Effects of nuclear re-interactions in quasi elastic neutrino-nucleus scattering

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The effects of nuclear re-interactions in the quasi elastic neutrino-nucleus scattering are investigated with a phenomenological model. We found that the nuclear responses are lowered and that their maxima are shifted towards higher excitation energies. This is reflected on the total ν-nucleus cross section in a general reduction of about 15% for neutrino energies above 300 MeV.

I. INTRODUCTION

Many works on the measurements of atmospheric and solar neutrino fluxes pointed out several anomalies which could hardly be explained without assuming the phenomenon of flavour oscillation [1,2]. In particular, the flux of solar ν's is shown to be depleted in the whole energy spectrum [2,3], while anomalies in the angular distributions and event rates come out from different experiments which measure the fluxes of ν's produced by π, K and μ decays in the cascades initiated by primary cosmic rays in the atmosphere [4–6]. The continuously growing interest in the physics of neutrino oscillations drove an increasing effort in the studies of the various aspects related to the aforementioned measurements. The correct analysis of the data requires a good knowledge of the ν–nucleon cross section in a wide range of energies.

The total cross section is usually calculated by summing three separated contributions each of them evaluated using different models which take into account the dominant physical effects in the various energy regions. At energies above 3 GeV the cross section is well described by deep inelastic scattering processes within the parton model. The intermediate energy region, from 1 to 3 GeV, is dominated by the nucleon resonances. In this region, the appropriate degrees of freedom to describe the cross section seem to be nucleons and mesons. At energies below 1 GeV it is possible to neglect the nucleon excitations but it is necessary to consider nuclear effects.

A deep knowledge of the cross section for ν-induced reactions on nuclear targets is needed for the correct analysis and interpretation of the experimental data [7,8]. The total inclusive cross section and, therefore, the expected number of events could change. Moreover the kinematics of the final state is modified by nuclear recoil, Fermi motion and re-scattering. Therefore the knowledge of these effects provides the ultimate limit to some experimental issues. One of these is the reconstruction of the direction of the incoming neutrino, and then its pathlength from the production point [9]. Another is the background rejection based on parameters like the missing momentum in the transverse plane, used for ν_τ detection/tagging on long/short baseline experiments [10].

Recently, the scattering off nuclei like ^{12}C, ^{16}O (i.e. the main constituents of scintillator and water Čerenkov detectors) has been the object of many investigations [11,12].

In this work we shall concentrate on the quasi elastic regime of ν charged current interactions

$$\nu_l + n \to l^- + p \qquad \text{and} \qquad \bar{\nu}_l + p \to l^+ + n. \tag{1}$$

The interest in these particular channels lies on two different aspects:

- The quasi elastic scattering is the dominant process at ν energy below 1 GeV. In long baseline or atmospheric ν experiments, this energy region is sensitive to low values of Δm^2 (i.e. $10^{-3} \div 10^{-2}\, eV^2$), which is indicated as the preferred solution for atmospheric ν anomaly.
- The kinematics of the final state can be easily reconstructed in several types of detectors and provides a very clean signal if compared to the case of deep inelastic scattering which produces showering events.

The quasi elastic regime is characterized by nuclear excitation energies, i.e. the energy lost by ν, whose values range from 30 MeV up to about 300 MeV. In this energy region the scattering is dominated by the direct interaction of the ν with a single nucleon, i.e. the elementary processes of (1), while the other nucleons act as spectators. Anyway the ν-nucleon cross section has to be corrected because the nucleon is not free but embedded in the nucleus.

At a first sight, the quasi elastic cross section seems to be well described by mean–field (MF) models, like Hartee–Fock, shell model and Fermi gas model. These models neglect the collective nuclear excitations which are important at excitation energies smaller than 30 MeV, but they are not present at higher energies. In MF models the electro-weak excitation of the full nuclear system is described as the transition of a nucleon from a state below to one above the Fermi surface. In this way the binding of the nucleon in the nucleus and the Pauli blocking are taken into account. A 20 years experience in the study of quasi elastic electromagnetic excitations has shown that this picture is unable to provide an accurate quantitative description of experimental data. The agreement with the experiment is obtained if the rescattering between the emitted nucleon and the rest nucleus is considered in addition to the MF effects [13].

