frac{\partial}{\partial t}\psi^*\right] \quad ; \quad \mathbf{J} = \left[-i\left(\psi^*\nabla c^2\psi - \psi\nabla c^2\psi^*\right)\right]$$

Soluzione di onda piana

$$\psi(\mathbf{r},t) = Ne^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$\left[-\frac{\partial^2}{\partial t^2} + \nabla^2 c^2\right] Ne^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = \frac{m^2}{\hbar^2}c^4 Ne^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$\left(\omega^2 - k^2c^2\right) = \frac{m^2c^4}{\hbar^2} \quad ; \quad \hbar^2\omega^2 - \hbar^2k^2c^2 = m^2c^4$$

$$E = \hbar\omega \quad ; \quad \mathbf{p} = \hbar\mathbf{k}$$

$$
\begin{aligned}
\rho &= iN^* e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}(-i\omega) N e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} - iNe^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}(i\omega) N^* e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \\
&= 2\omega |N|^2 = 2\frac{E}{\hbar}|N|^2
\end{aligned}
$$

$$
\mathbf{J} = 2\mathbf{k}|N|^2 = 2\frac{\mathbf{p}}{\hbar}|N|^2
$$

$$
E = \pm\sqrt{p^2c^2 + m^2c^4}
$$

Densità di probabilità negative

Equazione di Dirac

$$E\psi = i\hbar\frac{\partial}{\partial t}\psi = H\psi \quad ; \quad H = \boldsymbol{\alpha}\cdot(-i\hbar\nabla)c + \beta mc^2$$

$\boldsymbol{\alpha} \equiv (\alpha_1, \alpha_2, \alpha_3)$ e β costanti da determinare.
Per la particella libera

$$\begin{aligned}
& E^2 = p^2c^2 + m^2c^4 \\
= \ & \left[\boldsymbol{\alpha}\cdot\mathbf{p}c + \beta mc^2\right]\left[\boldsymbol{\alpha}\cdot\mathbf{p}c + \beta mc^2\right] \\
= \ & (\boldsymbol{\alpha}\cdot\mathbf{p})(\boldsymbol{\alpha}\cdot\mathbf{p})c^2 + \beta mc^2\boldsymbol{\alpha}\cdot\mathbf{p}c + \boldsymbol{\alpha}\cdot\mathbf{p}c\beta mc^2 + \beta^2m^2c^4 \\
= \ & (\sum_{i=1}^{3}\alpha_ip_i)(\sum_{j=1}^{3}\alpha_jp_j)c^2 + mc^3\beta\sum_{i=1}^{3}\alpha_ip_i + mc^3\sum_{j=1}^{3}\alpha_jp_j\beta + \beta^2m^2c^4 \\
= \ & \sum_i \alpha_ip_i\alpha_ip_ic^2 + \sum_{i\neq j}(\alpha_i\alpha_j + \alpha_j\alpha_i)p_ip_jc^2 \\
& + mc^3\sum_i(\beta\alpha_i + \alpha_i\beta)p_i + \beta^2m^2c^4 \\
= \ & \sum_i p_i^2c^2 + m^2c^4
\end{aligned}$$

Proprietà di α e β

$$\alpha_i^2 = 1 \quad ; \quad \alpha_i \alpha_j + \alpha_j \alpha_i = 0 \quad ; \quad \alpha_i \beta + \beta \alpha_i = 0 \quad ; \quad \beta^2 = 1$$

1) Non commutano, sono matrici.
2) Sono hermitiane perché H è hermitiano
$\alpha_i = \alpha_i^\dagger$, $\beta = \beta^\dagger$
3) Dato che $\alpha_i^2 = \alpha_i \alpha_i^\dagger = \mathbf{I}$ e $\beta\beta^\dagger = \mathbf{I}$,
sono unitarie, hanno autovalori ± 1.
4) Dato che le tracce sono nulle hanno dimensioni pari.

$$Tr[AB] = Tr[BA]$$
$$Tr[\alpha_i] = Tr[(\alpha_i\beta)\beta] = Tr[\beta(\alpha_i\beta)] = Tr[\beta(-\beta\alpha_i)] =$$
$$Tr[-\beta^2\alpha_i] = Tr[-\alpha_i] = 0$$

5) Per matrici 2×2 non c'è nessuna β che anticommuti con $\alpha_{1,2,3}$.
Si deve passare alle dimensioni successive 4×4.

