

Gogny interactions with tensor terms

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1 Introduction

The tensor terms of the effective nuclear interaction are usually neglected in mean-field calculations. Recently, the important role of this part of the interaction in the shell evolution has been recognised. For example, in 2004, Schiffer *et al.* [1] studied the trend of the energy difference for the two proton levels $1h_{11/2}$ and $1g_{7/2}$ out of the $Z = 50$ closed shell. They found that this difference increases with the neutron excess, and concluded that this effect is consistent with a decrease in the nuclear spin-orbit interaction. Gaudetroy *et al.* [2] studied the reduction of the spin-orbit splitting in the $N = 28$ neutron gap $1f_{7/2} - 2p_{3/2}$ going from ^{49}Ca to ^{47}Ar . They concluded that the reason for this variation is related to the proton-neutron tensor interaction. More recently, Burgunder *et al.* [3], by studying the neutron spin-orbit splitting of $2p$ levels in nuclei with $N = 20$, succeeded in isolating the effects of spin-orbit and tensor terms of the two-body interaction.

In Skyrme interactions, a zero-range tensor component was already present in the first formulation proposed in 1956 [4]. However, only in the last years, the groups of Colò [5] and Lesinski [6] have considered a tensor term of this type.

Concerning finite-range interactions we recall that, already in 1978, Onishi and Negele [7] proposed an effective two-body interaction similar to the Gogny force containing a finite-range tensor term of gaussian type. However, only in 2006 Otsuka and collaborators [8] considered for the first time a tensor term in a Gogny type interaction, which they named GT2 interaction. This force was built by adding, to the standard Gogny force, a finite-range tensor term, of gaussian type, only in the tensor-isospin channel, and by neglecting pairing correlations. The strength of this tensor term was chosen to reproduce the value of the volume integral of the analogous term of the micro-

scopic interaction AV8' [9]. For simplicity, the range of the only gaussian of this tensor term was chosen equal to the longest range of the central part of the Gogny force. In the GT2 interaction, the values of the free parameters have been changed with respect to those of Gogny interaction, since a new fit of the properties of symmetric nuclear matter and of the experimental binding energies of some magic nuclei, has been carried out, with the inclusion of this new tensor term.

We approach the problem of adding tensor terms to Gogny-like interactions from a different perspective. At present, our goal is to identify observable quantities selectively sensitive to the presence of tensor terms in the effective nucleon-nucleon interaction used in Hartree-Fock and RPA calculations which also consider pairing. To study unambiguously the role of the tensor interaction, we added tensor terms to well known parameterizations of the Gogny force, D1S [10] or D1M [11]. Our working strategy consists in comparing different quantities calculated with and without tensor terms.

In section 2 we present our model and the different types of tensor interactions we have proposed. In section 3 we show the results we have obtained for different quantities using those interactions and finally, in section 4 we draw our conclusions.

2 The model

In our work, we consider an effective nucleon-nucleon interaction of the type

$$V(\mathbf{r}_1, \mathbf{r}_2) = \sum_{p=1}^6 V_p(\mathbf{r}_1, \mathbf{r}_2) O_p(1, 2) + V_{\text{SO}}(\mathbf{r}_1, \mathbf{r}_2) + V_{\text{DD}}(\mathbf{r}_1, \mathbf{r}_2) + V_{\text{Coul}}(\mathbf{r}_1, \mathbf{r}_2), \quad (1)$$

where $O_p(1, 2)$ indicates the operatorial dependence $\mathbb{1}$, $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$, $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$, $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$, S_{12} , $S_{12} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ while all the V_p

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are scalar functions and have finite-range. The spin-orbit and density-dependent terms, V_{SO} and V_{DD} respectively, are considered of zero-range. In the above expressions, we have indicated with S_{12} the traditional tensor operator

$$S_{12}(\mathbf{r}) = 3 \frac{[\boldsymbol{\sigma}_1 \cdot \mathbf{r}][\boldsymbol{\sigma}_2 \cdot \mathbf{r}]}{r^2} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2. \quad (2)$$

We calculate the ground state properties of the system by solving the Hartree-Fock coordinate space,

$$-\frac{\hbar^2}{2m_k} \nabla_1^2 \phi_k(\mathbf{r}_1) + U(\mathbf{r}_1) \phi_k(\mathbf{r}_1) + \int d^3r_2 W(\mathbf{r}_1, \mathbf{r}_2) \phi_k(\mathbf{r}_2) = \epsilon_k \phi_k(\mathbf{r}_1), \quad (3)$$

where U refers to the direct, local, Hartree term and W to the exchange, non local, Fock-Dirac term. The solution of these equations, which is obtained iteratively by using the plane wave expansion method of Ref. [12], provides us with the single-particle (s.p.) energies ϵ_k and wave functions $\phi_k(r)$ which are the basic ingredients to calculate the various observables.