In this paper we investigate the effects of the nucleon re-interaction on the quasi elastic ν cross sections for all the types of neutrinos, using the phenomenological folding model developed in refs. [14,15]. In sect. II we present the folding model and we apply it to the nuclear Fermi gas model in sect. III. The results of the calculations are shown and discussed in sect. IV and conclusions are presented in sect. V. Since in its original formulation the folding model was constructed to correct response functions, we recall in the Appendix the relationships between response functions, Green's functions and cross sections.

II. RE-INTERACTION EFFECTS

A general analysis of charge–changing semi–leptonic weak interaction in nuclei can be done in close analogy with electron scattering off nuclei [16]. The hypotheses usually done in the derivation of the electron scattering cross section can also be applied to the case of the scattering of weak interacting probes. A first hypothesis consists in assuming that the process can be well described already at the first order Feynmann diagram i.e. by considering only those diagrams where a single gauge boson is exchanged. In a second hypothesis the exchanged boson is assumed to interact with a single nucleon in the nuclear interior. The nuclear transition amplitude is obtained by summing the transition amplitudes of the single nucleons. In the second quantization language this means that we consider one–body transition operators only. A further hypothesis consists in assuming that the nucleus makes a transition between states of definite angular momentum.

In the nuclear MF model, these states are Slater determinants of the single particle states. Since we have chosen to restrict ourselves to the case where there is only one-particle in the continuum the nuclear final state $|f>$ will be described by a pure one–particle one–hole (1p–1h) excitation:

$$|f> \rightarrow |\Phi_f> = a_p^+ a_h |\Phi_i>, \tag{2}$$

where $|\Phi_f>$ and $|\Phi_i>$ indicate the Slater determinants describing the initial and final MF states.

As already mentioned in the introduction, the application of the MF model to the description of quasi elastic electron scattering data is unsatisfactory. The great experimental work done in this field has allowed the separation of the two response functions, the charge and the current responses forming the inclusive electron scattering cross section (Rosenbluth separation). MF calculations overestimate the charge responses and underestimate the current ones (for a review see for example [13]).

The studies done to clarify this puzzle have shown that the main correction to the MF responses is coming from the re–interaction of the emitted nucleon with the rest of the nucleus. This effect is only partially taken into account by the Random Phase Approximation (RPA), a theory which describes collective excited states as linear combination of 1p-1h and 1h-1p excitations of the ground state. In the quasi elastic region, i.e. at nuclear excitation energies above ∼100 MeV, continuum RPA calculations done with a finite range effective nucleon–nucleon interaction produce responses which do not differ very much from the MF responses [15,18,19]. More important are those re–interaction effects beyond the RPA description which, in the electron scattering literature, are called Final State Interactions (FSI). The FSI take into account the possibility that, after the interaction with the probe, the nucleus remains in a highly excited state which can be described only in terms of many–particle many–hole excitations. In this case FSI lower by $15 \div 20\%$ the MF responses.

We think that FSI may play an important role also in the description of weak quasi elastic responses. We have considered FSI using the model developed in [14,15], which we shall briefly present in the following.

The full Hilbert space $\mathcal{H}$ can be divided in a subspace $\mathcal{H}_0$ composed by all the 1p-1h Slater determinants defined in eq. (2) and a complementary subspace $\mathcal{H}_c = \mathcal{H} - \mathcal{H}_0$. The 1p-1h excited states are eigenvectors of the MF hamiltonian H_0

$$H_0 |\Phi_f> = E_f^{MF} |\Phi_f> \;, \tag{3}$$

and lead to the MF expression of the response:

$$S^{MF}(|\mathbf{q}|, \omega) = \sum_f < \Phi_f | \mathcal{O}(\mathbf{q}) | \Phi_i >< \Phi_f | \mathcal{O}(\mathbf{q}) | \Phi_i >^\dagger \delta(E_f - \omega) \tag{4}$$

$$= -\frac{1}{\pi} Im < \Phi_i | \mathcal{O}^+(\mathbf{q}) G^0(\omega) \mathcal{O}(\mathbf{q}) | \Phi_i > \;\; . \tag{5}$$

where we have indicated with $G^0(\omega)$ the MF Green's function (see Appendix).