Equazione di Dirac

$$i\hbar\frac{\partial}{\partial t}\boldsymbol{\psi} = -i\hbar c\boldsymbol{\alpha}\cdot\nabla\boldsymbol{\psi} + \beta mc^2\boldsymbol{\psi} \quad ; \quad \boldsymbol{\psi} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix}$$

Continuità dell'equazione di Dirac

$$\begin{aligned} \boldsymbol{\psi}^\dagger\left[i\hbar\frac{\partial}{\partial t}\boldsymbol{\psi}\right] &= \boldsymbol{\psi}^\dagger\left[-i\hbar c\boldsymbol{\alpha}\cdot\nabla\boldsymbol{\psi} + \beta mc^2\boldsymbol{\psi}\right] \\ \left[-i\hbar\frac{\partial}{\partial t}\boldsymbol{\psi}^\dagger\right]\boldsymbol{\psi} &= \left[i\hbar c\nabla\boldsymbol{\psi}^\dagger\cdot\boldsymbol{\alpha} + mc^2\boldsymbol{\psi}^\dagger\beta\right]\boldsymbol{\psi} \end{aligned}$$

Sottraendo

$$\begin{aligned} i\hbar\left[\boldsymbol{\psi}^\dagger\frac{\partial}{\partial t}\boldsymbol{\psi} + \left(\frac{\partial}{\partial t}\boldsymbol{\psi}^\dagger\right)\boldsymbol{\psi}\right] &= -i\hbar\, c\left[\boldsymbol{\psi}^\dagger\alpha\cdot(\nabla\boldsymbol{\psi}) + (\nabla\boldsymbol{\psi}^\dagger)\cdot\alpha\boldsymbol{\psi}\right] \\ &+ mc^2\left[(\boldsymbol{\psi}^\dagger\beta\boldsymbol{\psi}) - (\boldsymbol{\psi}^\dagger\beta)\boldsymbol{\psi}\right] \end{aligned}$$

$$i\hbar\frac{\partial}{\partial t}\left(\boldsymbol{\psi}^\dagger\boldsymbol{\psi}\right) = -i\hbar\, c\,\nabla\cdot\left[\boldsymbol{\psi}^\dagger\alpha\boldsymbol{\psi}\right]$$

$$\rho = |\psi|^2 \quad ; \quad \mathbf{J} = c\boldsymbol{\psi}^\dagger\boldsymbol{\alpha}\boldsymbol{\psi}$$

Unità naturali $\hbar = c = 1$

$$\begin{aligned} E\boldsymbol{\psi} &= (\boldsymbol{\alpha}\cdot\mathbf{p} + \beta m)\,\boldsymbol{\psi} \\ i\frac{\partial}{\partial t}\boldsymbol{\psi} &= (-i\boldsymbol{\alpha}\cdot\nabla + \beta m)\,\boldsymbol{\psi} \\ \Big[i\frac{\partial}{\partial t} &+ i\boldsymbol{\alpha}\cdot\nabla - \beta m\Big]\boldsymbol{\psi} = 0 \end{aligned}$$

$\boldsymbol{\alpha}$ e β non hanno espressione univoca. Scelta comune.

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} \mathbf{I} & 0 \\ 0 & -\mathbf{I} \end{pmatrix} \qquad \alpha_i^2 = \beta^2 = 1$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Moltiplico a sinistra per β.

$$\Big[i\beta\frac{\partial}{\partial t} + i\beta\boldsymbol{\alpha}\cdot\nabla - \beta^2 m\Big]\boldsymbol{\psi} = 0$$

Definiamo le 4 componenti di un quadrivettore

$$\boldsymbol{\gamma} \equiv [\beta, \beta\boldsymbol{\alpha}] \equiv \Big[\gamma^0, \boldsymbol{\gamma}\Big]$$

Forma covariante dell'equazione di Dirac

$$\left[i\gamma^0\frac{\partial}{\partial t} + i\boldsymbol{\gamma}\cdot\nabla - m\right]\psi = 0$$

$$\gamma^i = \beta\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \qquad \gamma^0 = \begin{pmatrix} \mathbf{I} & 0 \\ 0 & -\mathbf{I} \end{pmatrix}$$

