

Il problema a molti corpi nucleare

Giampaolo Co'

Lecce 22 Aprile 2008

Luca Signorelli, Orvieto (1499-1504)

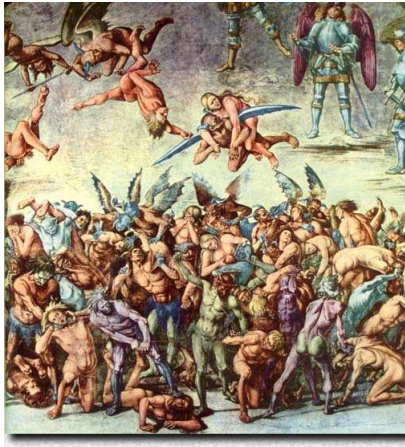
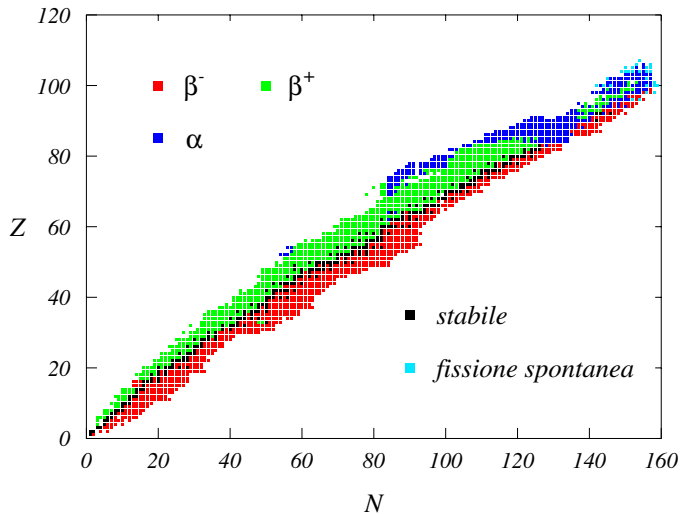


Tavola degli isotopi



Il solo sistema legato di due nucleoni

Formato da protone e neutrone

Dipendenza dall'isospin

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Momento angolare totale $J=1$

Dipendenza dallo spin

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Momento angolare totale $J=1$

Dipendenza dallo spin

$\mu_d = \mu_p + \mu_n - 0.00222$ magnetoni nucleari
 $Q=2.82$ mb

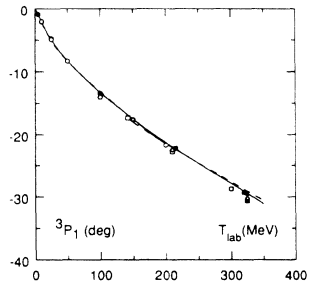
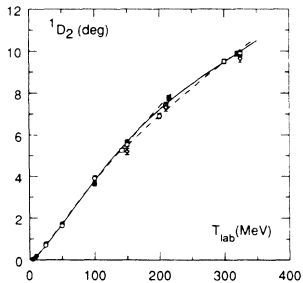
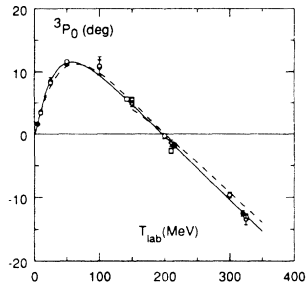
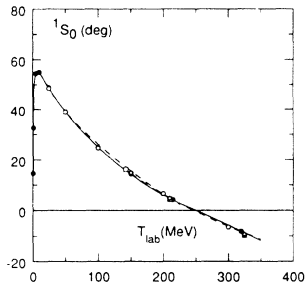
Forze non centrali. Dipendenza dal tensore