As already pointed out in the Introduction, we have analysed the role of the tensor force by adding to the traditional D1S and D1M parameterizations different tensor terms. We describe in the following the different parameterizations we have introduced up to now for these tensor terms and the fit procedure used in each case to select the values of the free parameters.

2.1 The D1ST and D1MT interactions

These two interactions have been proposed by our group in Ref. [13]. In this case, we considered only a tensor-isospin term which is based on the analogous term of the microscopic Argonne V18 interaction [14]. We constructed the tensor term of these interactions by taking the radial part of the AV18 interaction and multiplying it by a function which simulates the effect of short-range correlations

$$v_6(r) = v_{6, \text{AV18}}(r) \left(1 - e^{-b r^2}\right), \quad (4)$$

where r is the distance between the two interacting nucleons and b is the only free parameter. Since in the original parameterisations of Gogny-like forces the strength of the spin-orbit term is chosen to reproduce the experimental splitting of s.p. energies of the $1p_{1/2}$ and $1p_{3/2}$ neutron states in ^{16}O , we took this nucleus as reference.

The values of b selected for the D1ST and D1MT forces have been chosen by using an iterative procedure. We began with a HF calculation without the tensor term. This calculation produces a set of s.p. energies and wave functions to be used in Random Phase Approximation (RPA) calculations. We carried out RPA calculations, including the tensor term, to select the value of b in order to reproduce the energy of the first 0^- state in ^{16}O at 10.6 MeV. With this new interaction we recalculated the s.p. energies by changing the value of the spin-orbit strength W_{LS} to reproduce again the splitting of the neutron $1p$ states in ^{16}O . This procedure has been repeated until convergence. The values of the parameters are shown in Table 1.

Table 1. Parameters of the D1ST and D1MT interactions. The values of the other parameters are those of the corresponding D1S and D1M forces.

	$b \text{ [fm}^{-2}]$	$W_{\text{LS}} \text{ [MeV fm}^5]$
D1ST	0.60	134.0
D1MT	0.25	122.5

2.2 The D1ST2a and D1MT2a interactions

We proposed in Ref. [15] these two interactions to reproduce the experimental result obtained by Sorlin and Pourquet [16] concerning the behaviour of the energy gap in calcium isotopes. They observed that the difference between the energies of the s.p. neutron states $2p_{3/2}$ and $1f_{7/2}$ increases from ^{40}Ca to ^{48}Ca . This trend cannot be obtained by using the D1S or D1M the interaction, but the results with our D1ST and D1MT interactions were even worse.

The tensor term of the D1ST and D1MT forces considers the interactions between like and unlike nucleons with the same intensity and sign, i. e. they are both repulsive or attractive. To avoid this restriction, we added a pure tensor term, $p=5$ in Eq. (1), with a different strength with respect to that of the tensor-isospin term. This allows us to separately tune the neutron-proton and the like-nucleon tensor contributions of the interaction and also to reproduce the experimental observation of Sorlin and Pourquet [16].

The radial behaviour of the tensor terms of the D1ST2a and D1MT2a interactions is similar to that proposed by Onishi and Negele [7]

$$\begin{aligned} V_{\text{tensor}}(\mathbf{r}_1, \mathbf{r}_2) &= (V_{T1} + V_{T2} P_{12}^T) S_{12} e^{-(r_1 - r_2)^2 / \mu_T^2} \\ &= \left[\left(V_{T1} + \frac{1}{2} V_{T2} \right) + \frac{1}{2} V_{T2} \boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2) \right] \\ &\quad \times S_{12} e^{-(r_1 - r_2)^2 / \mu_T^2}, \end{aligned} \quad (5)$$

where we have indicated with P^T the usual isospin exchange operator. In Eq. (5) we have separated the pure tensor and tensor-isospin terms. With the aim of reducing the number of free parameters, we have considered for both tensor terms the same radial behaviour. We have chosen a Gaussian form, and for μ_T the value of the longest range of the original Gogny force, that is 1.2 fm for the D1S interaction and 1.0 fm for the D1M one.

