The Fermi Hypernetted Chain method for finite systems

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$$\delta E[\Psi] = \delta \left[\frac{< \Psi | H | \Psi >}{< \Psi | \Psi >} \right] = 0$$

$$\Psi(1,2,...,A) = F(1,2,...,A)\Phi(1,2,....,A)$$

$$F(1,2,...,A) = \prod_{i<j} f(r_{ij})$$

Two-body distribution function

$$g(x_1, x_2) = \frac{A(A-1)\int dx_3 \ldots dx_A \Psi^*(x_1, x_2, \ldots, x_A)\Psi(x_1, x_2, \ldots, x_A)}{\rho^2 \int dx_1 dx_2 \ldots dx_A \Psi^*(x_1, \ldots, x_A)\Psi(x_1, \ldots, x_A)}$$

$$< O > = \frac{1}{2}\rho^2 \int dx_1 dx_2 \, g(x_1, x_2) \, O(x_1, x_2)$$

$$g(x_1,x_2) = \frac{A(A-1)\int dx_3 \ldots dx_A \Phi^*(x_1,\ldots,x_A) F^* F \Phi(x_1,\ldots,x_A)}{\rho^2 \int dx_1 dx_2 \ldots dx_A \Phi^*(x_1,\ldots,x_A) F^* F \Phi(x_1,\ldots,x_A)}$$

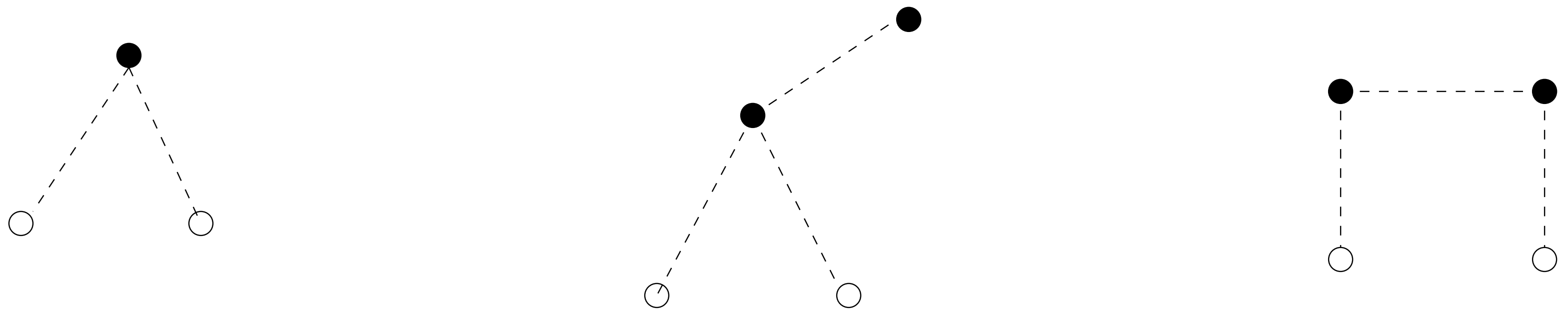
$$F^* F = \prod_{i<j} f^2(r_{ij})$$

$$f^2(r_{ij}) = 1 + h(r_{ij})$$

$$\prod_{i<j} f^2(r_{ij}) = f^2(r_{12})[1+h(r_{13})][1+h(r_{14})]....[1+h(r_{34})]....$$

$$\mathcal{N} \sim f^2(r_{12})\Big[1 + \mathcal{A}\sum_{j>2}\int dx_j h(r_{1j}) + \mathcal{B}\sum_{j>i>2}\int dx_i dx_j h(r_{ij}) + \ldots\Big]$$

