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**Recent progresses in
nuclear many–body theories**

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Lecce

Building a Many–Body Theory

Basic Elements

- ❶ Elementary components
- ❷ Interaction

Theoretical framework

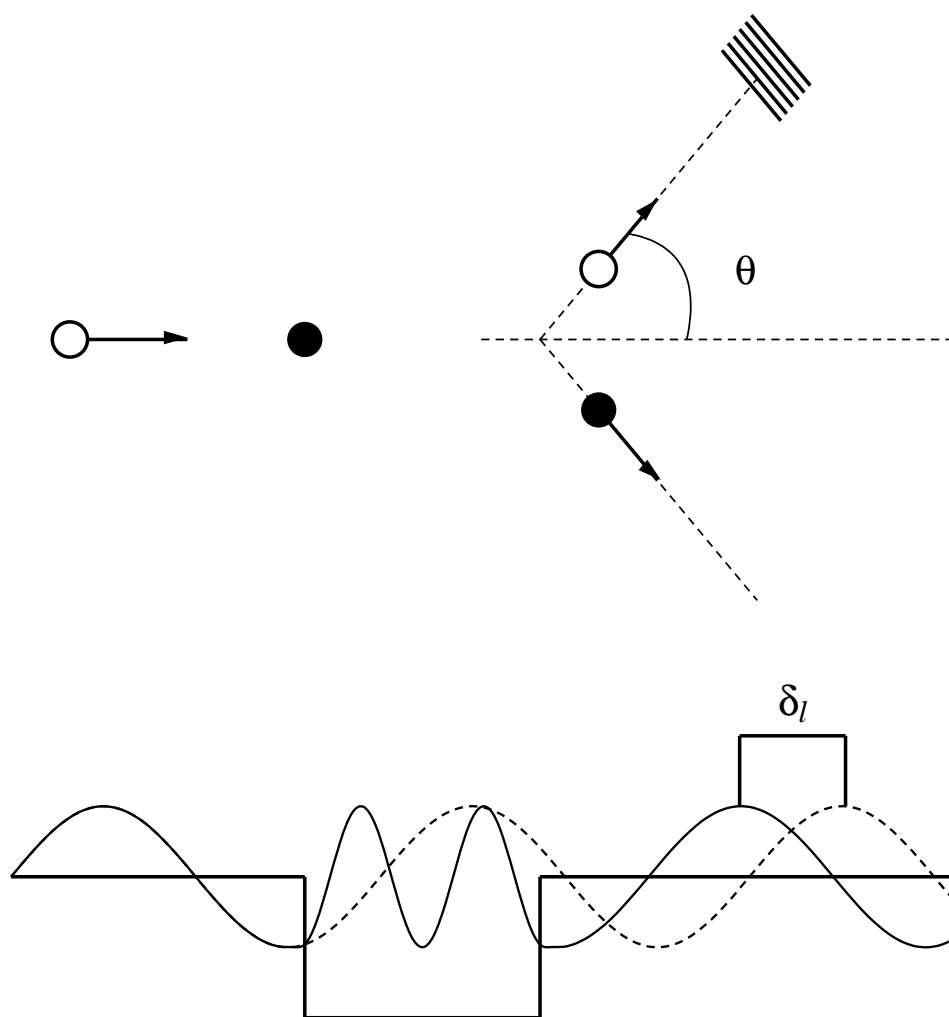
- ❶ Quantum or classical
- ❷ Relativistic or non-relativistic

Atomic Nuclei

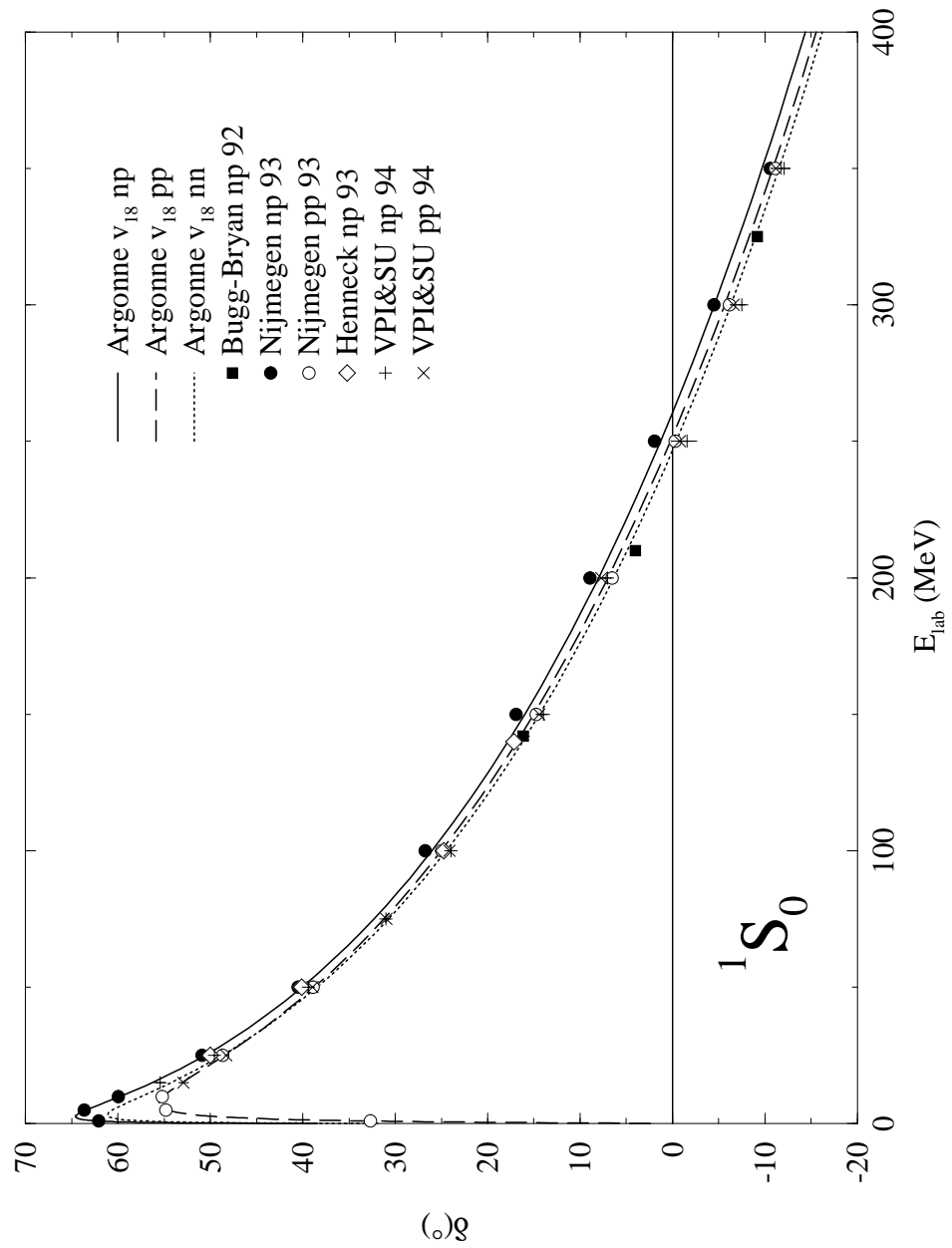
Elementary components: nucleons and mesons
Interactions: strong, e-m and weak interactions
Theoretical framework: non-relativistic quantum mechanics