$+2$



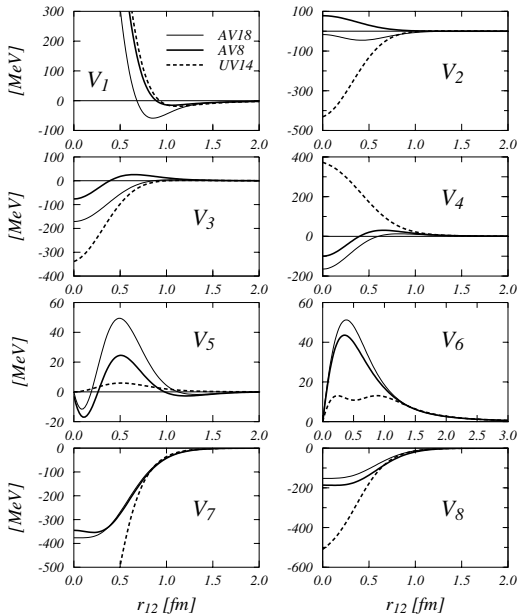
-1



$$V(i,j) = \sum_{p=1,18} v_p(r_{ij}) O_{ij}^p$$

$$\begin{aligned} O_{ij}^{p=1,14} &= [1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, (\mathbf{L} \cdot \mathbf{S}), \mathbf{L}^2, \mathbf{L}^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), (\mathbf{L} \cdot \mathbf{S})^2] \\ &\otimes [1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j] \end{aligned}$$

$$O_{ij}^{p=15,18} = [1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij},] \otimes [T_{ij}, \tau_{zi} + \tau_{zj}]$$



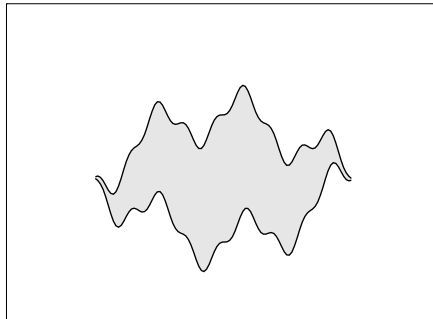
Green's function Monte Carlo

$$\Psi(\tau) = e^{-(H-E_0)\tau} \Psi_T = e^{-(H-E_0)\tau} \sum_N A_N \Psi_N = \sum_N e^{-(E_N-E_0)\tau} A_N \Psi_N$$

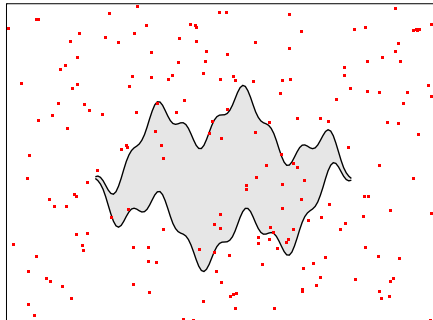
$$\Psi(\tau \rightarrow \infty) \rightarrow A_0 \Psi_0$$

$$\frac{\langle \Psi_T | e^{-(H-E_0)\tau/2} H e^{-(H-E_0)\tau/2} | \Psi_T \rangle}{\langle \Psi_T | e^{-(H-E_0)\tau/2} e^{-(H-E_0)\tau/2} | \Psi_T \rangle} \geq E_0$$

