RANDOM PHASE APPROXIMATION AND NEUTRINO-NUCLEUS CROSS SECTIONS [1]

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Abstract

The Random Phase Approximation theory is used to calculate the total cross sections of electron neutrinos on ^{12}C nucleus. The role of the excitation of the discrete spectrum is discussed. A comparison with electron scattering and muon capture data is presented. The cross section of electron neutrinos coming from muon decay at rest is calculated.

PACS numbers: 21.60.Jz; 23.40.Hc; 25.30.Dh; 25.30.Pt

The Random Phase Approximation (RPA) is an effective theory constructed to study the excitations of many-body systems. The RPA assumes that the exited states of these systems can be described as linear combinations of one-particle on-hole $(1p-1h)$ and one-hole one-particle $(1h-1p)$ excitations. The goal of the theory is to find the coefficients of the linear combinations for a given interaction between particles and holes.

In nuclear physics, the RPA has been applied to study excitations on a wide energy range, from a few MeV, the discrete spectrum, up to hundreds of MeV, in a regime called quasi-elastic where the emission of a single nucleon is the dominant process. One of the great successes of the RPA is the prediction of collective surface vibrations, called giant resonances, appearing at energies between 15 and 30 MeV in all the nuclei with more than 10 nucleons.

The inputs required by the RPA are the set of single particle energies and wave functions, and the effective interaction between particles and holes. In our calculations the single particle basis, which properly includes the continuum, has been obtained by solving the one-body Schrödinger equation with a spherical Woods-Saxon potential. The parameters have been taken from the literature [1], and have been fixed to reproduce the rms charge radii and the single particle energies close to the Fermi level. The theoretical uncertainty has been studied by using various *ph* interactions, specifically the LM1, LM2, and PP interactions of Ref. [1]. The LM1 and LM2 interactions are zero-range forces of Landau-Migdal type with slightly different values of the parameters. The PP interaction is a finite-range interaction. A common characteristic of the three interactions is that they have been rescaled to reproduce the excitation energy of the low lying 3^- state in ^{16}O at 6.13 MeV. Even though the three interactions produce the same excitation energy, they give different descriptions of the 3^- state. This is shown in

[1] Presented at the *XX Max Born Symposium*, Dec. 7-10 2005, Wrocław (Poland)

Fig. 1 where we compare the 3^- charge form factors with the data of Ref. [2] measured in inelastic electron scattering experiments. The difference between the various results indicates the magnitude of the theoretical uncertainty.

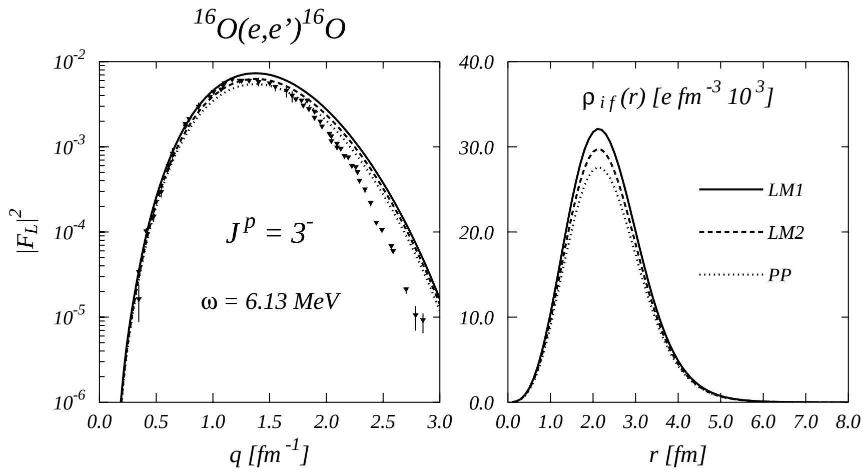

Figure 1: Charge form factors (left panel) and transition densities (right panel) of the low lying 3^- state in ^{16}O at 6.13 MeV calculated in RPA by using three different interactions which reproduce the excitation energy value. The data are form Ref. [2].

In recent years, we have used the continuum RPA to study neutrino induced excitations of the ^{12}C and ^{16}O nuclei in the discrete low-lying region, in the giant resonance and in the quasi-elastic regions. Since a detailed presentation of our results in the giant resonance and in the quasi-elastic regions can be found elsewhere [1, 3, 4, 5, 6], in this contribution we discuss the role of the discrete excitation region.