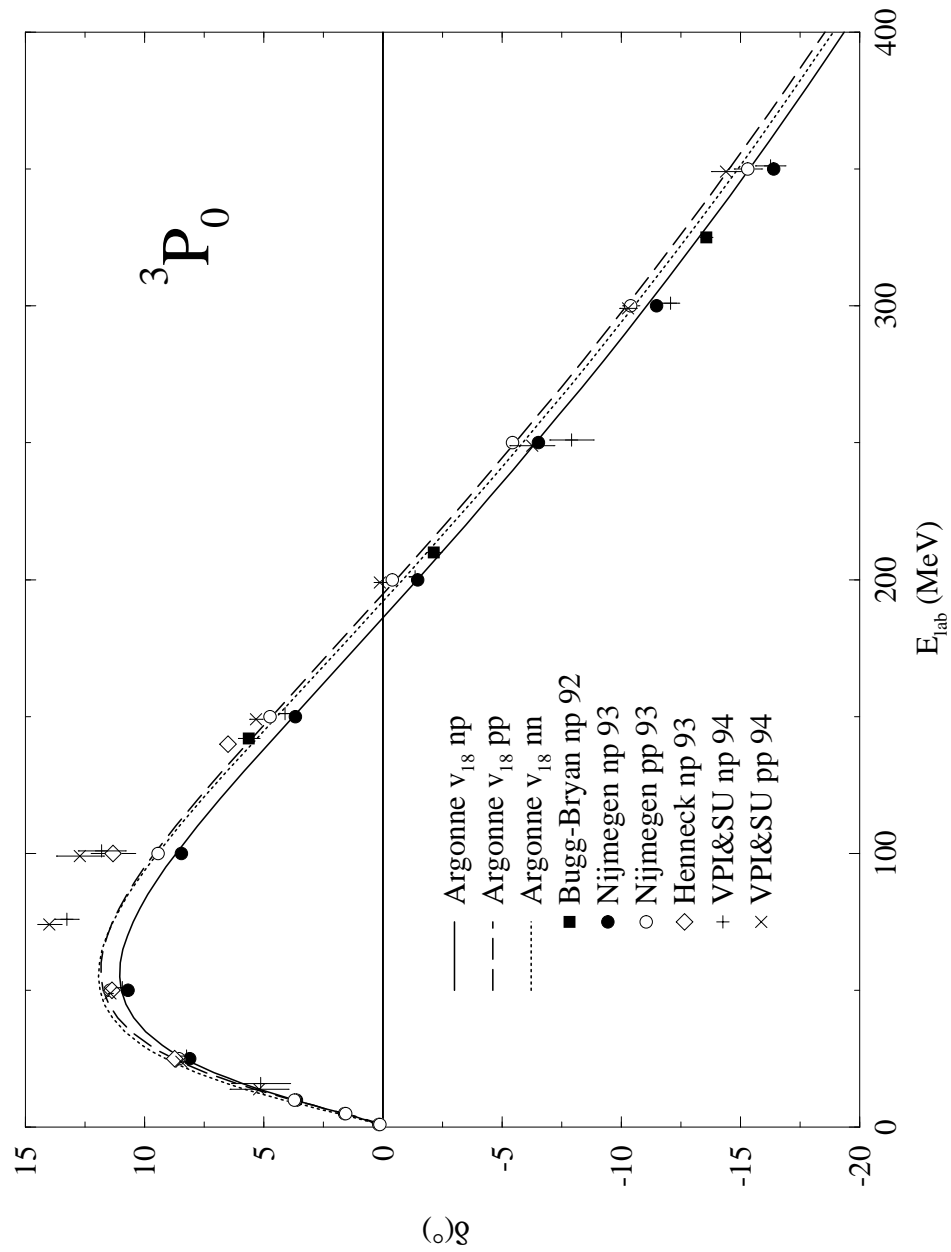
Many-Body Schrödinger Equation

$$H|\Psi\rangle = (T + V)|\Psi\rangle = E|\Psi\rangle$$



$$\frac{d\sigma}{d\theta} = \left| \sum_{L=0}^{\infty} (2L+1) e^{i\delta_L} \sin \delta_L P_L(\cos\theta) \right|^2$$