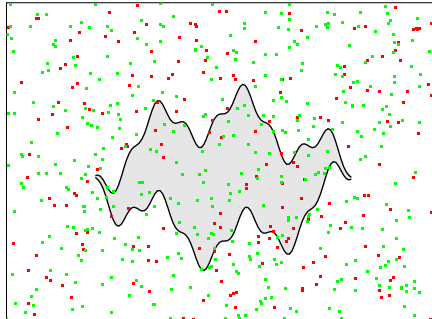
Green's function Monte Carlo



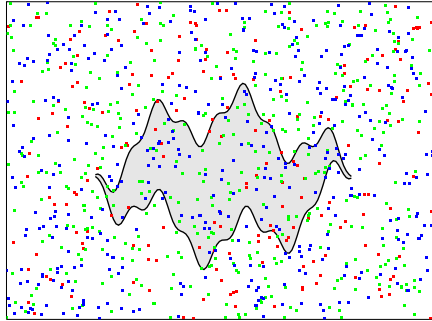
Green's function Monte Carlo



Green's function Monte Carlo



Green's function Monte Carlo

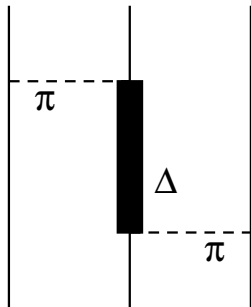


^3H Binding energy

Exp 8.48 MeV

2 N potential	2N
CD Bonn	7.953
Nijm II	7.709
Nijm I	7.731
Nijm 93	7.664
Reid 93	7.648
AV14	7.683
AV18	7.576

Forza a 3 corpi

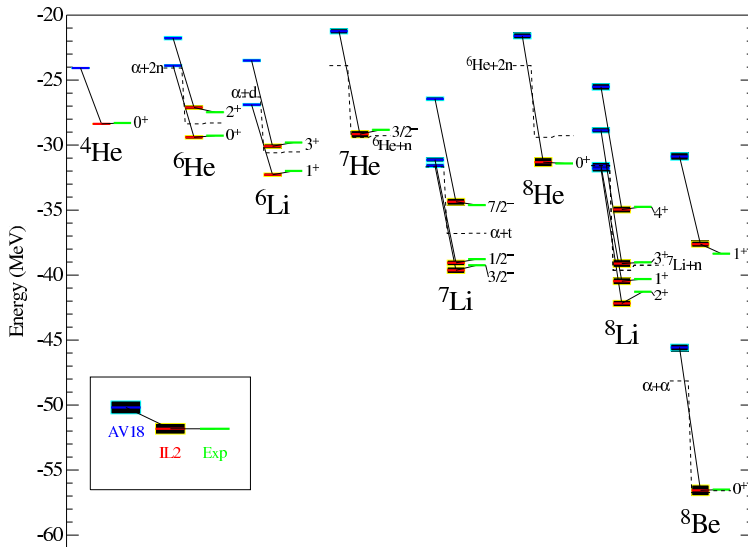


^3H Binding energy

Exp 8.48 MeV

2 N potential	2N	2N + 3N
CD Bonn	7.953	8.483
Nijm II	7.709	8.477
Nijm I	7.731	8.480
Nijm 93	7.664	8.480
Reid 93	7.648	8.480
AV14	7.683	8.480
AV18	7.576	8.479

S.C. Pieper and R.B. Wiringa, Ann. Rev. Nucl. Part. Sci. 51 (2001) 53.



Numero di configurazioni di spin e isospin per alcuni nuclei.

$$N_{conf} = 2^A \frac{A!}{Z!(A-Z)!}$$

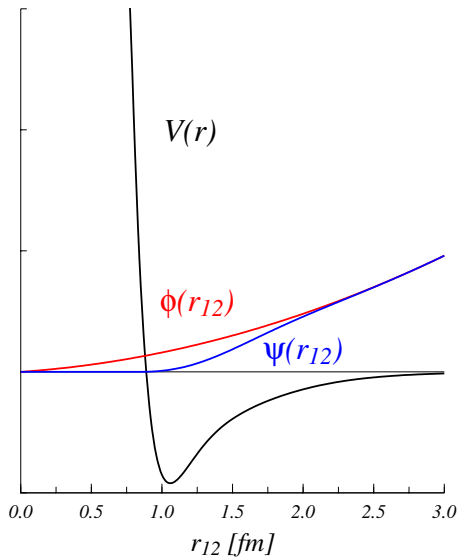
Nucleo	Z	N=A-Z	N_{conf}
${}^3\text{He}$	2	1	24
${}^4\text{He}$	2	2	96
${}^6\text{He}$	2	4	960
${}^6\text{Li}$	3	3	1280
${}^8\text{He}$	2	6	7168
${}^{12}\text{C}$	6	6	3,784,704
${}^{16}\text{O}$	8	8	$8.4 \cdot 10^8$
${}^{40}\text{Ca}$	20	20	$1.5 \cdot 10^{23}$
${}^{48}\text{Ca}$	20	28	$4.7 \cdot 10^{27}$