	^{12}C	^{12}N	^{12}B
LM1	17.2	20.2	14.3
LM2	18.8	21.7	15.9
PP	16.7	19.6	13.8
NI05	15.1	18.0	12.2
exp	15.1	17.3	13.4

Table 1: Energies, in MeV, of the isospin triplet 1^+ excited states referred to the ^{12}C ground state.

In the evaluation of the total neutrino cross section, the excitation of bound states cannot be neglected, even for neutrino energies well above the continuum emission threshold. As an example of the relevance of the discrete excitations, we consider here the case of the isovector triplet formed by the 1^+ excited states in the spectrum of ^{12}C.

The energies of both charge conserving and charge exchange excitations obtained in RPA are compared in Tab. 1 with the experimental values. None of the interactions above presented, well tuned to describe natural parity states, is able to reproduce the experimental energies. We have rescaled the LM1 interaction to obtain the energy of the charge conserving 1^+ state. Even with this interaction, that we called NI05, the RPA cannot reproduce the energies of the charge exchange states.

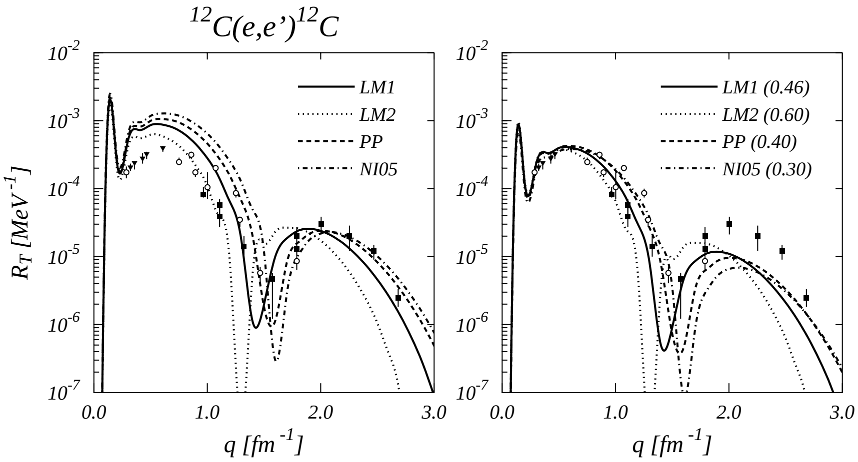

Figure 2: Magnetic form factors of the 1^+ in ^{12}C calculated with various interactions and compared with the data of Ref. [7]. The results of the left panel has been obtained by using the quenching factors indicated in the labels.

The difficulties of the RPA in the description of unnatural parity excitations are even better shown in Fig. 2 where we compare the magnetic form factors of the 1^+ state at 15.1 MeV with the data of Ref. [7] measured in inelastic electron scattering experiments. The bare RPA results are shown in the left panel. The spreading of these results is much larger than that shown in Fig. 1 for the 3^- state of ^{16}O. In any case, none of the curves reproduces the peak of the experimental form factor, where the data are more reliable. This is a common feature of almost all the electron scattering magnetic form factors in medium-heavy nuclei [8, 9]. The source of this deficiency of the RPA has been widely investigated and various studies indicate that the problem has to be ascribed to the restriction of the configuration space to $1p-1h$ excitations [10] rather than to the absence of correlations of short-range type [9].

We overcome this difficulty of the RPA by using quenching factors whose values are fixed to best fit the peak of the data. In the right panel of Fig. 2 we show the curves obtained in this manner. The values of the quenching factors are given in the labels.

The muon capture is an interesting case where the excitation of both bound and continuum states should be considered. In Fig. 3 we show the muon capture rate of the ^{12}C nucleus calculated with the various interactions, as a function of the nuclear excitation energy. The largest contribution to the total rate is given by the excitation of the continuum, dominated by the giant resonances, but the contribution of the bound states cannot be neglected. In the figure the result obtained by a mean-field (MF) calculation is represented by the full thin line. While all the other results predict a collective resonance behavior at about 19 MeV, the MF results does not show this characteristic.