We have chosen the values of the two free parameters, V_{T1} and V_{T2} , by considering two observables. The first one is the energy difference between the $1f_{5/2}$ and $1f_{7/2}$ s.p. neutron states in ^{48}Ca , whose experimental value is 8.8 MeV [17]. This observable depends only on the tensor interaction between like-nucleons, therefore it is ruled by the value of $V_{T1} + V_{T2}$. The other observable is the energy of the first 0^- state in ^{16}O , equal to 10.96 MeV [18]. In Table 2 we show the values of the parameters we have obtained for the D1ST2a and D1MT2a interactions, by using the analogous iterative procedure already utilised to define the D1ST and D1MT interactions.

Table 2. Parameters of the D1ST2a and D1MT2a interactions. The values of the other parameters are those of the corresponding D1S and D1M forces.

	V_{T1} [MeV]	V_{T2} [MeV]
D1ST2a	-135.0	115.0
D1MT2a	-310.0	260.0

This new form of the interaction improves the previous one since it allows the separated definition of the tensor force between like and unlike nucleons. In Ref. [15] we proposed another choice of the parameter values by changing one of the observables chosen to be reproduced. We called it D1ST2b, but we will not consider it here.

As we observed in the case of the D1ST and D1MT interactions, when a tensor term is added to the effective force it is very important to change the spin-orbit strength. For this reason, we have proposed a new parameterisation of the force where also changes of the spin-orbit term have been considered.

2.3 The D1ST2c and D1MT2c interactions

We constructed the D1ST2c and D1MT2c interactions by following the strategy inspired by the work of Zalewski *et al.*, [19]. We added to the usual D1S and D1M interactions the tensor term of Eq. (5) and we selected the parameter values of V_{T1} , V_{T2} and also of the spin-orbit strength W_{LS} . The procedure used to choose the values of these three parameters is described in detail in Ref. [20] and it can be summarised in the following steps.

- First, we reproduce the energy difference between the neutron $1f$ spin-orbit partner states in ^{40}Ca by modifying the value of W_{LS} . This quantity is very weakly dependent on the tensor force.
- Second, we reproduce the energy difference between the neutron $1f$ spin-orbit partner states in ^{48}Ca by choosing the strength of the like-particle tensor interaction ruled by the quantity $V_{T1} + V_{T2}$.
- Third, we reproduce the energy difference between the same s.p. states in ^{56}Ni to choose the strength of the unlike-particle tensor interaction, ruled by the parameter V_{T2} .

Table 3. Parameters of the D1ST2c and D1MT2c interactions. The values of the other parameters are those of the corresponding D1S and D1M forces.

	V_{T1} [MeV]	V_{T2} [MeV]	W_{LS} [MeV fm ⁵]
D1ST2c	-135.0	60.0	103.0
D1MT2c	-175.0	40.0	95.0

The values of the parameters we have obtained by following this strategy are given in Table 3.

3 Results

Using the interactions described above, we carried out various HF calculations to study different ground state quantities of interest such as binding and s. p. energies, charge radii and distributions. We also evaluated excited state properties in doubly closed shell nuclei by using the RPA approach both considering a discrete s.p. basis, DRPA, or the continuum part of s.p. spectrum, continuum RPA (CRPA). In particular, we studied the excitation of 0^- states in spherical nuclei, the isovector and isoscalar excitations in nuclei with $N=Z$, the 1^- excitation in nuclei with $N > Z$, unnatural parity states and charge-exchange excitations. Finally, we investigated the interplay between pairing and tensor effects by using a Hartree Fock plus Bardeen Cooper Schrieffer (HF+BCS) and Hartree Fock Bogolioubov (HFB) models. We present here below those cases where we found the relevant effects generated by the presence of the tensor force.

