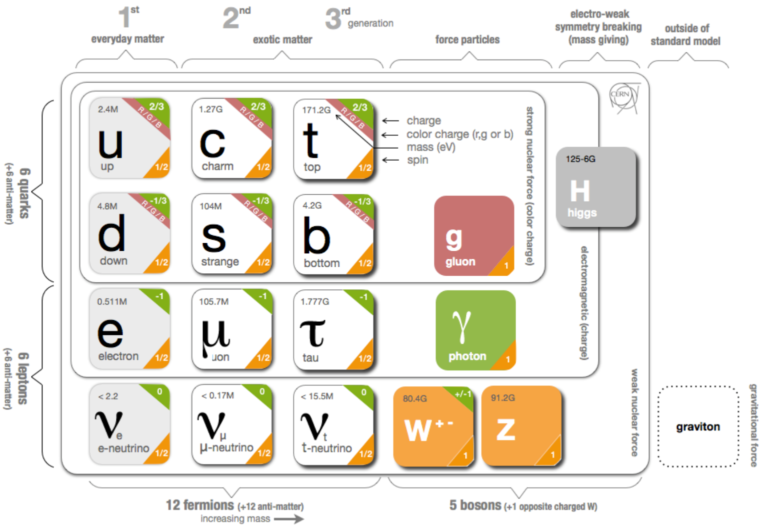
Il problema a molti corpi in meccanica quantistica

Giampaolo Co'

Lecce 14 Dicembre 2016

Luca Signorelli, Orvieto (1499-1504)



1st everyday matter
2nd exotic matter
3rd generation
force particles
electro-weak symmetry breaking (mass giving)
outside of standard model
6 quarks (+6 anti-matter)
6 leptons (+6 anti-matter)
2.4M 2/3 R/G/B u up 1/2
1.27G 2/3 R/G/B c charm 1/2
171.2G 2/3 R/G/B t top 1/2
charge
color charge (r,g or b)
mass (eV)
spin
strong nuclear force (color charge)
125-6G H higgs
4.8M -1/3 R/G/B d down 1/2
104M -1/3 R/G/B s strange 1/2
4.2G -1/3 R/G/B b bottom 1/2
g gluon 1
electromagnetic (charge)
0.511M -1 e electron 1/2
105.7M -1 μ uon 1/2
1.777G -1 τ tau 1/2
γ photon 1
weak nuclear force
< 2.2 0 ν e-neutrino 1/2
< 0.17M 0 ν μ-neutrino 1/2
< 15.5M 0 ν t-neutrino 1/2
80.4G +/-1 W+- 1
91.2G Z 1
graviton
gravitational force
12 fermions (+12 anti-matter)
increasing mass
5 bosons (+1 opposite charged W)

Quantità da definire

- Scala di energie
  (sistema non-relativistico).
- Particelle che formano il sistema
  (prive di struttura interna).
- Interazioni
  (effettive).

Equazione di Schrödinger per una particella

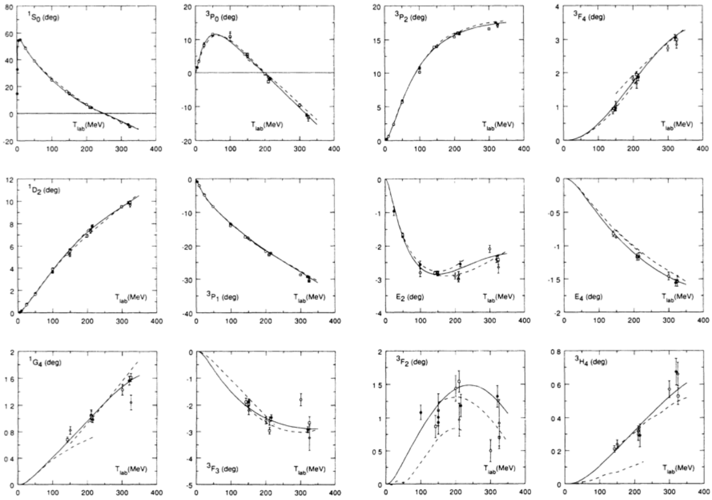
$$\Big[-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r})\Big]\phi_a(\mathbf{r}) = \epsilon_a \phi_a(\mathbf{r})$$