LINKED



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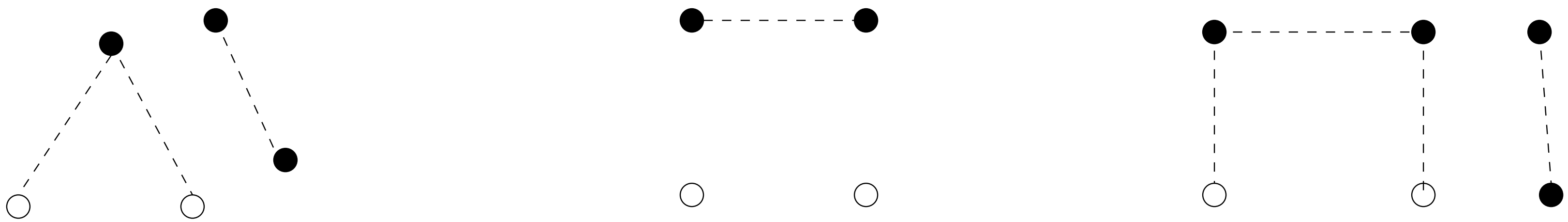
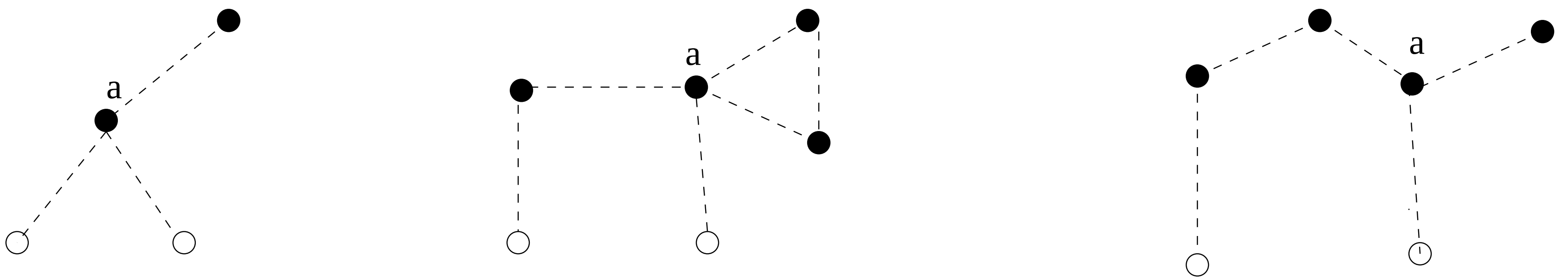
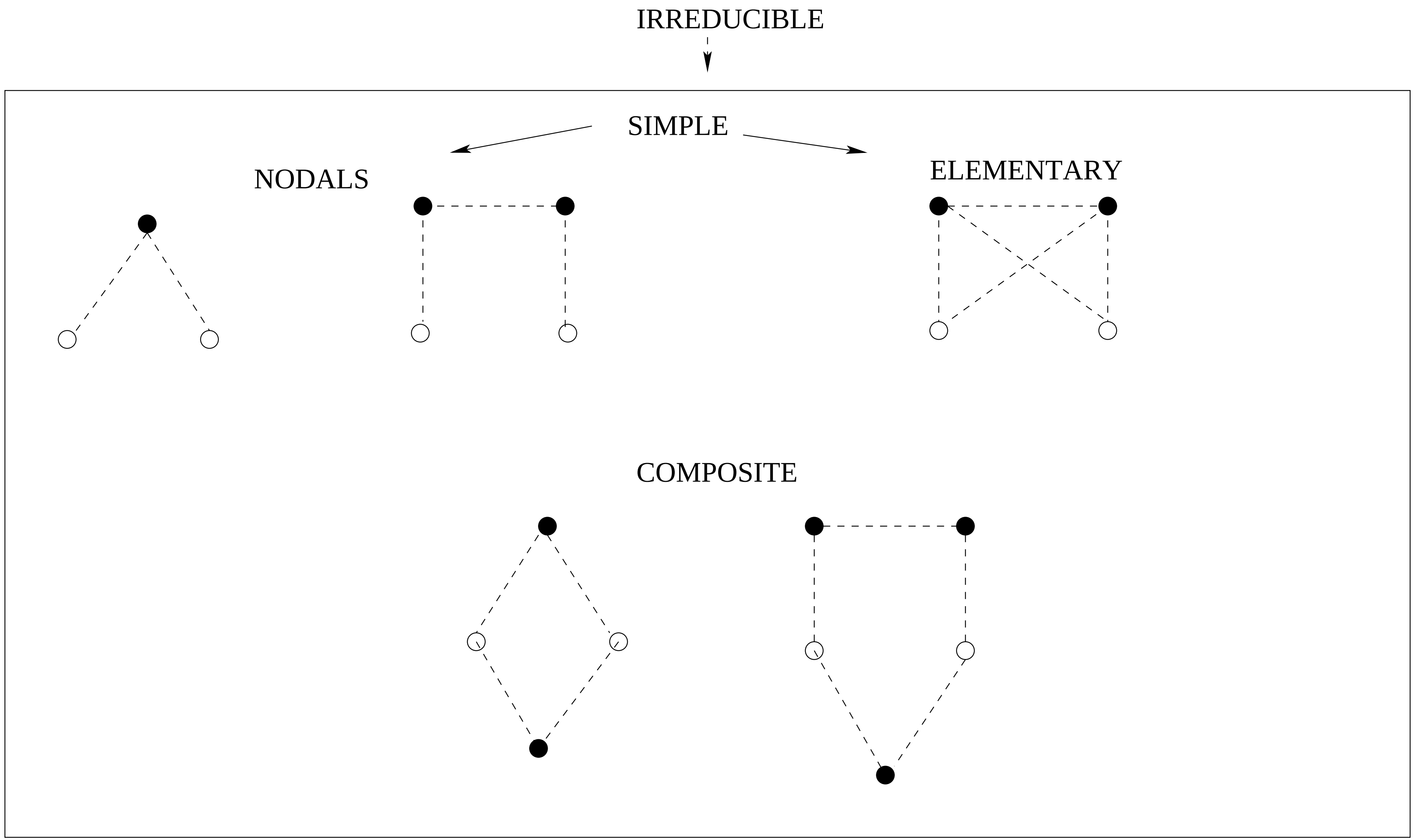
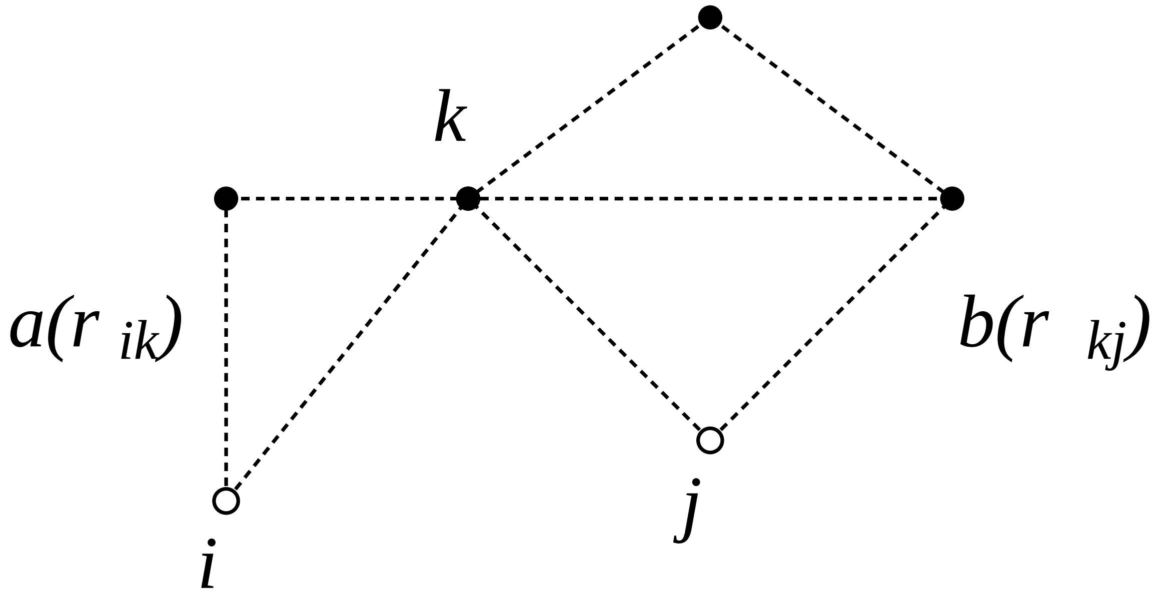