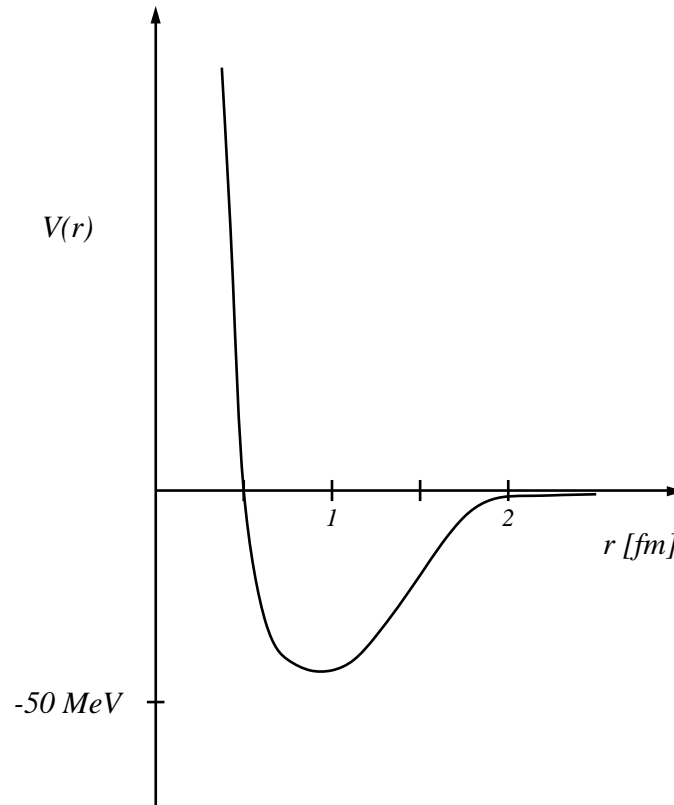


The nucleon - nucleon interaction

$$V(i, j) = \sum_{p=1,18} v_p(r_{ij}) O_{ij}^p$$

$$O_{ij}^{p=1,14} = [1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, (\mathbf{L} \cdot \mathbf{S}), \mathbf{L}^2, \mathbf{L}^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), (\mathbf{L} \cdot \mathbf{S})^2] \\ \otimes [1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j]$$

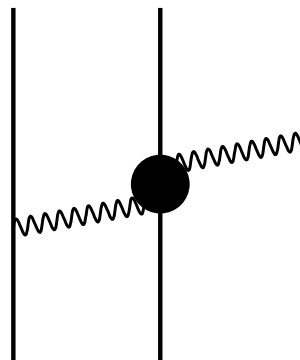
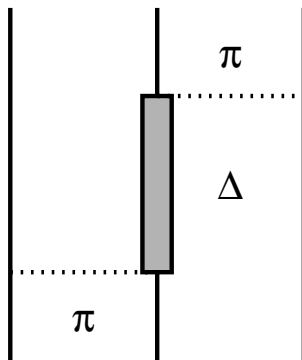
$$O_{ij}^{p=15,18} = [1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij},] \otimes [T_{ij}, \tau_{zi} + \tau_{zj}]$$



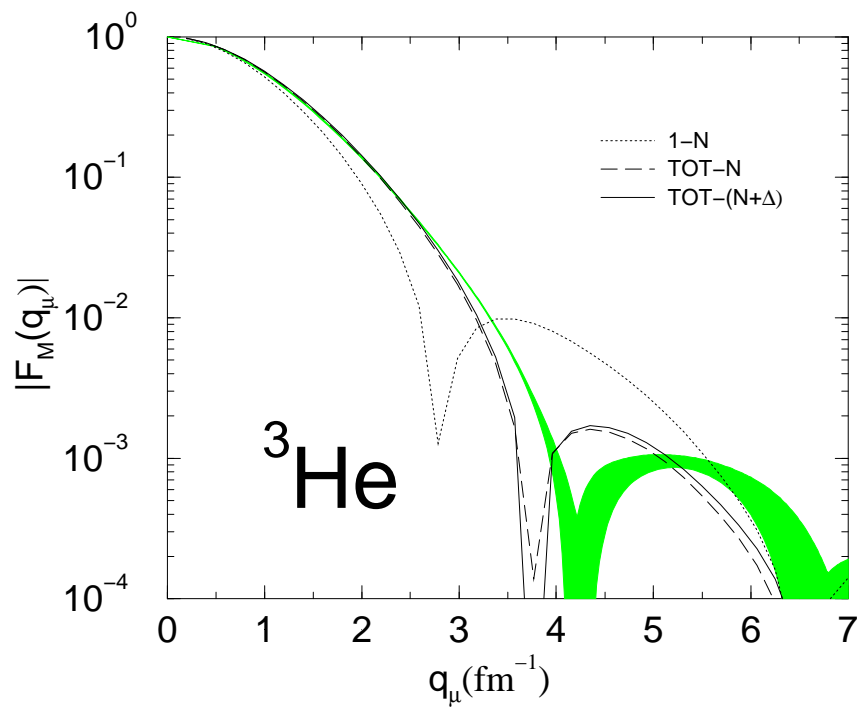
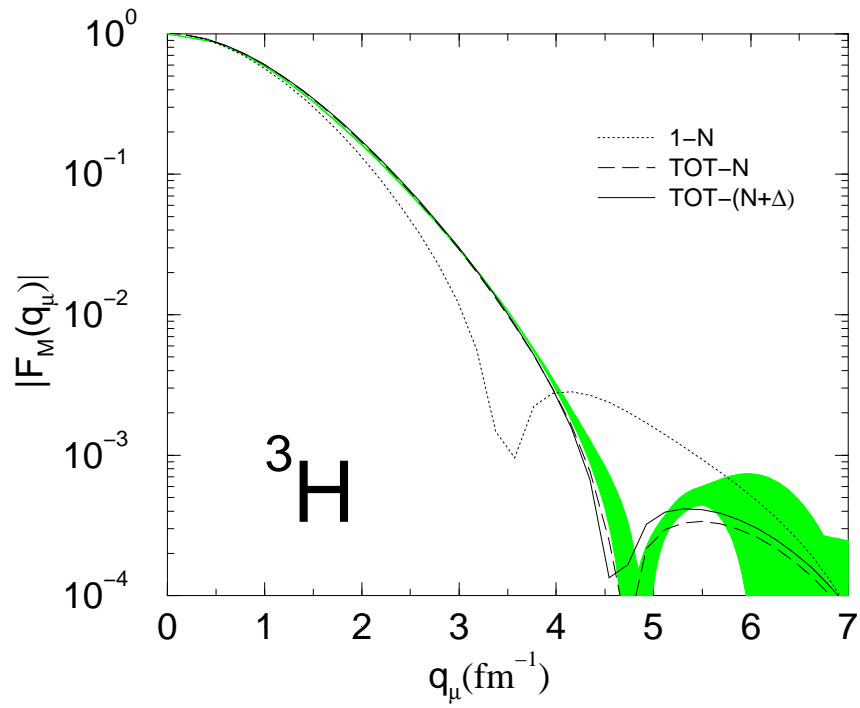
^3H Binding energy