Densità

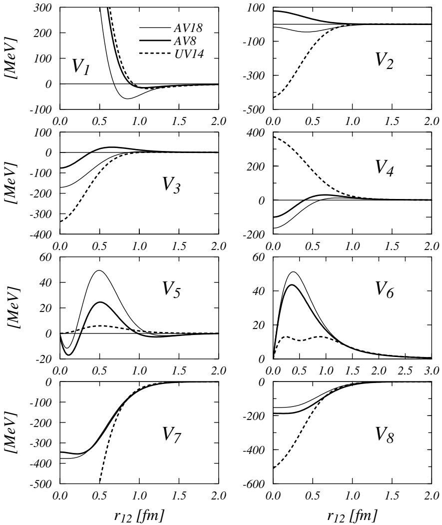
$$\rho(\mathbf{r}) = |\phi_a(\mathbf{r})|^2 \quad ; \quad \int d^3r\, \rho(\mathbf{r}) = 1$$

Osservabile

$$D_{a,b} \sim |\langle \phi_a | \mathcal{M} | \phi_b \rangle|^2 = \left| \int d^3r\, \phi_a^*(\mathbf{r}) \mathcal{M}(r) \phi_b(\mathbf{r}) \right|^2$$



$^1S_0$ (deg)
$^3P_0$ (deg)
$^3P_2$ (deg)
$^3F_4$ (deg)
$^1D_2$ (deg)
$^3P_1$ (deg)
E_2 (deg)
E_4 (deg)
$^1G_4$ (deg)
$^3F_3$ (deg)
$^3F_2$ (deg)
$^3H_4$ (deg)
T_{lab}(MeV)


AV18
AV8
UV14
V_1
V_2
V_3
V_4
V_5
V_6
V_7
V_8
[MeV]
r_{12} [fm]

Equazione di Schrödinger per molte particelle

$$\Big[\sum_{i=1}^{N}\Big(-\frac{\hbar^2}{2m}\nabla_i^2 + U(i)\Big) + \frac{1}{2}\sum_{i,j=1}^{N} V(i,j)\Big]\Psi(1,2,\cdots,N)$$
$$= \quad H\,\Psi(1,2,\cdots,N) = E\,\Psi(1,2,\cdots,N)$$

Densità

$$\rho(1,2,\cdots) = |\Psi(1,2,\cdots,N)|^2 \quad ; \quad \int d^3r_1 d^3r_2 \cdots \rho(1,2,\cdots) = 1$$

Osservabile

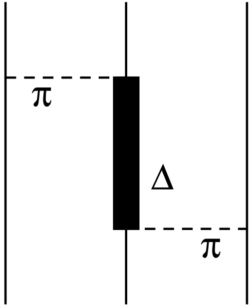
$$D_{A,B} \sim |\langle\Psi_A|\mathcal{M}|\Psi_B\rangle|^2$$
$$= \quad \left|\int d^3r_1 d^3r_2 \cdots \Psi_A^*(1,2,\cdots,N)\mathcal{M}(r)\Psi_B(1,2,\cdots,N)\right|^2$$

^3H Binding energy

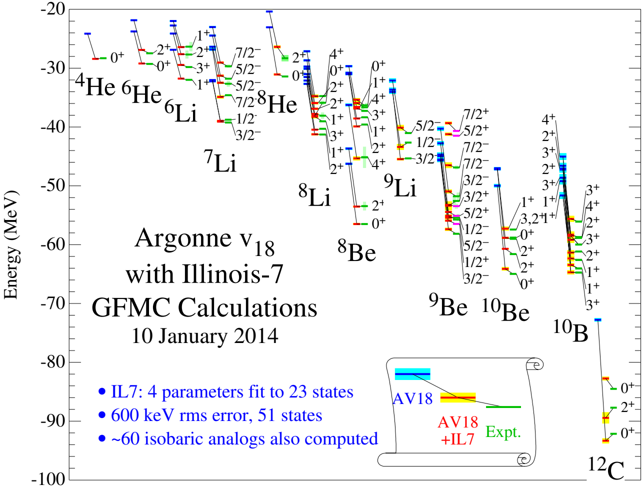
Exp 8.48 MeV

2 N potential	2N	2N + 3N
CD Bonn	7.953	8.483
Nijm II	7.709	8.477
Nijm I	7.731	8.480
Nijm 93	7.664	8.480
Reid 93	7.648	8.480
AV14	7.683	8.480
AV18	7.576	8.479