REDUCIBLE




IRREDUCIBLE
SIMPLE
NODALS
ELEMENTARY
COMPOSITE

Building of the nodal diagrams



$$\int d\vec{r}_k\, a(r_{ik})b(r_{kj}) \equiv (a(r_{ik})|b(r_{kj}))$$

$$
\begin{aligned}
g(r_{12}) &= f^2(r_{12})\, e^{[N(r_{12}) + E(r_{12})]} \\
&= [1 + h(r_{12})]\,[1 + N(r_{12}) + E(r_{12}) + \ldots] \\
&= [1 + N(r_{12}) + X(r_{12})]
\end{aligned}
$$

$$X(r_{12}) = g(r_{12}) - 1 - N(r_{12})$$

$$N(r_{12}) = (X(r_{1k})|\rho(\mathbf{r}_k)[N(r_{k2}) + X(r_{k2})])$$

Starting values

$$N(r_{12}) = 0 \qquad X(r_{12}) = f^2(r_{12}) - 1 = h(r_{12})$$

Steps to obtain HNC equations

1. Cluster expansion of the two-body distribution function.

2. Elimination of the unlinked diagrams.

3. Elimination of the reducible diagrams.

4. Composite diagrams are built as power sum of nodal and elementary diagrams.

5. Close expression to calculate the nodal diagrams

The elementary diagrams are inserted one by one.

Fermions

Slater Determinants

$$\Phi(x_1, ..., x_A) = \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_1(x_1) & \phi_1(x_2) & \ldots & \phi_1(x_A) \\ \phi_2(x_1) & \phi_2(x_2) & \ldots & \phi_2(x_A) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_A(x_1) & \phi_A(x_2) & \ldots & \phi_A(x_A) \end{vmatrix}$$

$$|\Phi(1, 2, \ldots, A)|^2 = \begin{vmatrix} \rho_0(x_1, x_1) & \rho_0(x_1, x_2) & \ldots & \rho_0(x_1, x_A) \\ \rho_0(x_2, x_1) & \rho_0(x_2, x_2) & \ldots & \rho_0(x_2, x_A) \\ \vdots & \vdots & \ddots & \vdots \\ \rho_0(x_A, x_1) & \rho_0(x_A, x_2) & \ldots & \rho_0(x_A, x_A) \end{vmatrix}$$