We have verified that the effect of the tensor term on the value of the binding energies is negligible [13]. This justifies our choice of building interactions with tensor terms by maintaining fixed the values of the parameters of the original D1S and D1M interactions which have been chosen mainly to reproduce binding energies.

The tensor terms of the interaction mainly affect the values of the s.p. energies and quantities related to them. We have studied the tensor effects on these quantities by considering the difference between the s.p. energies of spin-orbit partner levels

$$s = \epsilon_{l-1/2} - \epsilon_{l+1/2} . \quad (6)$$

We have analysed the difference between the values of s obtained by using interactions with and without tensor, D1S results compared with those obtained with D1ST, or D1M results compared with those of the D1MT force,

$$\Delta s = s_{D1\alpha T} - s_{D1\alpha} , \quad (7)$$

with $\alpha = S, M$.

We show in Fig. 1 the Δs values obtained for the $1p$, $1d$ and $1f$ proton spin-orbit levels for different isotopes of O, Ca, Ni, Zr, Sn and Pb. The open squares indicate the results obtained with the D1S interactions and the red solid circles those obtained with the D1M ones. The arrows show the nuclei where all the spin-orbit partner levels are fully occupied.

We first observe that the D1M and D1S forces provide very similar results. Second, we observe that the values of Δs are always negative. This means that the tensor force reduces the energy difference between spin-orbit partner levels.

We also observe that the tensor force does not affect the nuclei indicated by the arrows. These observations can be understood according to the interpretation proposed by Otsuka [8]. The effect of tensor force acting between an occupied neutron level with angular momentum $j_{>} \equiv l + 1/2$ with proton s.p. levels, consists in increasing the energies of the levels with $j'_{>} \equiv l' + 1/2$ and in lowering

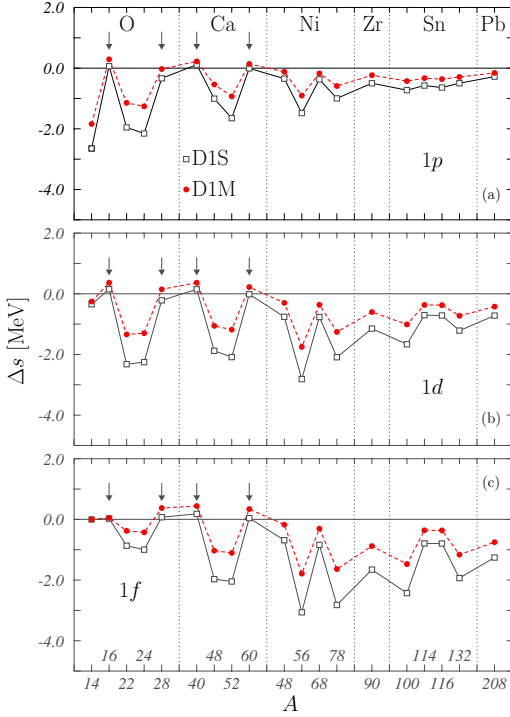


Figure 1. Differences between the s.p. energies of spin-orbit partners levels, according equation (6) and (7).

Table 4. Energies (in MeV) of the first 0^- state for different nuclei using D1S, D1ST, D1M and D1MT interactions, compared with the experimental ones [21].

Nucleus	exp.	D1S	D1ST	D1M	D1MT
^{12}C	18.40	19.63	14.42	18.83	15.27
^{16}O	10.96	13.95	10.94	13.08	10.96
^{40}Ca	10.78	12.22	9.57	11.56	9.60
^{48}Ca	8.05	14.10	11.63	12.85	11.26
^{208}Pb	5.28	8.27	7.93	8.24	7.92

those of the levels with $j'_< \equiv l' - 1/2$. Since the effect is reversed for the neutron level of $j_<$ type, when both neutron spin-orbit levels $j_<$ and $j_>$ are occupied, as in the nuclei indicated by the arrows, the two effects tend to cancel with each other, and the global effect is very small. For the nuclei with $N > Z$, Δs is negative since in all the cases there is, at least, one occupied neutron level with $j_>$, while the corresponding $j_<$ state is empty.