Indici $i, j, k = 1, 2, 3$ $\mu, \nu = 0, 1, 2, 3$

Proprietà delle matrici γ

$$\gamma^\nu\gamma^\mu = -\gamma^\mu\gamma^\nu \quad (\nu \neq \mu) \qquad (\gamma^i)^2 = -\mathbf{I} \qquad (\gamma^0)^2 = \mathbf{I}$$

$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 \qquad (\gamma^i)^\dagger = -\gamma^i \qquad (\gamma^0)^\dagger = \gamma^0$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \qquad (\gamma^5)^2 = \mathbf{I} \qquad (\gamma^5)^\dagger = \gamma^5 \qquad \gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$$

$$\gamma^5 = \begin{pmatrix} 0 & \mathbf{I} \\ \mathbf{I} & 0 \end{pmatrix}$$

Prodotto scalare

$$A^\mu \equiv \left(A_0, \mathbf{A}\right) \qquad A_\mu \equiv \left(A_0, -\mathbf{A}\right)$$

$$A_\mu B^\mu \equiv \sum_{\mu=0,1,2,3} A_\mu B^\mu = A_0 B^0 - \mathbf{A}\cdot\mathbf{B}$$

$$x^\mu \equiv (t, \mathbf{r}) \qquad x_\mu x^\mu = t^2 - \mathbf{r}^2$$

$$p^\mu \equiv (E, \mathbf{p}) \qquad p_\mu p^\mu = E^2 - \mathbf{p}^2 = m^2$$

Invarianti relativistici

$$\frac{\partial}{\partial x_\mu} \equiv \left(\frac{\partial}{\partial t}, -\nabla\right) \qquad p_\mu \equiv \left(i\frac{\partial}{\partial t}, i\nabla\right)$$

Equazione di Dirac

$$\left[i\gamma^\mu \frac{\partial}{\partial x_\mu} - m\right]\boldsymbol{\psi} = 0$$

Hamiltoniana NON commuta con $\mathbf{L}^2$ e L_z ma commuta con $\mathbf{J}^2$ e J_z dove $\mathbf{J} = \mathbf{L} + \boldsymbol{\sigma}/2$

Soluzione di onde piane

Sistema di riferimento a riposo $\mathbf{p} = 0$.

$$i\frac{\partial}{\partial t}\psi = \beta m \psi \quad ; \quad \beta = \begin{pmatrix} \mathbf{I} & 0 \\ 0 & -\mathbf{I} \end{pmatrix} \quad ; \quad \psi = \phi(\mathbf{r}, t) u$$

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad ; \quad u_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad ; \quad u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad ; \quad u_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\beta u_{1,2} = u_{1,2} \quad ; \quad \beta u_{3,4} = -u_{3,4}$$

$$i\frac{\partial}{\partial t}\phi(\mathbf{r}, t) u_{1,2} = m\phi(\mathbf{r}, t) u_{1,2} \quad ; \quad \phi(\mathbf{r}, t) = e^{-imt} \quad ; \quad \psi_{1,2} = e^{-imt} u_{1,2}$$

$$i\frac{\partial}{\partial t}\phi(\mathbf{r}, t) u_{3,4} = -m\phi(\mathbf{r}, t) u_{3,4} \quad ; \quad \phi(\mathbf{r}, t) = e^{imt} \quad ; \quad \psi_{3,4} = e^{imt} u_{3,4}$$

Autovalori dell'hamiltoniana sono $\pm$m.

Energie negative !!!

Invariante relativistico

$$mt = p_\nu x^\nu = E't' - \mathbf{p}' \cdot \mathbf{r}'$$

Le soluzioni dell'equazione si modificano in un sistema di riferimento in moto

$$\psi_{1,2} = e^{i(\mathbf{p}\cdot\mathbf{r}-Et)} u_{1,2} \quad ; \quad \psi_{3,4} = e^{-i(\mathbf{p}\cdot\mathbf{r}-Et)} u_{3,4}$$

$$i\frac{\partial}{\partial t}\psi + i\alpha \cdot \nabla\psi - \beta m \psi = 0$$

$$\psi = e^{i(\mathbf{p}\cdot\mathbf{r}-Et)} u = e^{i(\mathbf{p}\cdot\mathbf{r}-Et)} \begin{pmatrix} \phi \\ \chi \end{pmatrix}$$