Exp 8.48 MeV

2 N potential	2N	2N + 3N
CD Bonn	7.953	8.483
Nijm II	7.709	8.477
Nijm I	7.731	8.480
Nijm 93	7.664	8.480
Reid 93	7.648	8.480
AV14	7.683	8.480
AV18	7.576	8.479



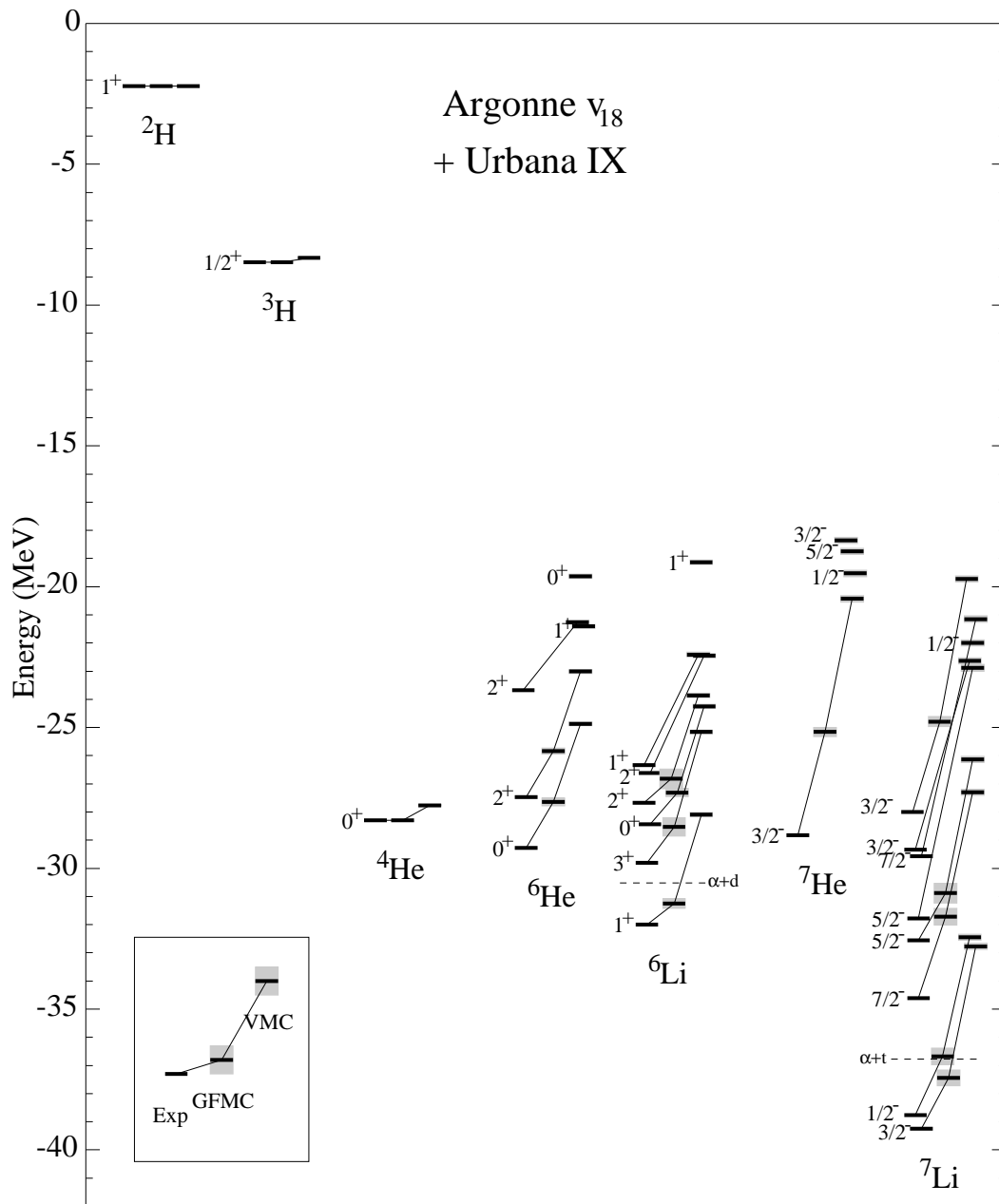
L. Marcucci, D. O. Riska and R. Schiavilla, Phys. Rev. C 58 (1998) 3069.