We have carried out DRPA calculations of the 0^- excitation for different magic nuclei by using D1S, D1M, D1ST and D1MT interactions. In Table 4 we compare the excitation energies obtained in our calculations with the experimental ones [21]. The inclusion of the tensor force always lowers the energy of the first 0^- state, and improves the agreement with the experimental values. The effect of the tensor force is larger in ^{12}C than in ^{208}Pb .

We studied the effects of the tensor force in magnetic dipole excitations within the CRPA framework [22]. The energy distributions of the $B(\mathcal{M}1)\uparrow$ strengths obtained

with the D1S and D1ST interactions are shown in Fig. 2 where we use a logarithmic scale to emphasise the peaks.

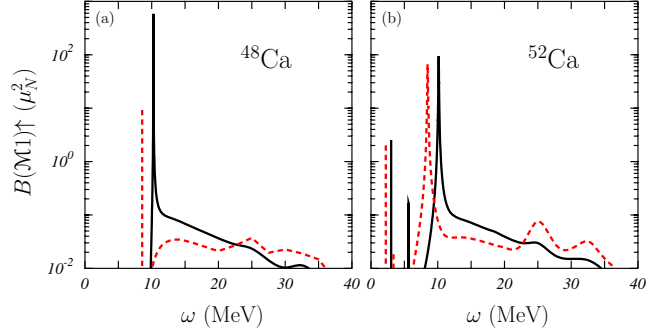


Figure 2. Energy distribution of the $B(\mathcal{M}1)\uparrow$ strength for ^{48}Ca and ^{52}Ca . Black solid and red dashed curves refer to the CRPA results obtained with D1S and D1ST interactions. The strength shown below the continuum threshold was obtained by a DRPA calculation.

The figure shows that the position of the peaks is lowered when tensor force is introduced. In case of ^{52}Ca the role of the tensor can be explained easily. The peak below the particle emission threshold and obtained with a DRPA calculation is due to the $[(2p_{1/2})(2p_{3/2})^{-1}]$ neutron transition. Considering that the neutron level $1f_{7/2}$ is completely full, it is clear that the energy of this transition must decrease, following the Otsuka's scheme described before. The main peaks, placed in the continuum, are instead dominated by the $[(1f_{5/2})(1f_{7/2})^{-1}]$ neutron transition. Now, taking into account that the neutron level $2p_{3/2}$ is full, we can argue again that the energy of this main transition lowers when tensor contribution is considered. We want to remark here that the contribution of the tensor active in this case is the particle-like one, that for the D1ST interaction has the same sign than the particle-unlike one, as we explained before. For that reason, the Otsuka's argument continues to be valid here.

We have studied the role of the tensor force in the Gamow-Teller (GT) excitations by extending the DRPA model to the treat charge-exchange excitations [23]. In Fig. 3 we show the GT strength distributions for ^{48}Ca , ^{90}Zr and ^{208}Pb . The discrete responses have been folded with a Lorentz function of 1 MeV width. We observe two peaks in all the cases. The tensor terms reduce the energy of the larger peaks by approximately 2 MeV, improving the agreement with the experimental data [24]. Nevertheless, our calculations do not describe well the experimental energy distributions. This is an intrinsic problem related to the limit of the RPA approximation, which considers only one-particle one-hole excitations.

By following the experimental results of Burgunder *et al.*, [3] which have investigated the energy difference between the neutron $2p$ spin-orbit partner levels in $N = 20$ isotones, ^{40}Ca , ^{36}S and ^{34}Si , we disentangled the effects induced by the tensor force from those generated by spin-orbit one. In particular, we studied the reduction of this energy difference in ^{36}S with respect to that of ^{40}Ca and

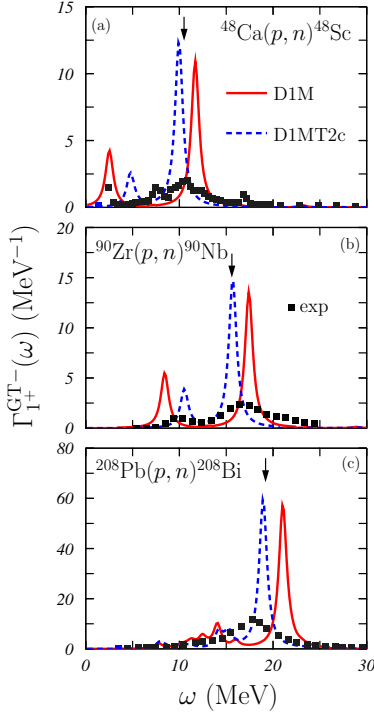


Figure 3. Energy distribution of the $I_{1+}^{GT-}(\omega)$ strengths. Red solid curves are the results with the D1M interaction and blue dashed curves those with the D1MT2c force. Arrows refer to the experimental energies of the main peaks [24]. Black squares are the experimental data, from [25].