$$\Big(E - \alpha \cdot \mathbf{p} - \beta m\Big) u = \left[\begin{pmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{I} \end{pmatrix} E - \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix} \cdot \mathbf{p} - \begin{pmatrix} \mathbf{I} & 0 \\ 0 & -\mathbf{I} \end{pmatrix} m\right] \begin{pmatrix} \phi \\ \chi \end{pmatrix} = 0$$

$$E\phi - \boldsymbol{\sigma} \cdot \mathbf{p}\chi - m\phi = 0 \qquad (E-m)\phi = \boldsymbol{\sigma} \cdot \mathbf{p}\chi$$

$$E\chi - \boldsymbol{\sigma} \cdot \mathbf{p}\phi + m\chi = 0 \qquad (E+m)\chi = \boldsymbol{\sigma} \cdot \mathbf{p}\phi$$

$$\phi = \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E-m}\chi \qquad \chi = \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E+m}\phi$$

$$\det \begin{vmatrix} E-m & -\boldsymbol{\sigma}\cdot\mathbf{p} \\ -\boldsymbol{\sigma}\cdot\mathbf{p} & E+m \end{vmatrix} = (E-m)(E+m)-(\boldsymbol{\sigma}\cdot\mathbf{p})^2 = E^2-m^2-\mathbf{p}^2 = 0$$

$$E = \pm\sqrt{\mathbf{p}^2+m^2}$$

Energie positive $E = |E|$

$$\phi = \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E-m}\chi$$

Regime non relativistico $|E| \simeq m$ quindi $\phi \gg \chi$

Scegliamo

$$\phi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \phi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Soluzioni ad energia positiva

$$\psi_+ = Ne^{i(\mathbf{p}\cdot\mathbf{r}-Et)}u_+ = Ne^{i(\mathbf{p}\cdot\mathbf{r}-Et)} \begin{bmatrix} \phi \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\phi \end{bmatrix}$$

Energie negative $E = -|E|$

$$\chi = \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E+m}\phi = \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{-|E|+m}\phi$$

Regime non relativistico $|E| \simeq m$ quindi $\chi \gg \phi$

Scegliamo

$$\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Soluzioni ad energia negativa

$$\psi_- = N e^{i(\mathbf{p}\cdot\mathbf{r}+|E|t)} u_- = N e^{i(\mathbf{p}\cdot\mathbf{r}+|E|t)} \begin{bmatrix} -\frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{|E|+m}\chi \\ \chi \end{bmatrix}$$