B.S. Pudliner, V.R. Pandharipande, J. Carlson, S.C. Pieper, R.B. Wiringa
Phys. Rev. C 56 (1997) 1720.

Argonne V18 plus Urbana IX interactions

	Exp	GFMC	VMC
^3H	8.48	8.47(1)	8.32(1)
^4He	28.30	28.30(2)	27.76(3)
^6He	29.27	27.64(14)	24.87(7)
^6Li	31.99	31.25(11)	28.09(7)
^6Be	26.92	25.52(11)	22.79(7)
^7He	28.82	25.16(16)	20.43(12)
^7Li	39.24	37.44(28)	32.78(11)



Number of spin–isospin configurations

$$N_{conf} = 2^A \frac{A!}{Z!(A-Z)!}$$

Nucleus	Z	N=A-Z	N_{conf}
3H	1	2	24
3He	2	1	24
4He	2	2	96
6He	2	4	960
6Li	3	3	1280
8He	2	6	7168
${}^{12}C$	6	6	3784704
${}^{16}O$	8	8	$\sim 8.4 \cdot 10^8$
${}^{40}Ca$	20	20	$\sim 1.5 \cdot 10^{23}$
${}^{48}Ca$	20	28	$\sim 4.7 \cdot 10^{27}$

Independent Particle Model

$$\begin{aligned}
 H &= \sum_i t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} v_{ijk} + \sum_i (u_i - u_i) \\
 &= \sum_i (t_i + u_i) + \sum_{i < j} v_{ij} + \sum_{i < j < k} v_{ijk} - \sum_i u_i \\
 &= H_0 + V_{res}
 \end{aligned}$$

$$E_0 |\Phi\rangle = H_0 |\Phi\rangle = \sum_i h_i |\Phi\rangle$$

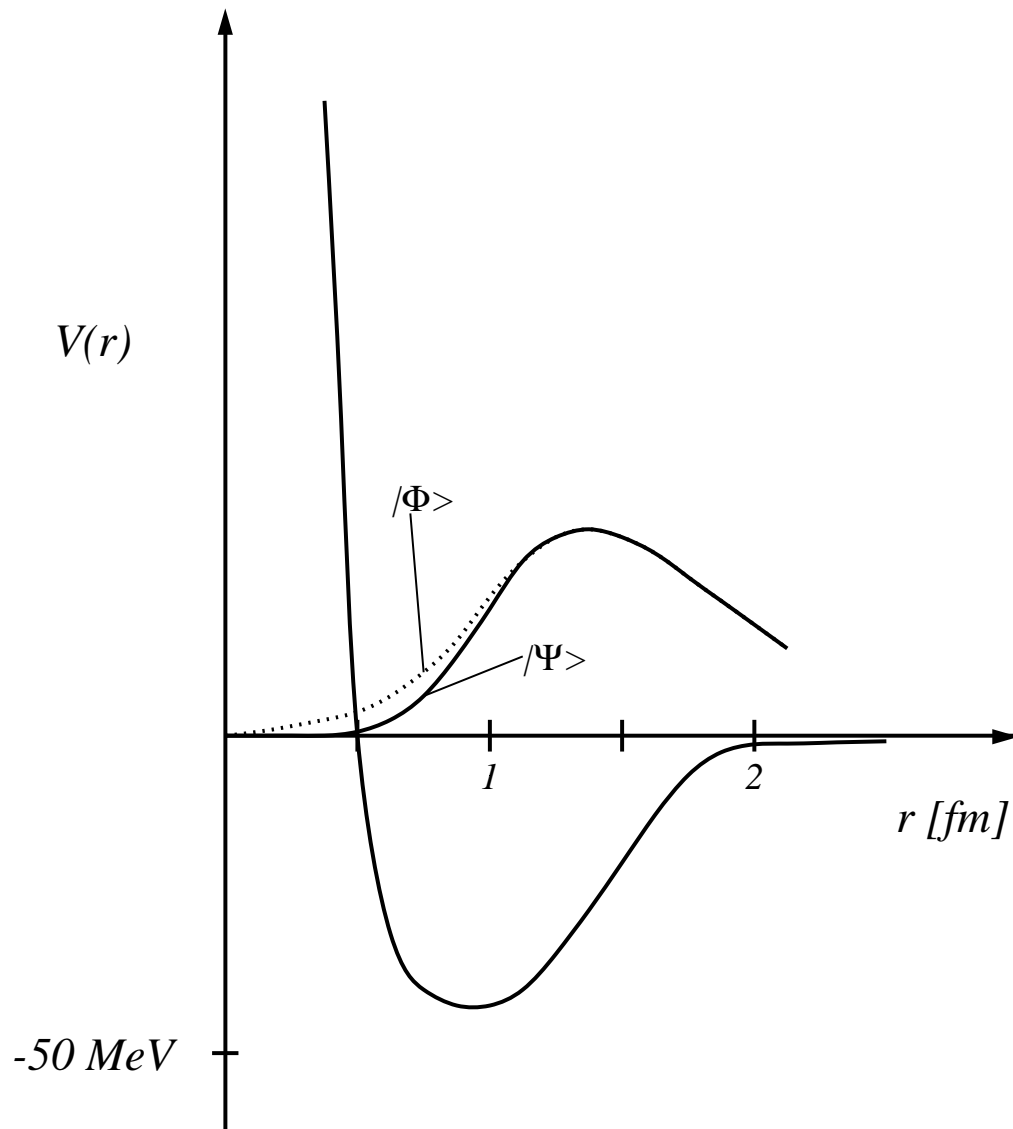
$$|\Phi\rangle = \det \left[|\phi_i\rangle \right]$$

$$h_i |\phi_i\rangle = \epsilon_i |\phi_i\rangle$$

Many-body problem \longrightarrow many one-body problems

Perturbation Theory

$$E = E_0 + \langle \Phi | V_{res} \sum_{n=0}^{\infty} \left(\frac{1}{E_0 - H_0} V_{res} \right)^n | \Phi \rangle_c$$