Experimentally, it has been possible to disentangle the contribution of various ^{12}C excited states to the total capture rate. In Tab. 2. we compare our capture rates with the data quoted in Ref. [11]. Our calculations describe reasonably well the contribution of the continuum but they overestimate the capture rates of the discrete states. The main source of error is due to the 1^+ excitation. For this reason we show in the rows labeled with (q) the results obtained by multiplying the 1^+ states with the quenching factors of Fig. 2. The use of the quenching factors slightly improves the agreement with the data.

By looking to the total rates only, the mean field calculation provides the value that is in better agreement with the experimental one. However, a more detailed analysis of the various contributions shows that this result is obtained as a sum of a too large discrete contribution with the too small

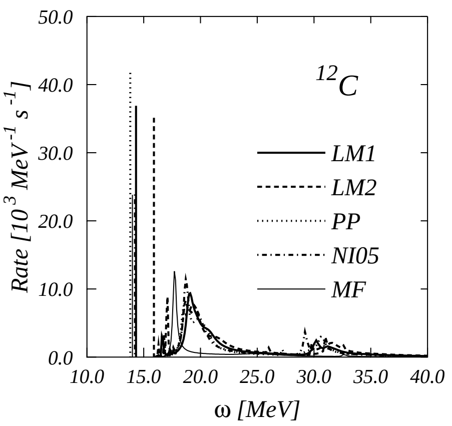

Figure 3: Muon capture rates calculated with various interactions as a function of the nuclear excitation energy.

	LM1	LM2	PP	NI05	MF	exp
1^+	34.93	33.86	34.56	31.57	23.83	6.04
1^+(q)	16.07	20.31	13.82	9.47		
2^+	0.26	0.20	0.35	0.39	0.46	0.21
2^-	1.40	0.79	0.52	8.09	9.04	0.18
1^-	0.32	0.29	6.31	0.98	1.13	0.62
dis	36.90	35.13	41.73	41.03	34.46	7.05
dis(q)	18.04	21.48	20.99	18.93		
con	31.35	37.09	28.28	31.16	12.48	30.04
tot	68.25	72.22	70.53	72.19	46.94	37.09
tot(q)	50.39	58.67	49.79	50.09		

Table 2: Muon capture rates of ^{12}C in 10^3 s^{-1}. The rows labeled with (q) shows the results obtained by using the quenching factors given in Fig. 2. The experimental values have been taken from Ref. [11].

contribution of the continuum.

In Fig. 4 we show the total neutrino cross sections for charge conserving and charge exchange reactions as a function of the neutrino energy. The lower lines have been obtained by considering only the excitation to the continuum, while the upper curves include also the excitation of the discrete states, whose main contribution is given by the 1^+ state. The lines of the left panel are those obtained with the bare RPA calculations, while in those of the right panels the discrete contributions have been multiplied by the quenching factors of Fig. 2. Even in this case the contribution of the discrete excitation is not negligible. The spreading of the various results is a measure of the theoretical uncertainty.

As an example of the consequences of this uncertainty, we have calculated the ^{12}C$(\nu_e, e^-)^{12}$B total sections for neutrinos emitted by μ^+ at rest, since this quantity has been measured [12]. We consider the electron neutrinos coming form the decay

$$\mu^+ \rightarrow \overline{\nu}_\mu \ \ + \nu_e + \ \ e^+ \ .$$

The ν_e are emitted with the energy distribution shown in the upper panel of Fig. 5. The cross sections as a function of the neutrino energy are calculated by multiplying $W(\epsilon_i)$ with the cross sections of panels (b)