$$\rho_0(x_i, x_j) = \sum_a \phi_a^*(x_i)\phi_a(x_j)$$

$$\int dx_j \rho_0(x_i, x_j)\rho_0(x_j, x_k) = \rho_0(x_i, x_k)$$

Steps to obtain FHNC equations

S. Fantoni and S. Rosati, Nuov. Cim. 25 (1975) 593.

1. Cluster expansion of the two-body distribution function.

2. Elimination of the unlinked diagrams.

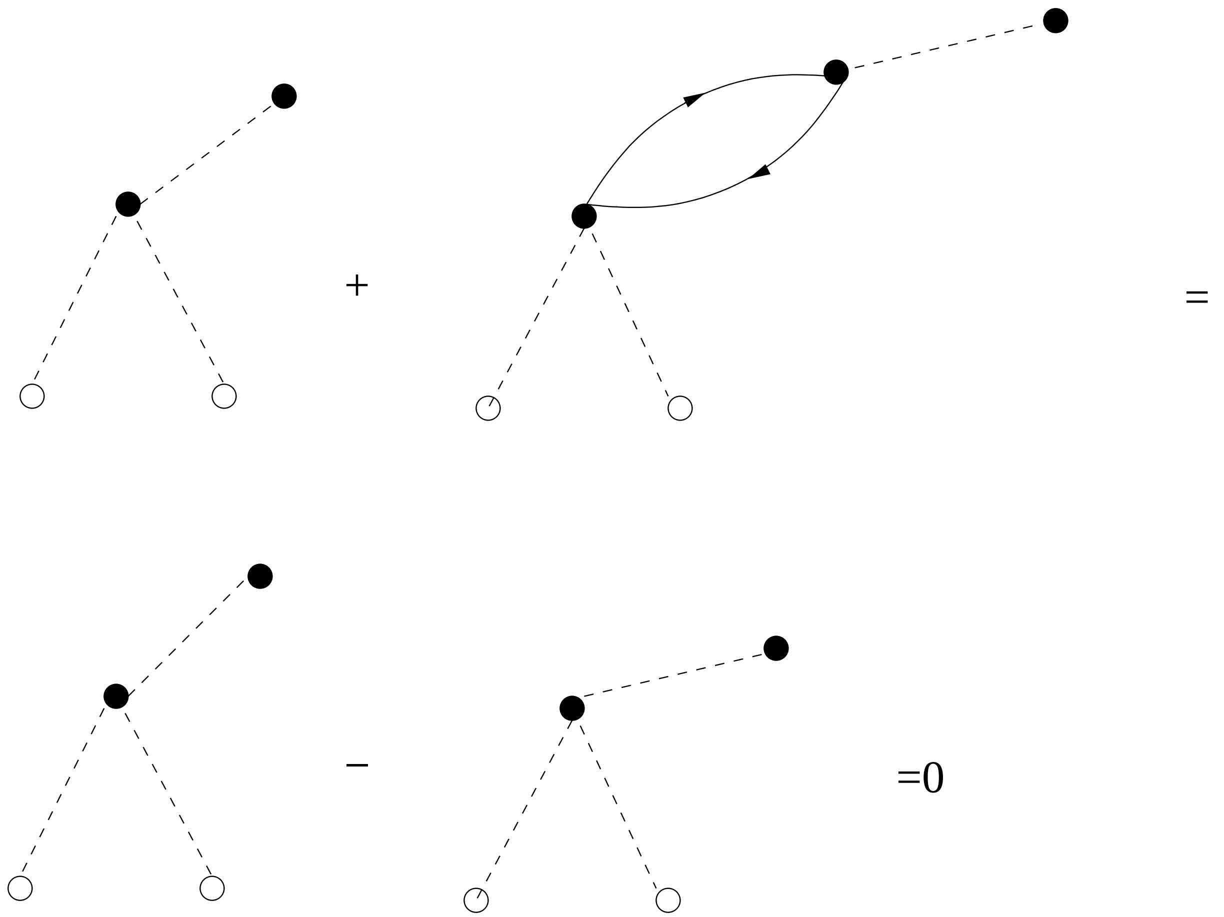
3. Elimination of the reducible diagrams

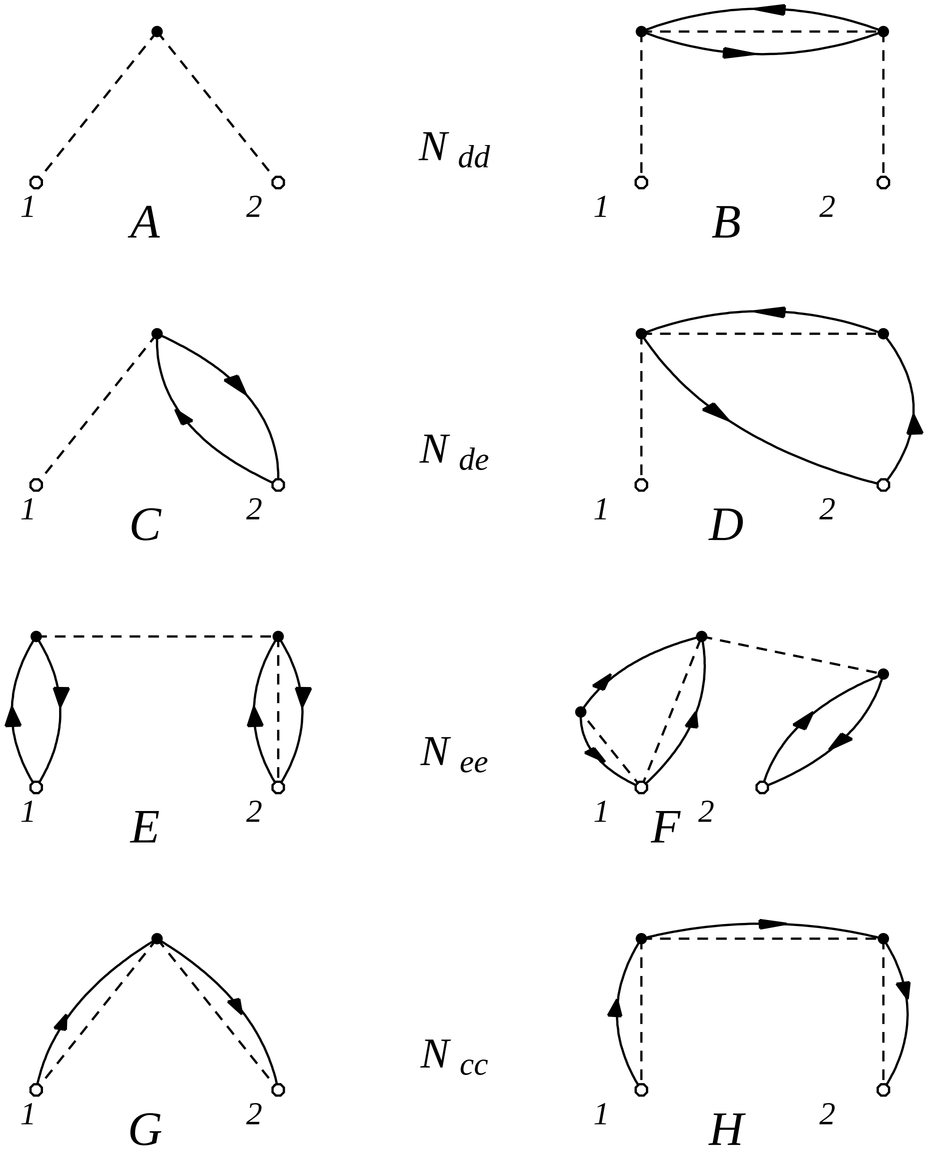
4. Composite diagrams are built as a power sum of nodal and elementary diagrams.