that of ^{34}Si with respect to that of ^{36}S [26]. We show in table 5 the percentile reductions of the $2p$ energy difference obtained in HF calculations with three interactions.

Table 5. Percentile reductions of the energy difference between the neutron $2p$ spin-orbit partner levels.

$^{36}\text{S} / ^{40}\text{Ca}$	$^{34}\text{S} / ^{36}\text{Si}$
D1S	D1S
13%	43%
D1ST2a	D1ST2a
40%	39%
D1ST2c	D1ST2c
27%	42%

In the extreme shell model the proton state $1d_{3/2}$ is full in ^{40}Ca and totally empty in ^{36}S . In this case, the main source of the reduction of the neutron $2p$ energy difference is due to the tensor interaction between unlike nucleons. This interpretation is supported by the fact that the D1ST2a interaction gives the maximum value of this reduction. The V_{T2} parameter ruling this part of the tensor interaction is larger in D1ST2a than in D1ST2c.

While the energy differences discussed so far strongly depend on the tensor force, they are almost independent when we compare ^{36}S with ^{34}Si . In the extreme shell model the difference between these two nuclei is given by the fact

that the proton level $2s_{1/2}$ full in ^{36}S is totally empty in ^{34}Si . The reduction of the energy difference between the $2p$ spin-orbit partner levels is only due to the spin-orbit strength, that it is almost the same in all three interactions considered.

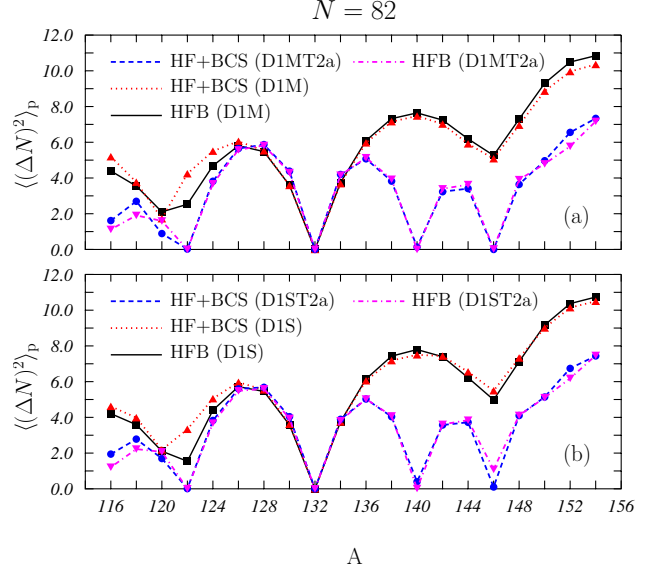


Figure 4. Proton number fluctuation $\langle (\Delta N)^2 \rangle_p$ for various $N = 82$ isotones with D1M and D1MT2a (panel (a)) and D1S and D1ST2a interactions (panel (b)). In each panel HF+BCS results are shown by red dotted lines (without tensor), and blue dashed lines (with tensor). The HFB results are indicated by the black solid lines (without tensor) and the magenta dashed-dotted lines (with tensor).

We conclude our discussion by presenting the results of our study of the interplay between the tensor force and the pairing correlations. This investigation has been carried out by using HF+BCS [27] and HFB [28] calculations using as effective interactions the traditional D1S, D1M and the new ones including tensor D1ST2a, D1MT2a. In both kind of calculations we have considered the tensor contribution only at the HF level. We have studied the proton number fluctuation for $N = 82$ isotones and our results are shown in figure 4. Since $N = 82$ is a magic number there is not neutron number fluctuations in the isotones we have considered.