Forza a 3 corpi



S.C. Pieper and R.B. Wiringa, Ann. Rev. Nucl. Part. Sci. 51 (2001) 53.



Numero di configurazioni di spin e isospin per alcuni nuclei.

$$N_{conf} = 2^A \frac{A!}{Z!(A-Z)!}$$

Nucleo	Z	N=A-Z	N_{conf}
^3He	2	1	24
^4He	2	2	96
^6He	2	4	960
^6Li	3	3	1280
^8He	2	6	7168
^{12}C	6	6	3,784,704
^{16}O	8	8	$8.4 \cdot 10^8$
^{40}Ca	20	20	$1.5 \cdot 10^{23}$
^{48}Ca	20	28	$4.7 \cdot 10^{27}$

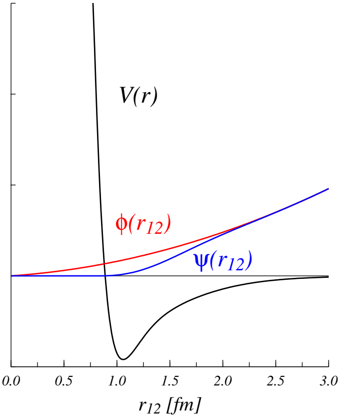
Teoria perturbativa

$$H = H_0 + H_1$$

$$H_0|\Phi_0\rangle = E_0|\Phi_0\rangle$$

$$H_0|\Phi_0\rangle = \sum_i h_i \prod_{k=1,A} |\phi_k\rangle = \sum_i \epsilon_i \prod_{k=1,A} |\phi_k\rangle$$

$$\begin{aligned} E &= \langle\Phi_0|H_0|\Phi_0\rangle + \langle\Phi_0|H_1 \sum_{n=0}^{\infty} \left(\frac{1}{E_0 - H_0} H_1 \right)^n |\Phi_0\rangle_c \\ &= E_0 + \langle\Phi_0|H_1|\Phi_0\rangle + \langle\Phi_0|H_1 \frac{1}{E_0 - H_0} H_1 |\Phi_0\rangle_c + \cdots \end{aligned}$$



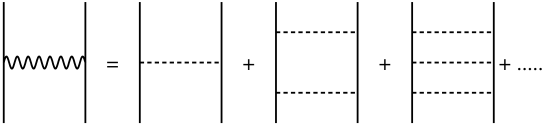
$$\lim_{V\to\infty} V(r)\,\psi(r) \to \textit{finito}$$

$$\lim_{V\to\infty} V(r)\,\phi(r) \to \infty$$

Interazione effettiva

$$G|\Phi_0\rangle = V|\Psi_0\rangle$$

$$\begin{aligned}\langle\Phi_0|G(\omega)|\Phi_0\rangle &= \langle\Phi_0|H_1|\Phi_0\rangle \\ &+ \sum_{p,q>F} \frac{\langle\Phi_0|H_1|\Phi_{pq}\rangle\langle\Phi_{pq}|G(\omega)|\Phi_0\rangle}{\omega - \epsilon_p - \epsilon_q}\end{aligned}$$