5. Detailed classification of nodal diagrams.

6. Close expression to calculate the nodal diagrams.

Infinite system of Fermions
Translational invariance.



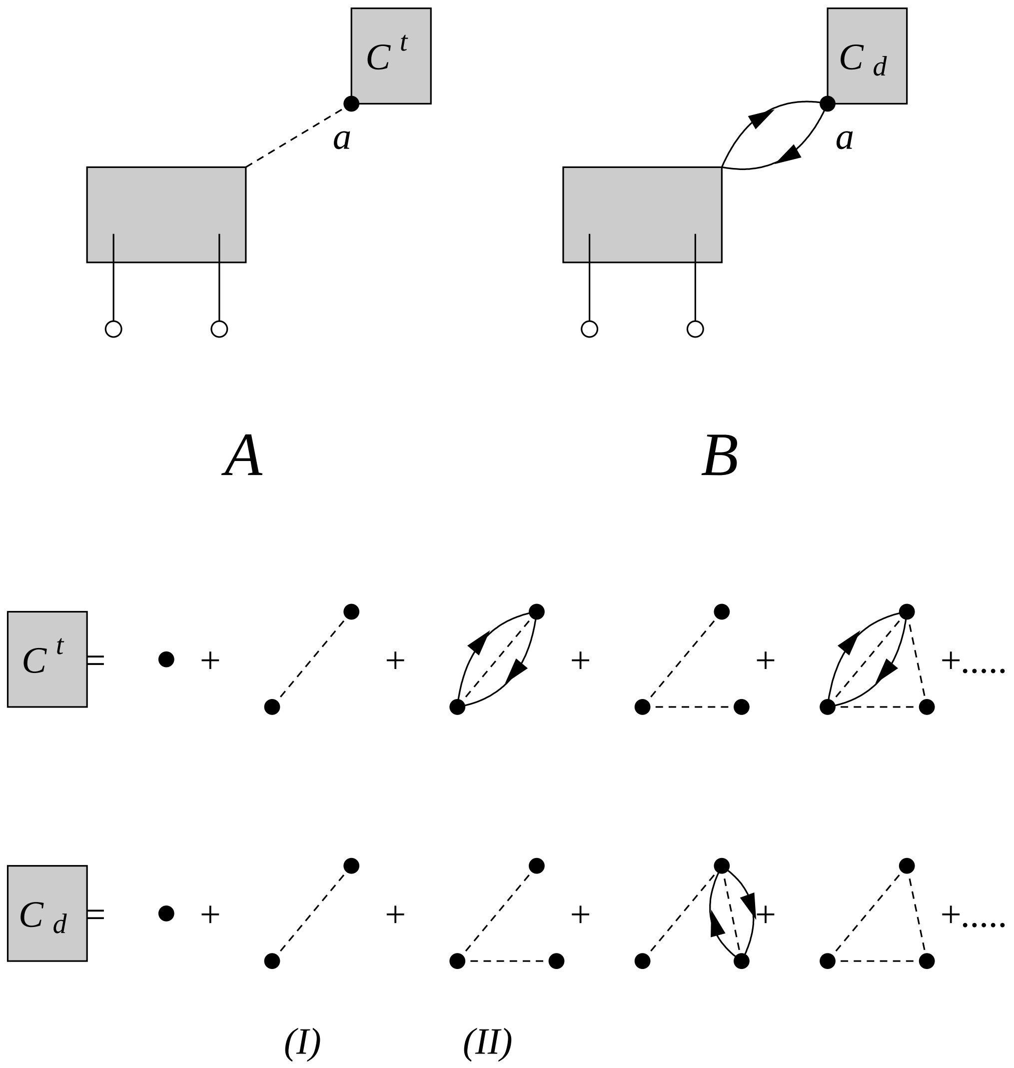


1
2
A
N_{dd}
1
2
B
1
2
C
N_{de}
1
2
D
1
2
E
N_{ee}
1
2
F
1
2
G
N_{cc}
1
2
H

Steps to obtain FHNC equations for finite fermion systems.