To evaluate the modifications of S^{MF} due to presence of $\mathcal{H}_c$ we introduce projection operators P onto $\mathcal{H}_0$ and Q onto $\mathcal{H}_c$. The effects of $\mathcal{H}_c$ can be taken into account by considering the effective hamiltonian:

$$H_{eff}(\omega) = PHP - PHQ\ \frac{1}{QHQ - \omega - i\eta}\ QHP = PHP - U(\omega)\ . \tag{6}$$

If the hamiltonian H_0 is composed by one-body hamiltonians, like in the MF case, only the two–body term $V = H - H_0$ can connect the $\mathcal{H}_0$ and $\mathcal{H}_c$ Hilbert subspaces, therefore the many-particle many–hole excited states forming $\mathcal{H}_c$ act only as doorway states. Since the operators $\mathcal{O}(\mathbf{q})$ are one–body operators they act only on the $\mathcal{H}_0$ subspace.

Using the effective hamiltonian in the Green's function (eq. (31) in the appendix) and inserting a full set of eigenstates of H_0, we obtain the following expression for the response function:

$$\begin{aligned} S(|\mathbf{q}|,\omega) = \frac{1}{\pi} Im \sum_{f,f'} < i|\mathcal{O}^+(\mathbf{q})|\Phi_f > & \left[\frac{1}{E_f - \omega - \Sigma_{ff'}(\omega) - i\eta} \right. \\ & \left. + \frac{1}{E_f + \omega - \Sigma_{ff'}(-\omega) + i\eta} \right] < \Phi_{f'}|\mathcal{O}(\mathbf{q})|i > \ , \end{aligned} \tag{7}$$

where we have used $< \Phi_f|PHP|\Phi_{f'} > = < \Phi_f|H_0|\Phi_{f'} > = E_f \delta_{ff'}$ and we have defined $\Sigma_{ff'}(\omega) = < \Phi_f|U(\omega)|\Phi_{f'} >$.

We introduce the assumption that in quasi elastic energy range the term $\Sigma_{ff'}$ does not strongly depend on the individual MF states $|\Phi_f >$ but rather on the full phase space available at the energy ω. In other words we make the assumption:

$$\Sigma_{ff'}(\omega) \longrightarrow \Sigma(\omega)\delta_{ff'}\ , \tag{8}$$

where $\Sigma(\omega)$ is a complex function of the transferred energy:

$$\Sigma(\omega) = \Delta(\omega) - \frac{i}{2}\Gamma(\omega)\ . \tag{9}$$

Using the above expressions in eq.(7) we obtain:

$$S(|\mathbf{q}|,\omega) = \sum_f | < \Phi_f|\mathcal{O}(\mathbf{q})|i > |^2 \left[\rho(E_f,\omega) + \rho(E_f,-\omega)\right]\ , \tag{10}$$

where we have defined:

$$\rho(E,\omega) = \frac{1}{2\pi} \frac{\Gamma(\omega)}{[E - \omega - \Delta(\omega)]^2 + [\Gamma(\omega)/2]^2}\ . \tag{11}$$

In the expression (10) the nuclear ground state is still eigenstate of the full hamiltonian H, but it is usually replaced by the nuclear ground state provided by H_0. Making this substitution and considering that the quasi elastic region is in the continuum excitation region of the nucleus, we can rewrite the full response function as a convolution of the MF response (5) with the lorentzian functions $\rho(E,\omega)$:

$$S(|\mathbf{q}|,\omega) = \int_0^\infty dE\, S^{MF}(|\mathbf{q}|,E) \left[\rho(E,\omega) + \rho(E,-\omega)\right] \tag{12}$$

The FSI produce three effects on the MF response: a lowering of the maximum value of the response, a widening of the width and a shift of the position of the peak due to the $\Delta(\omega)$ term.

The functions $\Delta(\omega)$ and $\Gamma(\omega)$ are connected by a dispersion relation:

$$\Delta(\omega) = \frac{1}{2\pi} P \int_{-\infty}^{+\infty} d\omega' \frac{\Gamma(\omega')}{\omega' - \omega}\ , \tag{13}$$

where we have indicated with P the principal value integral. Therefore our calculations would only require the knowledge of $\Gamma(\omega)$ which can be related to the imaginary part of the single particle self-energy.