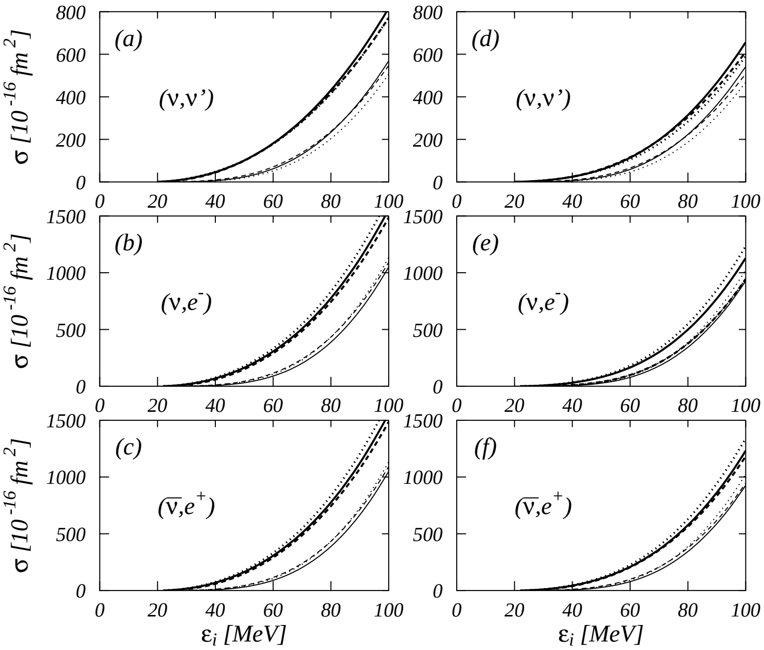

Figure 4: Total neutrino cross sections as a function of the neutrino energies, calculated with various interactions. The full lines show the results obtained with the LM1, the dashed lines with the LM2, the dotted lines with the PP interaction respectively. The lower, thinner, lines have been obtained by considering only the excitation to the continuum. The upper, thicker lines include also the contribution of the discrete excitation. The left panels show the bare RPA results, the right panels the results obtained by using the quenching factors of Fig. 2 for the discrete excitations.

and (e) of Fig. 4. These cross sections are shown in the lower panel of Fig. 5, where the upper, thinner, lines indicate the bare RPA cross sections, and the lower, thicker, lines the quenched cross sections. The total cross sections are obtained by integrating these curves. We obtain the values of 36.0, 42.0 and 46.0 10^{-16} fm^2 for the bare RPA calculations, and 7.0, 19.2 and 21.3 10^{-16} fm^2 for the quenched calculations. These results should be compared with the experimental value of 14.0 ± 1.2 10^{-16} fm^2 [12]. The spreading of the various theoretical results is much larger than the experimental uncertainty.

In summary, we can conclude that the contribution of the excitation of discrete states to the total neutrino-nucleus cross section is not negligible, even for neutrino energies of a few hundred MeV. There are large theoretical uncertainties in the description of the low-lying discrete spectrum of medium-heavy nuclei, especially for unnatural parity states. These uncertainties have large effects on the cross sections of neutrinos of energies up to several tens of MeV, such as the neutrinos coming from muon decay or supernovae neutrinos [1].

In order to obtain a satisfactory description of the low-lying discrete spectrum, and, hopefully, to reduce the theoretical uncertainties, it is necessary to use theories which goes beyond the RPA framework by considering more complicated excitations, such as $2p-2h$ degrees of freedom [10, 13, 14]. Furthermore, in the specific case of the ^{12}C nucleus, there are indications that deformation effects are important [15].

This work has been partially supported by the MURST through the PRIN: *Teoria della struttura dei nuclei e della materia nucleare.*

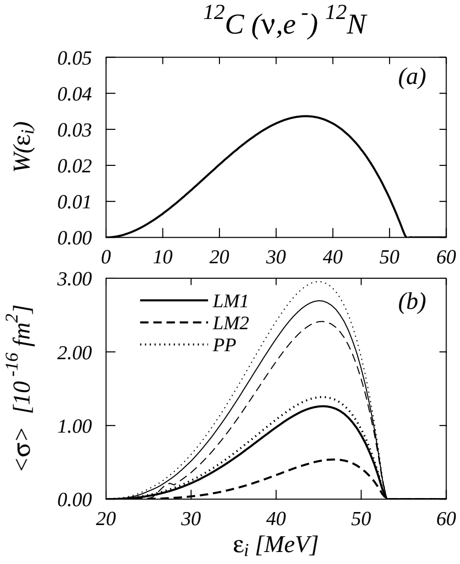

Figure 5: The upper panel shows the energy distribution of the electron neutrinos coming from muon decay at rest, Michel spectrum. In the lower panel we show the neutrino the cross sections obtained by multiplying the energy distribution $W(\epsilon_i)$ with the total cross sections of the panels (b) and (e) of Fig. 4. The upper, thinner, lines show the RPA results, the lower, thicker, lines have been obtained by using the quenching factors of Fig. 2.

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