S. Fantoni and S. Rosati, Nucl. Phys. A 328 (1979) 478

1. Cluster expansion of the two-body distribution function.
2. Elimination of the unlinked diagrams.
3. Elimination of the reducible diagrams
4. Detailed classification of nodal diagrams
5. Composite diagrams are built as a power sum of nodal and elementary diagrams.
6. Close expression to calculate the nodal diagrams.



$$C(\mathbf{r}) = exp[U_d(\mathbf{r})][\rho_0(\mathbf{r}) + U_e(\mathbf{r})] = \rho(\mathbf{r})$$

Realistic nucleon-nucleon interaction

$$V(x_{ij}) = \sum_{p=1}^{8} V_p(r_{ij}) O_{ij}^p$$

$$O_{ij}^{p=1,8} = 1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j), S_{ij}, S_{ij}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j),$$

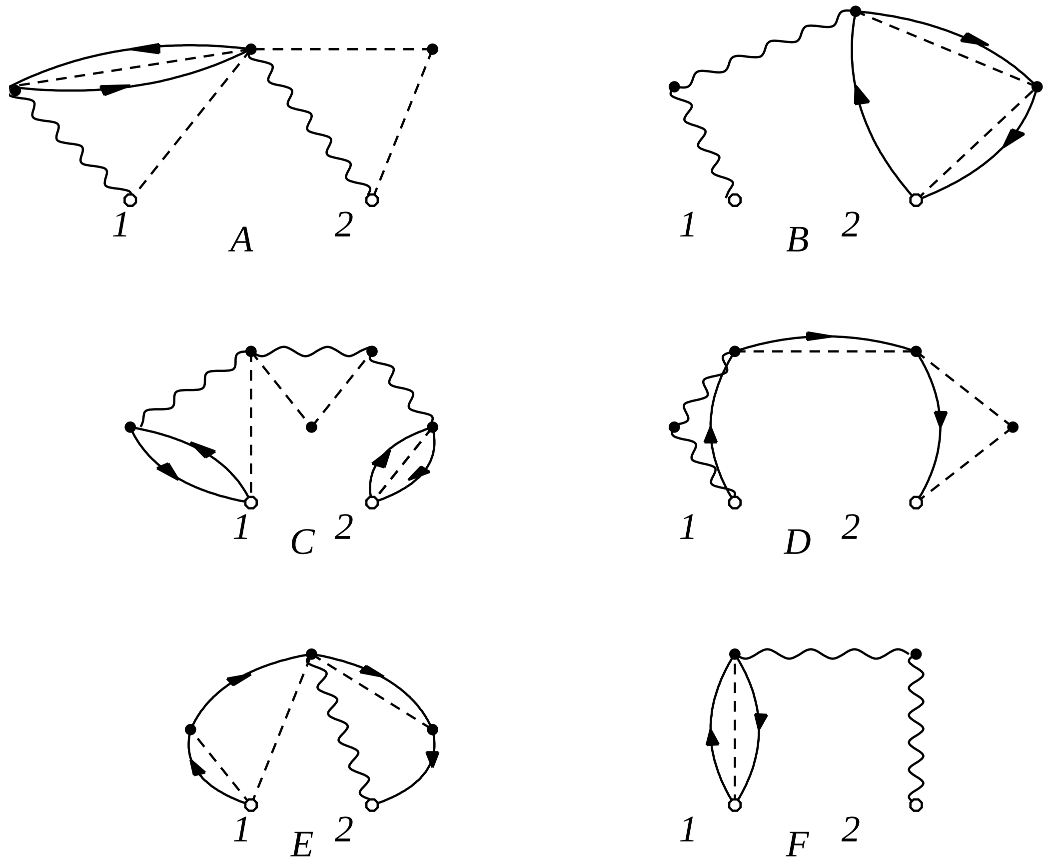
$$\mathbf{L}_{ij} \cdot \mathbf{s}_{ij}, \mathbf{L}_{ij} \cdot \mathbf{s}_{ij}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j).$$

$$S_{ij} = 3(\boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{ij})(\boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij}) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