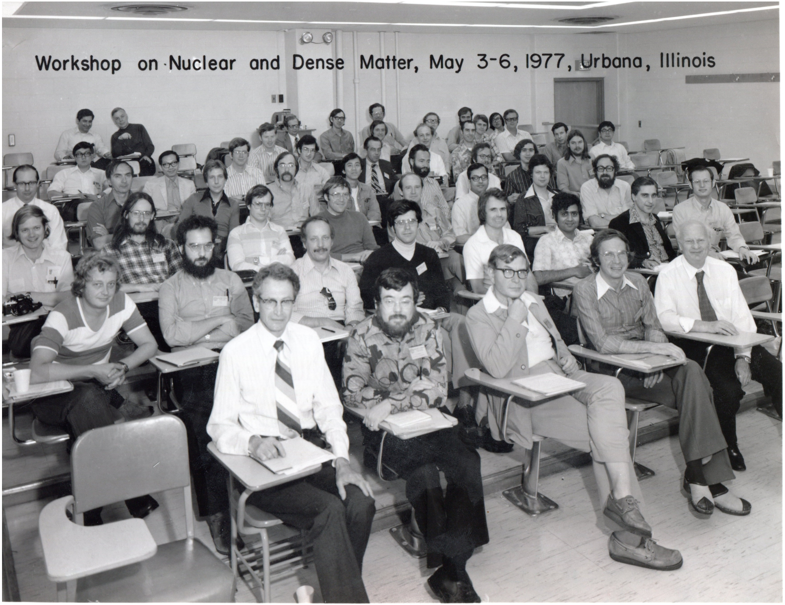


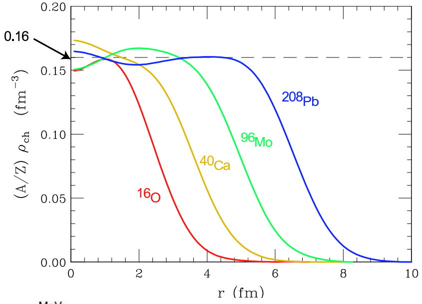
G

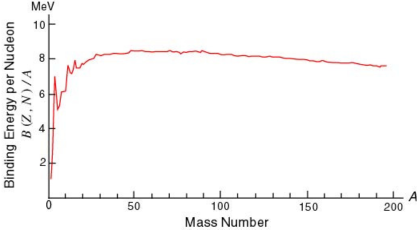
Principio Variazionale

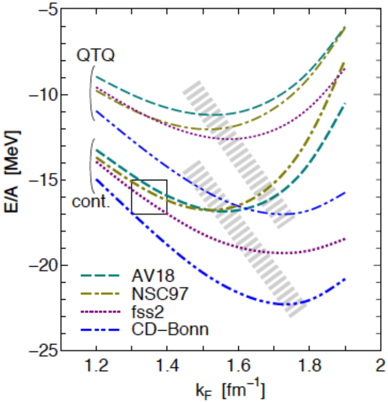
$$\Psi(1,2,...,A) = F(1,2,...,A)\Phi(1,2,....,A)$$

$$\delta E[\Psi] = \delta \left[\frac{<\Psi|H|\Psi>}{<\Psi|\Psi>} \right] = \delta \left[\frac{<\Phi|F^\dagger H F|\Phi>}{<\Psi|\Psi>} \right] = 0$$

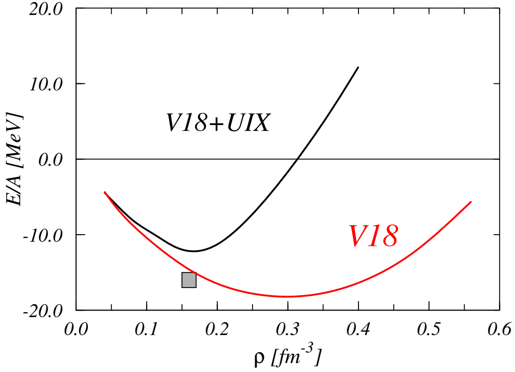

Workshop on Nuclear and Dense Matter, May 3-6, 1977, Urbana, Illinois


0.16
0.20
0.15
0.10
0.05
0.00
(A/Z) ρ_{ch} (fm^{-3})
0
2
4
6
8
10
r (fm)
208Pb
96Mo
40Ca
16O


MeV
10
8
6
4
2
Binding Energy per Nucleon
$B(Z,N)/A$
0
50
100
150
200
A
Mass Number



QTQ
cont.
E/A [MeV]
k_F [fm^{-1}]
-5
-10
-15
-20
-25
1.2
1.4
1.6
1.8
2
AV18
NSC97
fss2
CD–Bonn