N costante di normalizzazione. Per $\psi^\dagger \psi = 1$

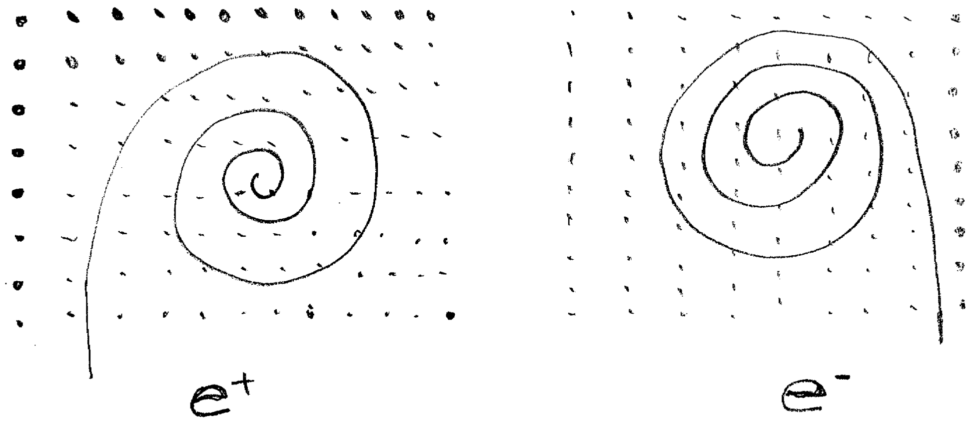
$$N = \sqrt{\frac{m+|E|}{2|E|}}$$

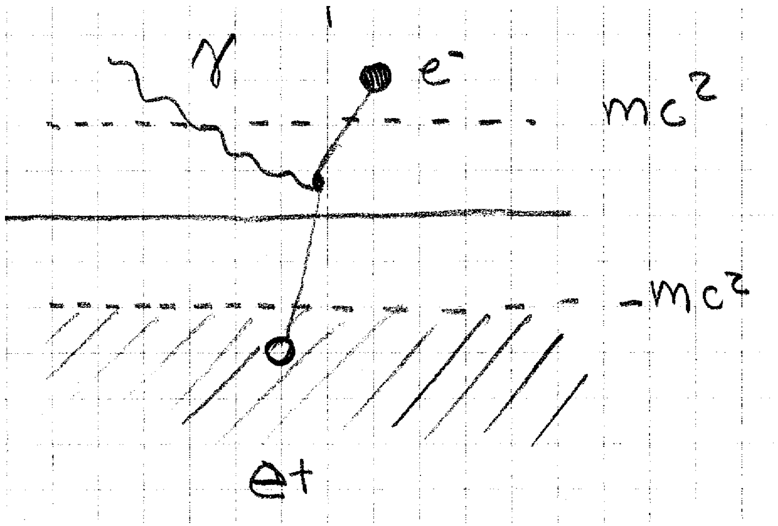
Spinori in onde piane

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ \frac{p_z}{|E|+m} \\ \frac{p_x+ip_y}{|E|+m} \end{bmatrix} \quad ; \quad u_2 = \begin{bmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{|E|+m} \\ \frac{-p_z}{|E|+m} \end{bmatrix}$$

$$u_3 = \begin{bmatrix} -\frac{p_z}{|E|+m} \\ -\frac{p_x+ip_y}{|E|+m} \\ 1 \\ 0 \end{bmatrix} \quad ; \quad u_4 = \begin{bmatrix} -\frac{p_x-ip_y}{|E|+m} \\ \frac{p_z}{|E|+m} \\ 0 \\ 1 \end{bmatrix}$$

Antimateria





Antiparticelle

Proprietà che NON variano

massa, spin, isospin, vita media

Proprietà uguali in valore assoluto e segno opposto

carica elettrica, 3^a componente isospin, momento magnetico, n^o leptonico, n^o barionico, stranezza, charm, beauty

Fermioni identificati con una barra $\overline{p}$.
Tranne leptoni $e^{\pm}, \mu^{\pm}, \tau^{\pm}$.

Equazione di Klein-Gordon per densità di carica

$$-e\left[-\frac{\partial^2}{\partial t^2}+\nabla^2c^2-m^2\right]\psi=0 \qquad \psi=Ne^{-i(\mathbf{p}\cdot\mathbf{r}-Et)}$$

Carica unitaria con segno -.

$$\rho(e^-)=-e2E|N|^2 \qquad \mathbf{J}(e^-)=-e2\mathbf{p}|N|^2$$

Cambio carica

$$\rho(e^+)=e2E|N|^2 \qquad \mathbf{J}(e^+)=e2\mathbf{p}|N|^2$$

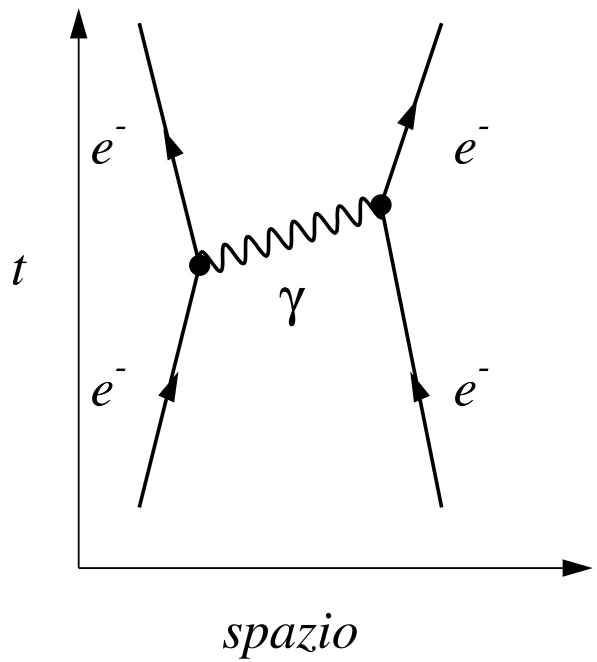
Posso ottenere il cambio carica anche cambiando $E\rightarrow -E$ e $\mathbf{p}\rightarrow -\mathbf{p}$. Nella soluzione ottengo lo stesso risultato cambiando $t\rightarrow -t$, che cambia $\mathbf{p}\rightarrow -\mathbf{p}$.