Operator dependent correlations

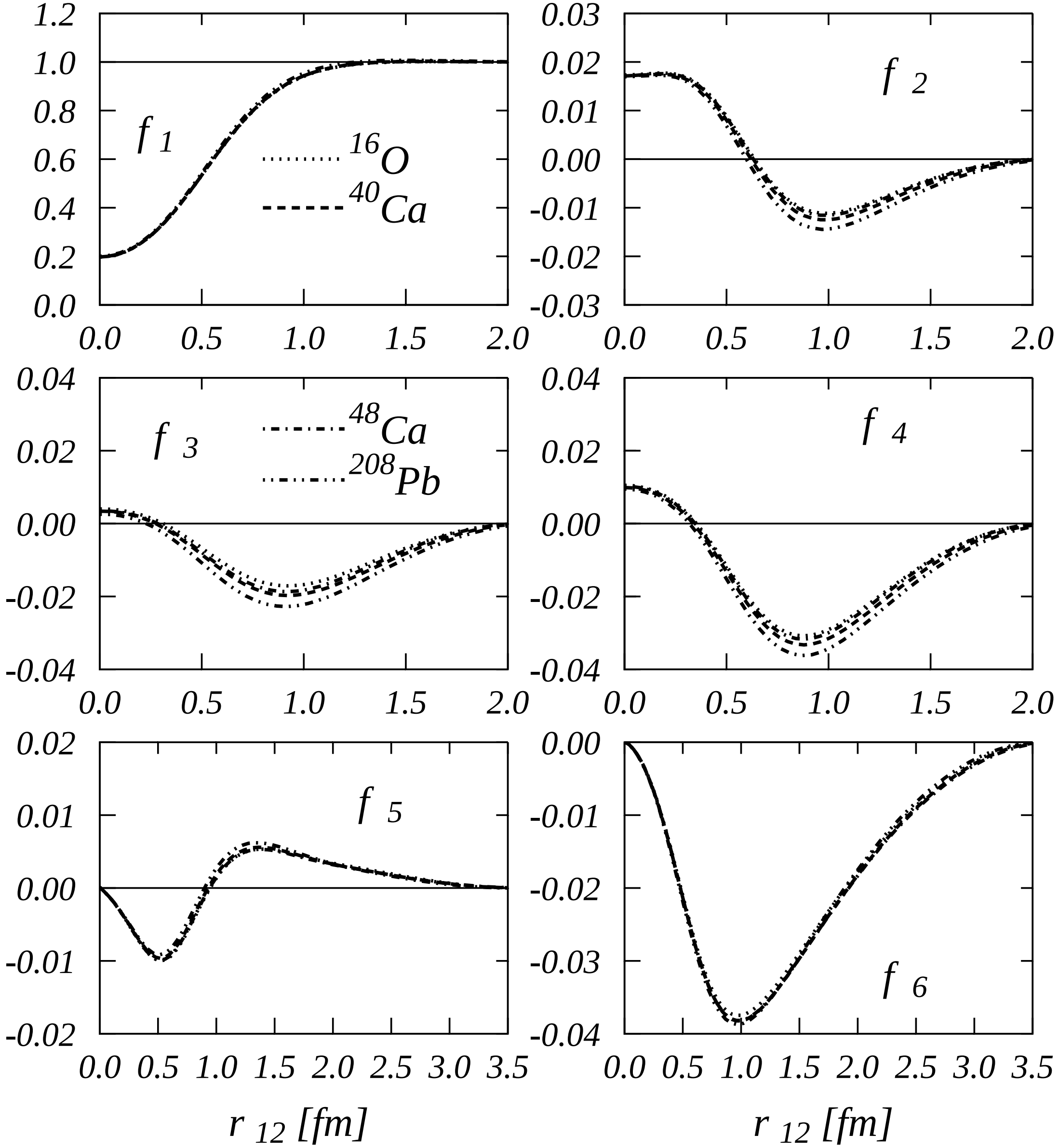
$$\mathcal{F}(1, ..., A) = \mathcal{S}(\prod_{j>i=1}^{A} F_{ij}) = \mathcal{S}[\prod_{j>i=1}^{A} \sum_{p=1}^{6} f_p(r_{ij}) O_{ij}^p]$$

Single Operator Chain



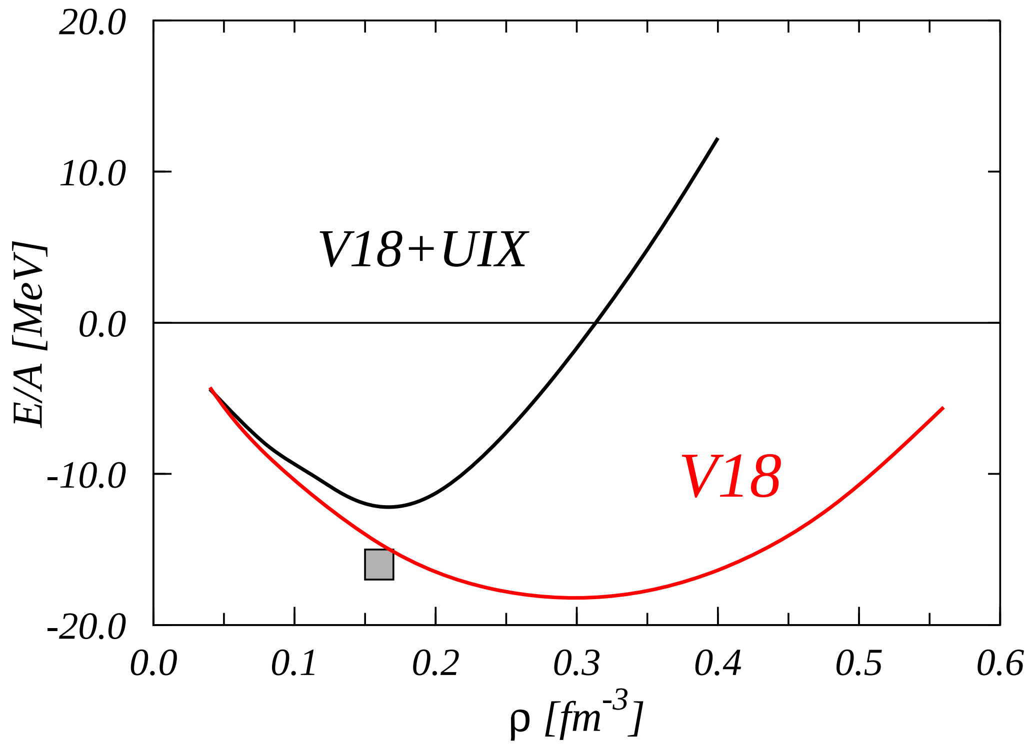
- Doubly magic nuclei.
- Different ϕ for protons and neutrons.
- jj coupling scheme.
- Argonne V8' + Urbana IX
- Correlations up to $p = 6$.
- Minimization of the correlations with two healing distances.
- No minimisation on the ϕ.
- Calculations for ^{16}O, ^{40}Ca, ^{48}Ca e ^{208}Pb.