G - Matrix

$$\begin{aligned}
 G &= V + V \frac{Q}{e} G \\
 &= V + V \frac{Q}{e} \left(V + V \frac{Q}{e} (V + \dots) \right)
 \end{aligned}$$

$$\left| \begin{array}{c} G \\ \text{~~~~~} \end{array} \right| =$$

$$\left| \begin{array}{c} V \\ \text{-----} \end{array} \right| + \left| \begin{array}{c} V \\ \text{-----} \\ V \end{array} \right| + \left| \begin{array}{c} \text{-----} \\ \text{-----} \\ \text{-----} \end{array} \right| + \dots$$

CBF FHNC–SOC

$$\delta E[\Psi] = \delta \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0$$

$$\Psi(1, 2...A) = G(1, 2...A)\Phi(1, 2...A)$$

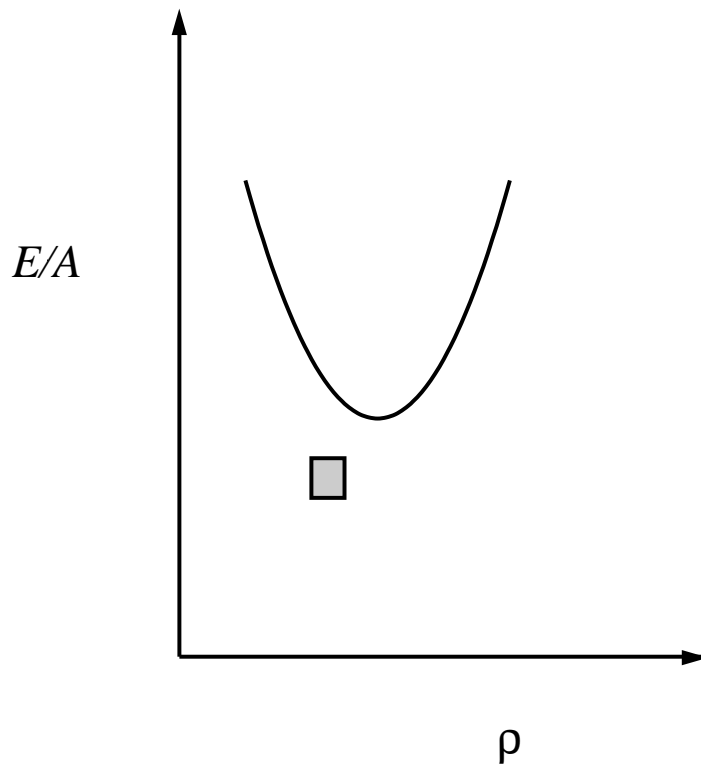
$$G(1, 2...A) = \prod_{i < j} F_{ij}$$

$$F_{ij} = \sum_{p=1,8} f^p(r_{ij}) O_{ij}^p$$

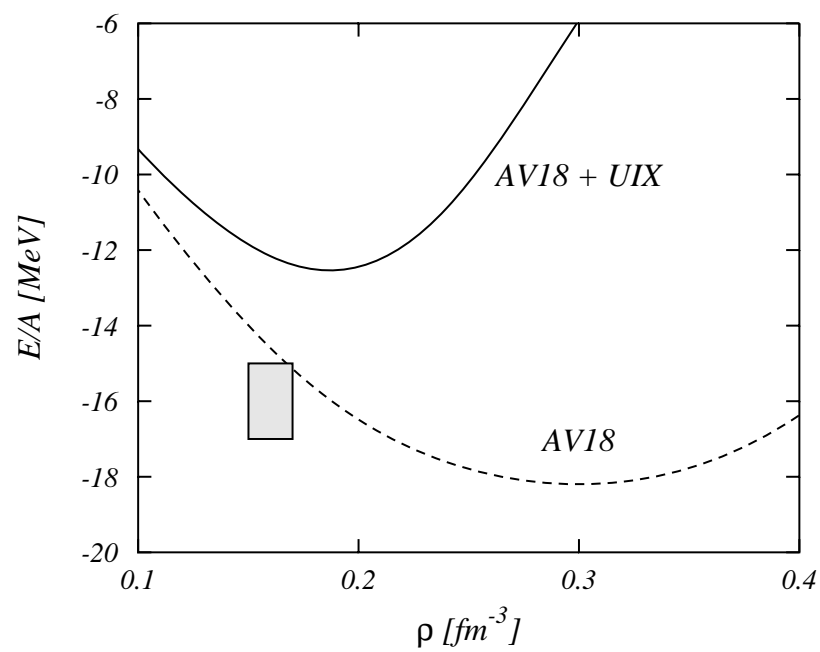
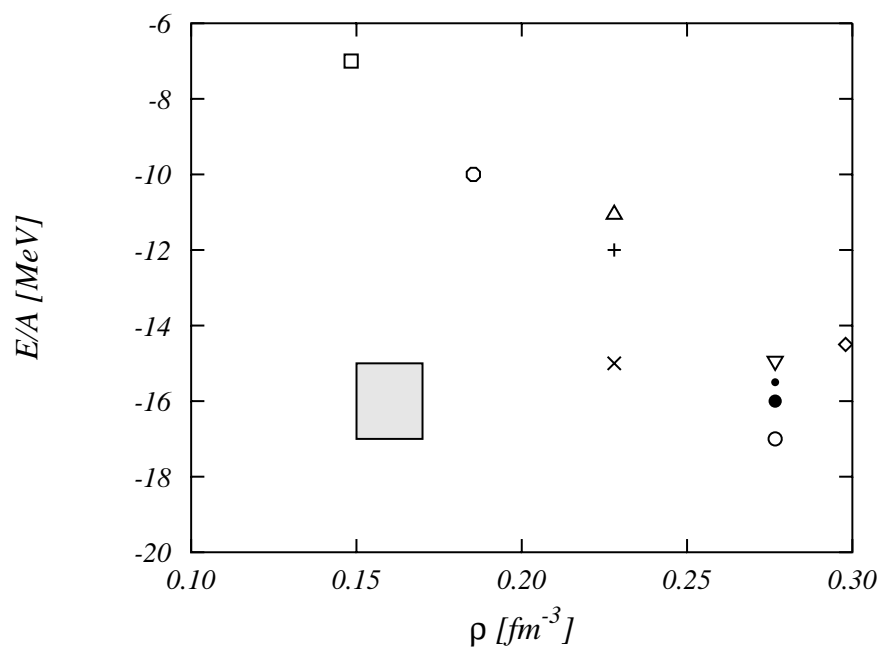
$$O_{ij}^{p=1,8} = [1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, (\mathbf{L} \cdot \mathbf{S})_{ij}] \otimes [1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j]$$