Our description of the nuclear excitation does not consider the non locality of the mean field. A simple way to take into account this important correction is the introduction of a q–dependent nucleon effective mass $M^*(q)$ [20]. For a given value of M^*/M the following scaling relation holds:

$$S_{M^*}(|\mathbf{q}|,\omega) = \frac{M^*}{M} S_M(|\mathbf{q}|, \frac{M^*}{M}\omega)\ . \tag{14}$$

III. APPLICATION TO THE FERMI GAS MODEL

In the previous outline of the folding model we have not specified the characteristics of the MF hamiltonian H_0 (the only requirement was that it should be a sum of single particle hamiltonians).

Finite nuclear systems are realistically described by MF hamiltonians of Hartree–Fock and shell model type where the single particle wave functions are calculated solving the single particle Schrödinger equation for a spherically symmetric potential. On the other hand, the excitation energy and 3-momentum transfer values characterizing the quasi elastic region are such that surface and collective excitations of the nucleus are not important. Therefore it is plausible to use in this region a simpler MF model, the Fermi gas (FG) model, which describes the nucleus as a translationally invariant system composed by an infinite number of nucleons whose momentum distribution is given by:

$$n(|\mathbf{p}|) = \frac{\mathcal{T}}{\frac{4}{3}\pi k_F^3}\Theta(k_F - |\mathbf{p}|) \ , \tag{15}$$

where Θ is the Heaviside function, $\mathcal{T}$ = Z or N, k_F is the Fermi momentum and $\mathbf{p}$ the nucleon 3-momentum.

In this system the full hamiltonian is the sum of free single particle hamiltonians and the single particle wave functions used to form Slater determinants are plane waves. The nucleon single particle energies $\epsilon(|\mathbf{p}|)$ are related to their 3-momentum by

$$\epsilon(|\mathbf{p}|) = \sqrt{|\mathbf{p}|^2 + M^2} - w \ , \tag{16}$$

where $p = (p_0, \mathbf{p})$ is the nucleon 4-momentum, M its rest mass and we have subtracted a constant binding energy w. The nucleus excitation energy is given by the difference between the particle and hole single particle energies. In the Fermi gas model the observable quantities are calculated per unit of volume and per nucleon. Their values should remain constant in the limit of infinite volume and nucleon number.

The relevance of nuclear finite size in the quasi elastic region has been studied in [21] by comparing electromagnetic shell model and FG responses. It has been found that the FG model generates nuclear responses rather close to those of the shell model as long as the value of the Fermi momentum is taken as the average value with respect to the nuclear matter density $n(r)$:

$$< k_F >= \left(\frac{3}{2}\pi^2\right)^{1/3} \frac{\int dr\, r^2\, [n(r)]^{4/3}}{\int dr\, r^2\, n(r)} \ . \tag{17}$$

Neutrino–nucleus quasi elastic cross section was first evaluated in a FG model by Smith and Moniz [22], and the final expression contains the various responses (as it does the expression given in eq. (28)). Since the interaction matrix elements show a very weak dependence on the initial nucleon 3-momentum $\mathbf{p}$, we make the common approximation of writing the cross section in a factorized form:

$$\left(\frac{d\sigma(|\mathbf{q}|,\omega)}{d\Omega_l dE_l}\right)_{FG} = \sum_{i=1}^{A} \left(\frac{d\sigma(|\mathbf{q}|,\omega)}{d\Omega_l}\right)_{(\nu,N_i)} R^i_{FG}(|\mathbf{q}|,\omega) \ , \tag{18}$$

where A is the number of nucleons forming the target nucleus, $d\Omega_l$ is the differential solid angle around the direction of the outgoing lepton, and

$$\left(\frac{d\sigma(|\mathbf{q}|,\omega)}{d\Omega_l}\right)_{(\nu,N)} \tag{19}$$

is the neutrino cross section on free nucleons. In eq. (18) the nuclear effects, Fermi motion, binding and Pauli blocking, are taken into account by the $R_{FG}(\mathbf{q},\omega)$ function whose expression is [23]:

$$R^i_{FG}(|\mathbf{q}|,\omega) = \frac{1}{\frac{4}{3}\pi k_F^3} \int \frac{d^3p M_i^2}{p_0 p_0'} \delta(p_0 + \omega - p_0') \Theta(k_F - |\mathbf{p}|) \Theta(|\mathbf{p}'| - k_F) \ , \tag{20}$$

where $p = (p_0, \mathbf{p})$ and $p' = (p_0', \mathbf{p}' = \mathbf{p} + \mathbf{q})$ are the initial and final nucleon 4-momenta. The validity of the approximation leading to the factorized form of the cross section has been checked in ref. [23] where it was found that in the quasi elastic region the expression (18) numerically differs from the full expression given in ref. [22] by a few percent.