F. Arias de Saavedra, C. Bisconti, G. Co', A. Fabrocini, Phys. Rep. 450 (2007) 1.



		^{16}O	^{40}Ca	^{48}Ca	^{208}Pb
$v_8' + UIX$	T	32.33	41.06	39.64	39.56
	V^6_{2-body}	-38.15	-48.97	-46.60	-48.43
	V_{LS}	-0.70	-0.85	-0.79	-0.80
	V_{Coul}	0.86	1.96	1.57	3.97
	$T + V(2)$	-5.66	-6.83	-6.24	-5.80
	V_{3-body}	0.86	1.76	1.61	1.91
	E	-4.80	-5.05	-4.62	-3.78
	E_{exp}	-7.97	-8.55	-8.66	-7.86

A. Akmal, V. R. Pandharipande and D. G. Ravenhall, Phys. Rev. C 58 (1998) 1804, (Tab. VI)



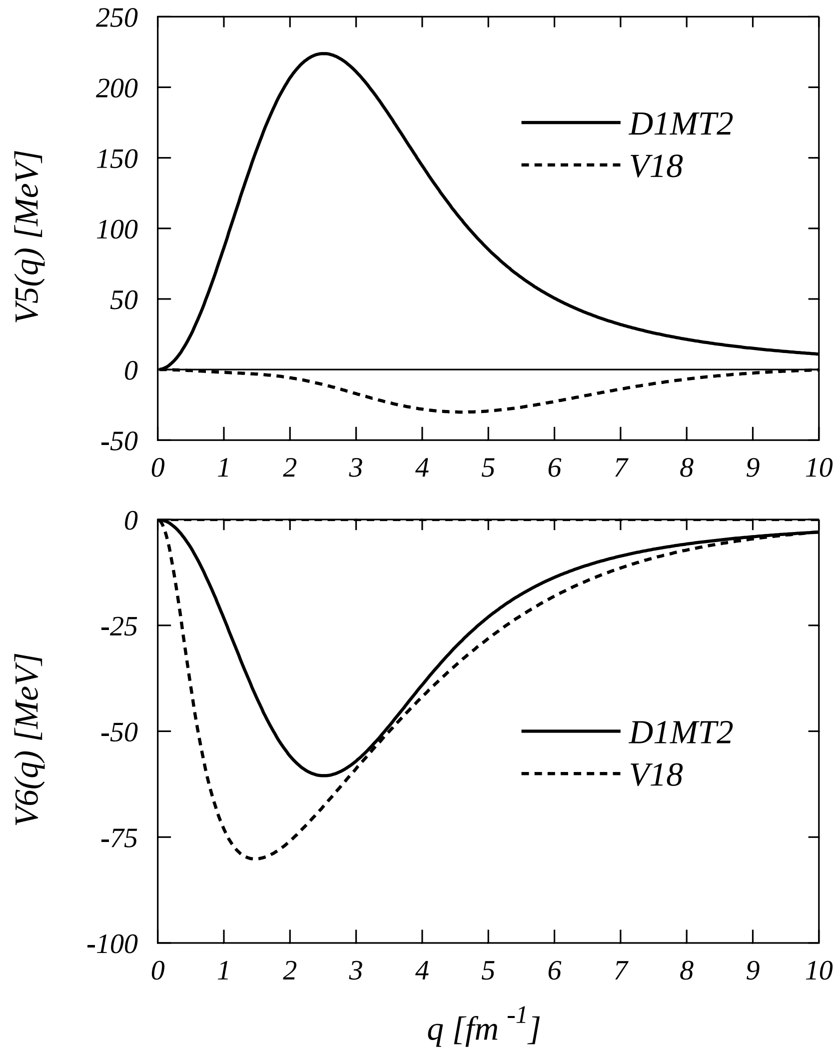
$$E[\Psi] = \left[\frac{< \Phi|F^\dagger H F|\Phi >}{< \Phi|F^\dagger F|\Phi >}\right] \equiv \left[\frac{< \Phi|H^{\mathrm{eff}}|\Phi >}{< \Phi|F^\dagger F|\Phi >}\right]$$

S. Cowell, V. R. Pandharipande, Phys. Rev. C 67 (2003) 035504.

A. Lovato, C. Losa, O. Benhar, Phys. Rev. C 83 (2011) 054003.

A. Lovato, *Ab initio calculations on nuclear matter properties including the effects of three-nucleons interaction* Ph. D. Thesis, SISSA - Trieste, Italy (2014).

M. Anguiano, M. Grasso, C. Co', V. De Donno, A. M. Lallena, Phys. Rev. C 86 (2012) 0543202.


V5(q) [MeV]
250
200
150
100
50
0
-50
D1MT2
V18
0 1 2 3 4 5 6 7 8 9 10
V6(q) [MeV]
0
-25
-50
-75
-100
D1MT2
V18
0 1 2 3 4 5 6 7 8 9 10
q [fm -1]