Nuclear matter

- Infinite system of nucleons
- $N=Z$
- No Coulomb interaction



$$\frac{E}{A} = -16.0 \pm 1.0 \text{ MeV} \quad \rho = 0.16 \pm 0.01 \text{ fm}^{-3}$$



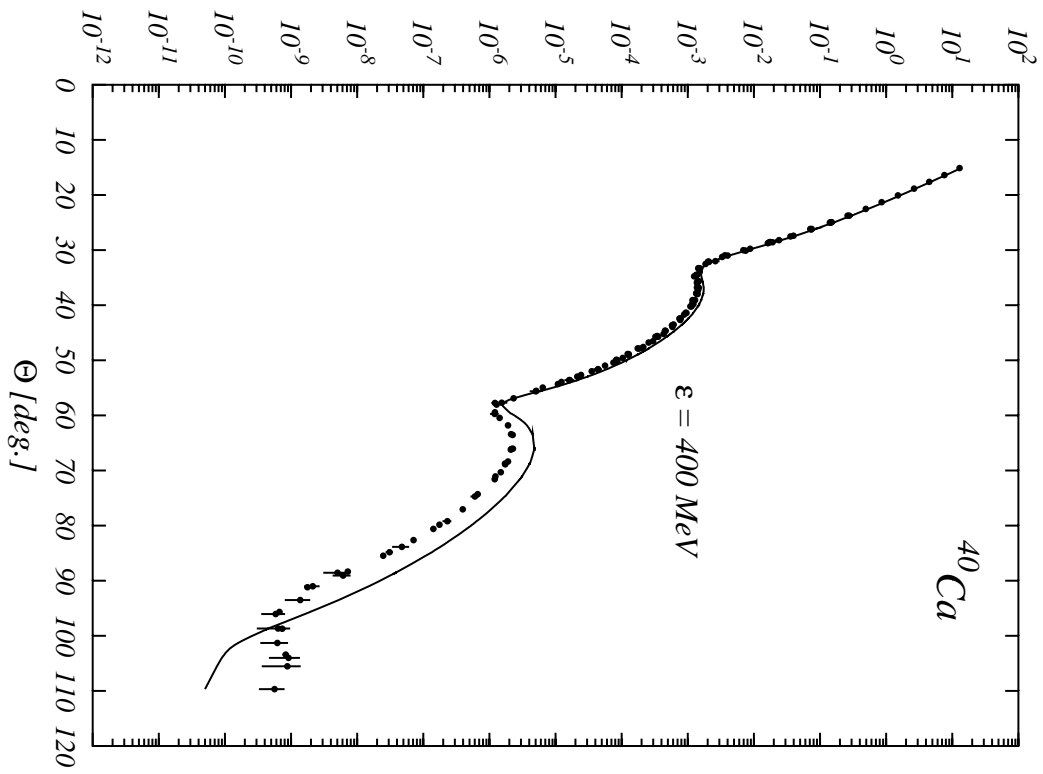
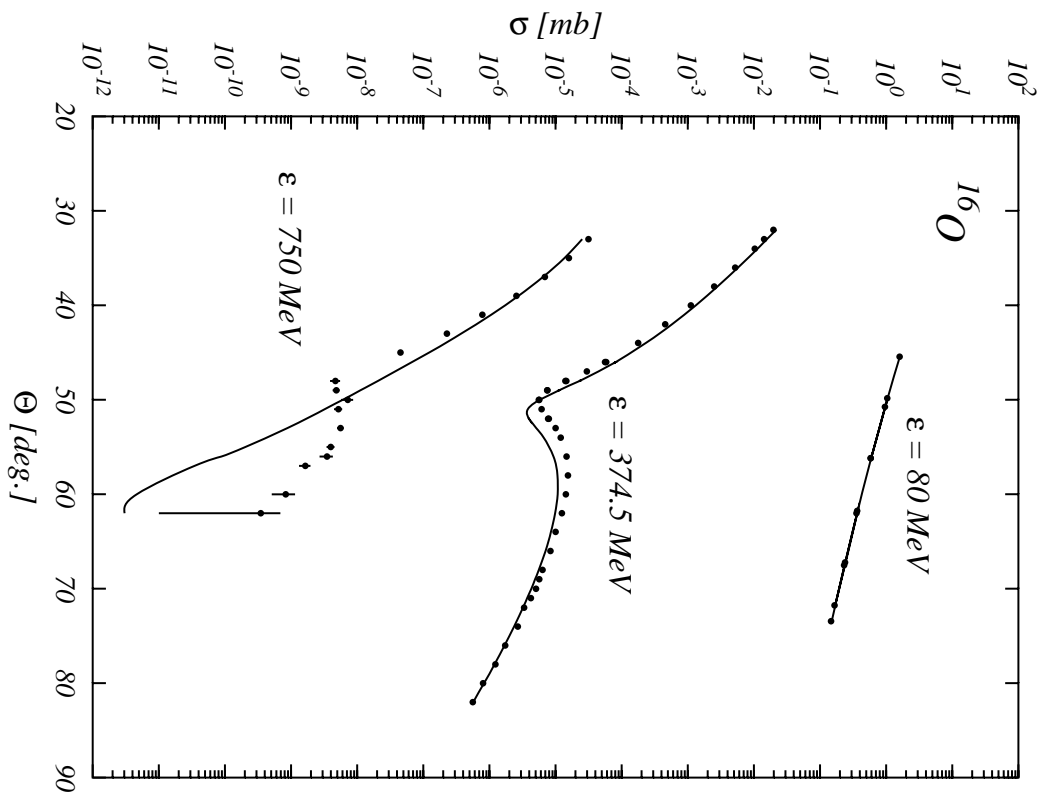
Finite nuclei