The folding model described in the previous section should be applied to the response functions using different folding functions $\rho(E,\omega)$ for every response function according to their spin and isospin dependence. In the quasi elastic region these differences among responses are rather small, therefore we have used the same folding function for all them. Since the cross section is a sum of responses, we have applied the folding directly to the cross section:

$$\begin{aligned}\left(\frac{d\sigma(|\mathbf{q}|,\omega)}{d\Omega_l dE_l}\right)_{M^*} &= \int_0^\infty d\tilde{E}\left(\frac{d\sigma(|\mathbf{q}|,\tilde{E})}{d\Omega_l dE_l}\right)_{FG}\left[\rho(\tilde{E},\omega)+\rho(\tilde{E},-\omega)\right] \\ &= \sum_{i=1}^{A}\left(\frac{d\sigma(|\mathbf{q}|,\omega)}{d\Omega_l}\right)_{(\nu,N_i)} R^i(|\mathbf{q}|,\omega) \end{aligned} \tag{21}$$

following eqs.(12) and (14), with the definition $\tilde{E}=(EM^*)/M$.

IV. RESULTS AND DISCUSSION

The only free parameter required by the FG model is the value of the Fermi momentum which can be related to the average density of the real nuclear system through eq. (17). In all our calculations we have used $k_F = 220$ MeV/c which corresponds to the average density of nuclei with mass number $15 < A < 50$.

Once the FG cross section has been calculated, the other inputs we need are related to correction for the re–interaction: the $\Gamma(\omega)$ function of eq. (9) and the effective nucleon mass M^*. From a theoretical point of view $\Gamma(\omega)$ can be evaluated by considering many–particle many–hole nuclear excitations, however we have used an estimate based on a comparison with experiments. The data we have considered for positive values of ω are those related to the imaginary part of the nuclear mean field whose parameters have been fixed to fit nucleon–nucleus elastic scattering cross sections. For negative values of ω we have considered the energy width of the single particle levels, measured in knock–out reactions like $(e,e'p)$ or (p,d). We have obtained $\Gamma(\omega)$ by making the average of the single particle energy width $\gamma(\omega)$:

$$\Gamma(\omega)=\frac{1}{\omega}\int_0^\infty d\epsilon\,[\gamma(\epsilon+\omega)+\gamma(\epsilon-\omega)]\ . \tag{22}$$

In order to reproduce the empirical values of the single particle widths, we have used the expression:

$$\gamma(\epsilon)=a\cdot\frac{\epsilon^2}{\epsilon^2+b^2}h(\epsilon)\ , \tag{23}$$

with $a = 11.5$ and $b = 18$ [20]. Unfortunately the data are limited to $|\epsilon| < 100\,$MeV, therefore the high energy behaviour, controlled by $h(\epsilon)$, is affected by strong uncertainties. We have used two extreme parameterizations compatible with the data:

$$h(\epsilon)=1 \tag{24}$$

$$h(\epsilon)=\frac{c^2}{\epsilon^2+c^2} \tag{25}$$

with $c = 110$.

The value of the effective nucleon mass is another input of our calculation. It has been determined by using the expressions proposed in the polarization potential model of ref. [24]:

$$\frac{M^*}{M}=\frac{1}{1+2M\Delta U/(q^2+q_0^2)} \tag{26}$$

with

$$q_0^2=\frac{2M\Delta U}{M/M_0^*-1}\ \ , \tag{27}$$

where ΔU=50 MeV is the depth of the potential well. The above expressions have been built such as $M^*/M = 1$ in the limit for $q\to\infty$ and $M^* = M_0^*$ for $q \to 0$. Various nuclear structure studies [20,24,25] indicate that M_0^*/M=0.8, and this is the value we have used in our calculations.