$$H = H_0 + H_1$$

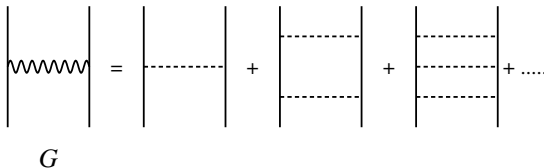
$$H_0|\Phi_0\rangle = E_0|\Phi_0\rangle$$

$$H_0|\Phi_0\rangle = \sum_i h_i \prod_{k=1,A} |\phi_k\rangle = \sum_i \epsilon_i \prod_{k=1,A} |\phi_k\rangle$$

$$\begin{aligned} E &= \langle\Phi_0|H_0|\Phi_0\rangle + \langle\Phi_0|H_1 \sum_{n=0}^{\infty} \left(\frac{1}{E_0 - H_0} H_1 \right)^n |\Phi_0\rangle_c \\ &= E_0 + \langle\Phi_0|H_1|\Phi_0\rangle + \langle\Phi_0|H_1 \frac{1}{E_0 - H_0} H_1|\Phi_0\rangle_c + \dots \end{aligned}$$



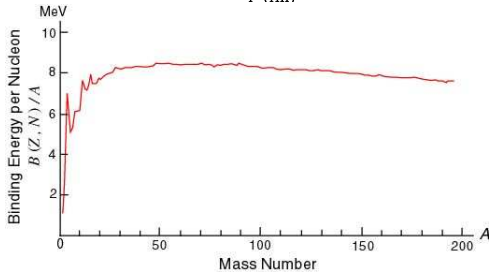
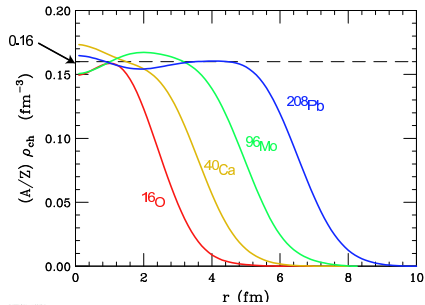
$$\begin{aligned} \langle \Phi_0 | G(\omega) | \Phi_0 \rangle &= \langle \Phi_0 | H_1 | \Phi_0 \rangle \\ &+ \sum_{p,q > F} \frac{\langle \Phi_0 | H_1 | \Phi_{pq} \rangle \langle \Phi_{pq} | G(\omega) | \Phi_0 \rangle}{\omega - \epsilon_p - \epsilon_q} \end{aligned}$$

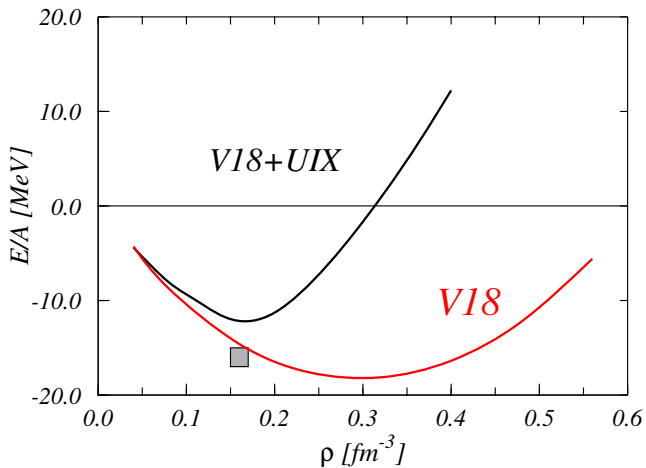


Principio Variazionale

$$\Psi(1, 2, \dots, A) = F(1, 2, \dots, A)\Phi(1, 2, \dots, A)$$

$$\delta E[\Psi] = \delta \left[\frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} \right] = \delta \left[\frac{\langle \Phi | F^\dagger H F | \Phi \rangle}{\langle \Psi | \Psi \rangle} \right] = 0$$