Energies per nucleons in MeV

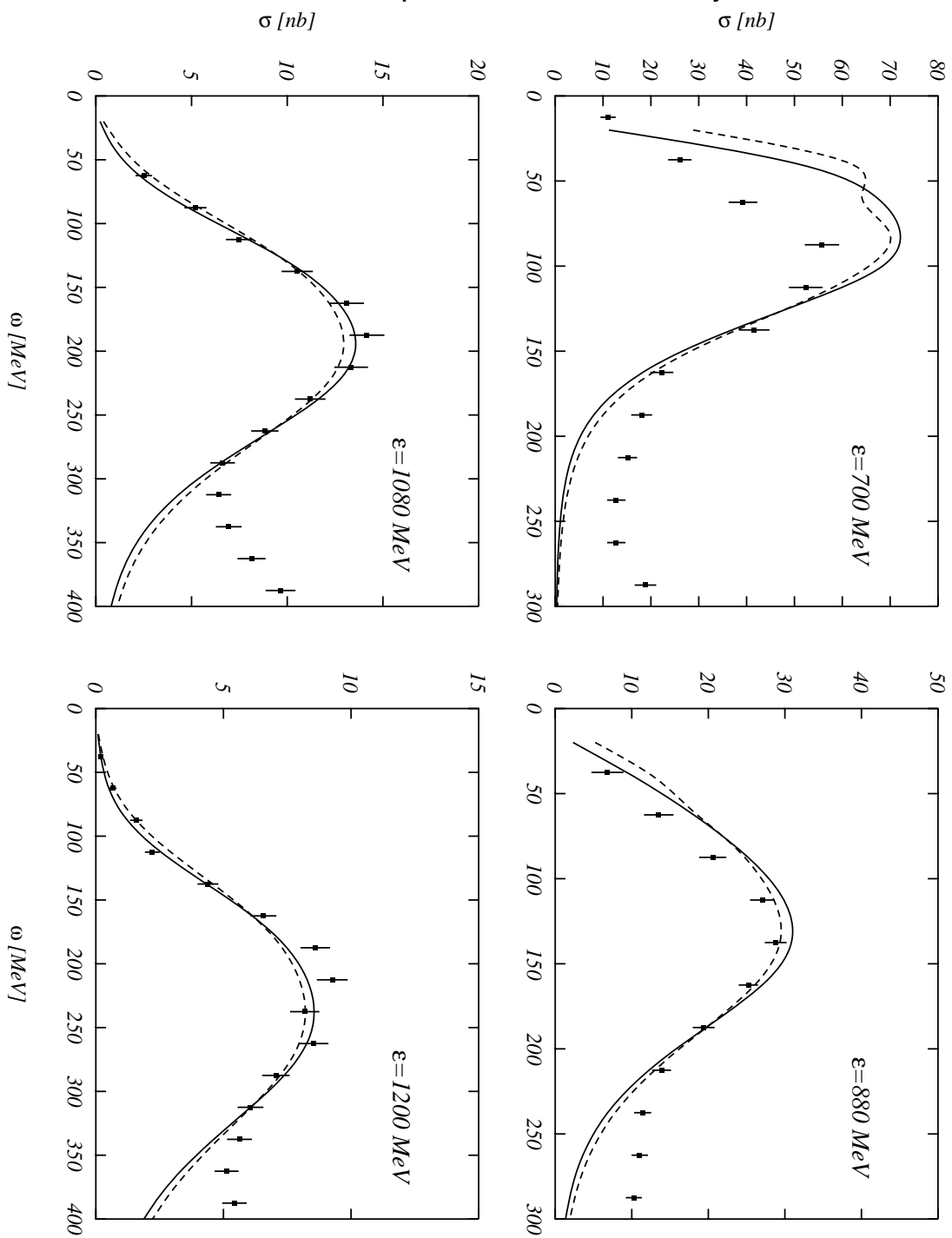
	^{16}O	^{40}Ca
$\langle T \rangle$	32.64	38.15
$\langle v_{8,ij} \rangle$	-37.79	-46.34
$\langle v_{ijk}^{2\pi} \rangle$	-2.36	-2.98
$\langle v_{ijk}^R \rangle$	3.00	3.94
E_{cm}	0.64	0.23
$\langle v_{Coul} \rangle$	0.94	2.10
ΔE	-1.21	-1.28
E	-5.41	-6.64
$\langle R \rangle$ [fm]	2.67	3.39
E_{exp}	-7.97	-8.55
$\langle R \rangle$ [fm]	2.73	3.48

A. Fabrocini, F. Arias de Saavedra, and G. C.

Phys. Rev. C 61 (2000) 044302



G. C. and A.M. Lallena, to be published on Ann. Phys.



G. C. and A.M. Lallena, to be published on Ann. Phys.