20.0
10.0
0.0
-10.0
-20.0
E/A [MeV]
V18+UIX
V18
0.0
0.1
0.2
0.3
0.4
0.5
0.6
ρ [fm-3]

F. Arias de Saavedra, C. Bisconti, G. Co' and A. Fabrocini

Renormalized Fermi hypernetted chain approach in medium-heavy nuclei. Phys. Rep. 450 (2007) 1

		^{12}C	^{16}O	^{40}Ca	^{48}Ca	^{208}Pb
v'_8 + *UIX*	T	27.13	32.33	41.06	39.64	39.56
	V^6_{2-body}	-29.13	-38.15	-48.97	-46.60	-48.43
	V_{LS}	-0.51	-0.70	-0.85	-0.79	-0.80
	V_{Coul}	0.67	0.86	1.96	1.57	3.97
	$T + V(2)$	-1.84	-5.66	-6.83	-6.24	-5.80
	V_{3-body}	0.66	0.86	1.76	1.61	1.91
	E	-1.17	-4.80	-5.05	-4.62	-3.78
	E_{exp}	-7.68	-7.97	-8.55	-8.66	-7.86

Hartree-Fock

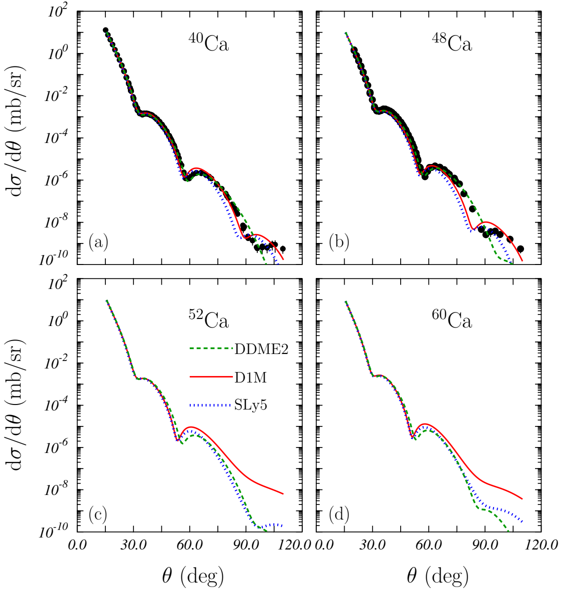
$$h(i)\,\phi_i(x_i) = -\frac{\hbar^2}{2m_i}\nabla_i^2\phi_i(x_i) + \mathcal{U}(x_i)\,\phi_i(x_i) - \int \mathcal{W}(x_i, x_k)\,\phi_i(x_k) d\,x_k = \epsilon_i\,\phi_i(x_i)$$

Hartree

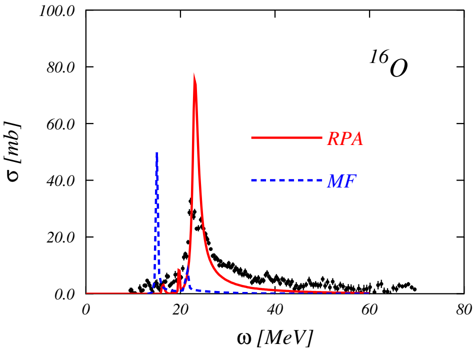
$$\mathcal{U}(x_i) = \sum_k \int \phi_k^*(x_k) V_{\text{eff}}(x_i, x_k) \phi_k(x_k) d\,x_k$$

Fock-Dirac

$$\mathcal{W}(x_i, x_k) = \sum_k \phi_k^*(x_i) V_{\text{eff}}(x_i, x_k) \phi_k(x_k)$$



^{40}Ca
^{48}Ca
^{52}Ca
^{60}Ca
(a)
(b)
(c)
(d)
DDME2
D1M
SLy5
$d\sigma/d\theta$ (mb/sr)
θ (deg)


100.0
80.0
60.0
40.0
20.0
0.0
σ [mb]
0
20
40
60
80
ω [MeV]
16O
RPA
MF