Risultati

		^{12}C	^{16}O	^{40}Ca	^{48}Ca	^{208}Pb
$\nu_8' + \text{UIX}$	T	27.13	32.33	41.06	39.64	39.56
	$V_{2\text{-body}}^6$	-29.13	-38.15	-48.97	-46.60	-48.43
	V_{LS}	-0.51	-0.70	-0.85	-0.79	-0.80
	V_{Coul}	0.67	0.86	1.96	1.57	3.97
	$T + V(2)$	-1.84	-5.66	-6.83	-6.24	-5.80
	$V_{3\text{-body}}$	0.66	0.86	1.76	1.61	1.91
	E	-1.17	-4.80	-5.05	-4.62	-3.78
$\nu_{14} + \text{UVII}$	T	24.63	29.25	37.70	36.47	36.48
	$V_{2\text{-body}}^6$	-27.08	-35.84	-47.16	-44.86	-46.87
	V_{LS}	-0.06	-0.10	-0.10	-0.09	-0.08
	V_{Coul}	0.68	0.88	2.02	1.59	4.03
	$T + V(2)$	-1.83	-5.81	-7.54	-6.89	-6.44
	$V_{3\text{-body}}$	0.54	0.69	1.28	1.15	1.41
	E	-1.29	-5.12	-6.26	-5.74	-5.03
	E_{exp}	-7.68	-7.97	-8.55	-8.66	-7.86

$$\begin{aligned}\delta E[\Psi] &= \delta \left[\frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} \right] = \delta \left[\frac{\langle \Phi | F^\dagger H F | \Phi \rangle}{\langle \Psi | \Psi \rangle} \right] \\ &\simeq \delta \left[\frac{\langle \Phi | H_{eff} | \Phi \rangle}{\langle \Phi | \Phi \rangle} \right] = 0\end{aligned}$$

$H_{eff} = \sum_i h_i$ fenomenologica

$$\Phi(x_1 \dots x_A) = \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_{\nu_1}(x_1) & \dots & \phi_{\nu_A}(x_1) \\ \vdots & & \vdots \\ \phi_{\nu_1}(x_A) & \dots & \phi_{\nu_A}(x_A) \end{vmatrix}$$

$$\begin{aligned}
H &= \sum_i T_i + \sum_{i < j} V_{ij} + \sum_{i < j < k} V_{ijk} \\
&= \sum_i T_i + \sum_{i < j} V_{ij} + \sum_{i < j < k} V_{ijk} + \sum_i U_i - \sum_i U_i \\
&= \sum_i (T_i + U_i) + \sum_{i < j} V_{ij} + \sum_{i < j < k} V_{ijk} - \sum_i U_i \\
&= \sum_i h_i + \sum_{i < j} V_{ij} + \sum_{i < j < k} V_{ijk} - \sum_i U_i \\
&= H_{\text{eff}} + V_{\text{res}}
\end{aligned}$$

$$\Delta = 100 \frac{|B_{HF} - B_{exp}|}{B_{exp}}$$

	B_{HF} [MeV]	B_{exp} [MeV]	Δ
^{16}O	8.275	7.975	3.76
^{40}Ca	8.679	8.552	1.48
^{48}Ca	8.742	8.667	0.86
^{90}Zr	8.707	8.710	0.034
^{132}Sn	8.307	8.355	0.57
^{208}Pb	7.747	7.867	1.52