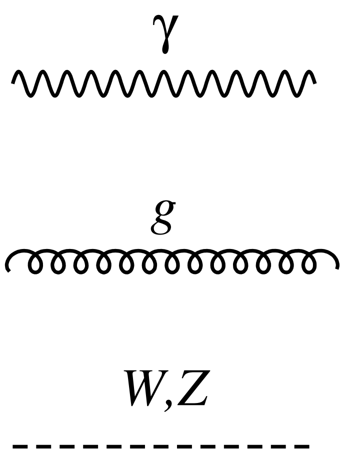
$$\psi=Ne^{-i\big((-\mathbf{p})\cdot\mathbf{r}-E(-t)\big)}$$

Interpetazione di Feynmann-Stückelbeg

Le antiparticelle sono particelle che simuovono all'indietro nel tempo.

Diagrammi di Feynmann





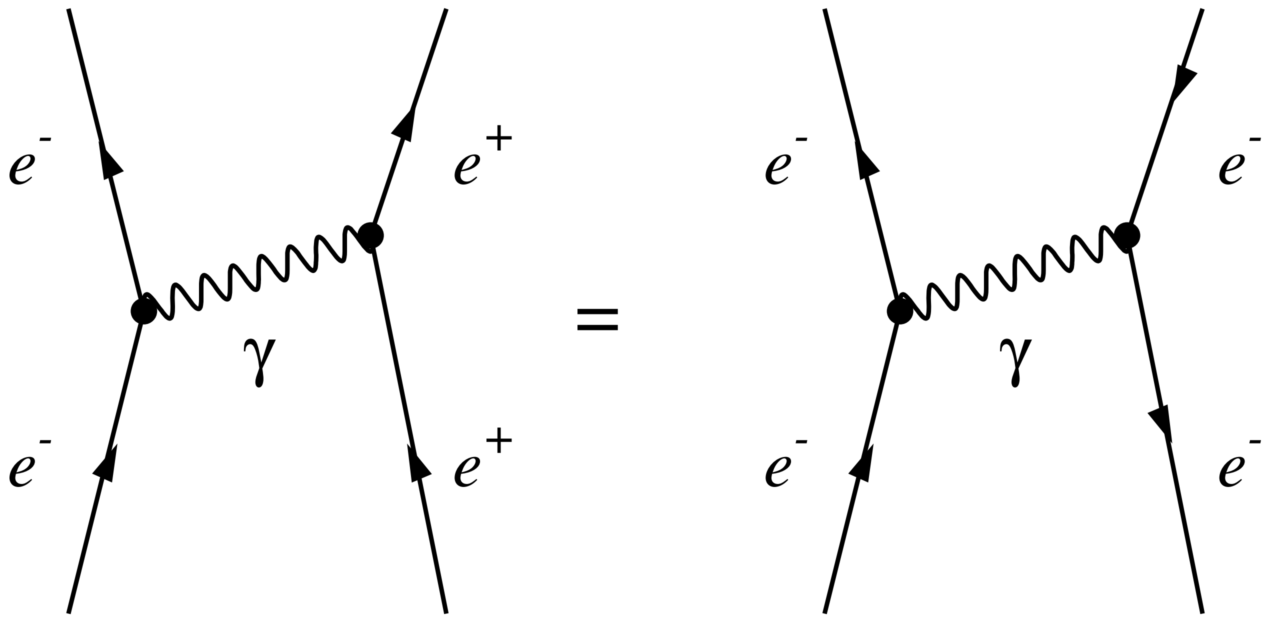
Particelle scambiate virtuali. Non soddisfano $E^2 = \mathbf{p}^2c^2 + m^2c^4$.

I punti che uniscono 3 o più linee sono vertici.

Costante di accoppiamento dell'interazione.

Propagatore $1/(\mathbf{q}^2 - \omega^2 + M^2c^2)$.

Mie convenzioni



Bibliografia

Approfondimenti degli argomenti presentati possono essere trovati nei cap. 1, 2 e 3 del libro

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