The figure show that HF+BCS and HFB results are very similar, indicating the validity of the first and simpler model. The effect of the tensor force on the proton number fluctuation is relevant for the with $A > 136$. The tensor force produces a lowering of the pairing correlations for these nuclei. This is consequence of the increase of the energy difference between the proton $2d_{5/2}$ and $1g_{7/2}$ states, which are those of interest for the filling up of the protons in this region. We show in figure 5 the evolution of the proton s.p. energies of these two states. The energy of the $2d_{5/2}$ proton level does not change very much when tensor is introduced, but that of the $1g_{7/2}$ becomes more negative when the tensor force is taking into account. This is due

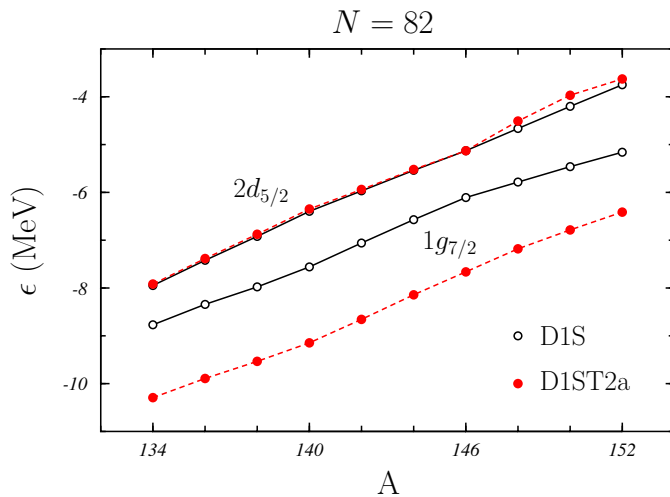


Figure 5. Single particle energies, ϵ (in MeV) for the proton states $2d_{5/2}$ and $1g_{7/2}$. Open (solid) circles have been obtained with the D1S (D1ST2a) interaction.

to the interaction between the proton on these two levels with the neutrons of the $1h_{11/2}$ level, that is full for the $N = 82$ isotones, while the $1h_{9/2}$ level is empty.

4 Conclusions

In this work we have described the parameterisations of Gogny type interactions including a tensor term that have been introduced in the last years by our group. We have specified the form of the tensor term considered, the values of the parameters and the fit procedure used in each case. Some of these parameterisation, D1ST, D1MT, include only a tensor-isospin term. This implies that the particle-like and particle-unlike contributions of the tensor force have the same strength and sign, which does not allow to describe various properties at the same time. For this reason we introduced parameterisations containing a pure tensor and a tensor isospin terms, allowing to define in a separate way the particle-like and particle-unlike parts of the tensor contribution. This is the case of the interactions D1ST2a, D1MT2a, D1ST2c and D1MT2c. We have presented some results for various observables, comparing to the results obtained using the traditional D1S and D1M Gogny interactions with those obtained with the forces which consider the tensor terms.

We have chosen observables for which an important contribution of tensor force has been observed. Some of them are related to mean-field calculations, s. p. energies and pairing correlations, and other ones to excitations, energy of the first 0^- states, magnetic dipole states and Gamow-Teller excitations.

The tensor force affects in a very important way the values of the s.p. energies, strongly modifying the energy difference between spin-orbit partner states. We have observed that there is an important effect of the tensor interaction in the description of magnetic states and charge exchange excitations, varying in a non negligible way the position of the peaks in the energy distributions.

From the study of the energy difference of the $2p$ spin-orbit partner levels in $N = 20$ isotones we disentangled the effects of the tensor and spin-orbit terms. From the study of the proton number fluctuation of $N = 82$ isotones we found a relation between tensor effects and pairing correlations. We observed that for the isotones with $A > 136$ the effect of the pairing correlations decrease in a drastic way when the tensor force is taken into account. This can have consequences on some properties related to the pairing, for example deformation.

The information obtained in this study can be used to choose the values of all the five free parameters of our approach, the strengths V_{T1} , V_{T2} , and the ranges μ_T , μ_{Tt} of the tensor terms defined in Eq. (1) and the strength of the spin-orbit force W_{LS} . Even in the case of global fit of the Gogny interaction including a tensor force, the results we have presented will be very helpful in order to choose observables of interest to the fit procedure.

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