The effects of the re-scattering are evident in fig.1 where we show $R(\mathbf{q},\omega)$ of eq. (21) calculated for $|\mathbf{q}|$=600 MeV/c as a function of ω. The dotted line represents $R_{FG}(\mathbf{q},\omega)$, eq.(18). The other two lines have been obtained performing the folding with $h(\epsilon)$ given by eq. (25), the dashed line with $M^*/M = 1$, and the full line using the value of the effective mass given by the procedure discussed above. A comparison between dashed and dotted lines shows that the re–interaction is moving strength from the peak region towards both lower and higher energy regions. The total strength is conserved i.e. the areas underlined by the two curves in the figure are equal. The peak position is slightly shifted towards higher energy values owing to the presence of $\Delta(\omega)$ in the denominator of the folding $\rho(E,\omega)$ functions. The differences between dashed and full lines are due to the nucleon effective mass which produces a further spreading, a lowering of the peak value and a shift of the peak to a higher energy.

The sensitivity of our results to the expression chosen to set $\Gamma(\omega)$ and to the effective mass, can be seen in fig.2, where we show the ν_μ–nucleus cross sections per nucleon as a function of the neutrino energy. The dashed lines have been obtained setting $M^*/M = 1$ while the full lines have been calculated using the expression (26). In both cases the upper curves have been obtained using $h(\epsilon) = 1$ in eq. (23) while the lower lines have been produced with the expression (25). As expected the use of the effective mass lowers the values of the cross section at all energies. It is also clear that the use of $h(\epsilon) = 1$ enhances the effect of the folding, since this means that the re-scattering is active whatever is the energy of the emitted nucleon. In the other choice the re-scattering effects are switched off when the energy of the emitted nucleon is above 200 MeV. We think that the calculations with M^* given by eq.(26) are the most realistic ones. On the other hand, we do not have any good reason to prefer one of the two possible choices for $h(\epsilon)$. The expression (25) would be favored by the speculations about the so-called color transparency [26] claiming that the nucleus is transparent to high energy emitted nucleons. However these speculations do not have, so far, experimental support. The results presented in figs.4 and 5 have been obtained using effective mass and eq. (25). We should remark, in any case, that the maximum difference between the upper and lower full lines in fig.2 is of 5%.

Our study has been made under the assumption of no uncertainties in the elementary ν–nucleon cross section. This hypothesis, common to all the calculations of this type, is not fully correct, since this elementary cross section depends upon the electromagnetic and axial nucleon form factors. In our calculations we have used the dipole form for the electromagnetic form factors, and we have checked that changes produced by the more sophisticated parameterization of ref. [27] were within the numerical accuracy of the calculations. Since we have adopted the dipole expression, the only free parameter is the axial mass M_A. In the calculations presented so far we used the axial mass value of $M_A = 1.00$ GeV/c^2 which has been adopted also in ref. [28]. This value is compatible with the analysis of the neutrino nucleus total cross section data of refs. [29,30]. On the other hands, these studies, and also other ones done on different target nuclei, show an uncertainty on the axial mass value of 15%. To test the sensitivity of the cross section to the axial form factor we have calculated the ν_μ-nucleus cross section by changing the axial mass value by a 15%. These results are presented in fig. 3 where it is shown that the changes on the cross section are of the same order of magnitude of the changes on the axial mass. This is not a specific feature of our model, but it is a systematic error which affects all the theoretical evaluations of the ν-nucleus cross sections by shifting all the predicted values. Comparisons between different models have to be done by using the same value of the axial mass.

The total quasi elastic neutrino and antineutrino cross sections for the three flavours are presented in fig.4 where the dashed lines show the FG results and the full lines have been obtained with our folding model. As expected, the effect of the FSI is analogous for all the cases, since it depends on the nuclear structure and not on the details of the interaction.

In Fig.5 we compare the ν_μ-nucleus cross section data [29–36] with the FG result [28] and with that of our model including FSI. The contributions of the nucleon resonances and the deep inelastic scattering to the total cross section (calculated in the framework of the FG model) are also shown. The 15% lowering of the quasi elastic peak, due to Final State Interactions is clearly visible.

Finally a crude estimate of the effect of the lowering of the quasi-elastic peak on the total upward muon flux, produced by atmospheric neutrinos, has also been done. Above a μ energy threshold of 100 MeV, the integral muon flux is reduced by $\sim 5\%$.

V. SUMMARY AND CONCLUSIONS

The ν-nucleus scattering in the quasi elastic region leaves the nucleus in a highly excited state which decays mainly by nucleon emission. This decay is not properly treated in MF nuclear models, where the emitted nucleon does not interact with the residual nucleus. We have presented a phenomenological model to correct the MF cross sections for this effect. The re-interaction model has been applied to FG differential cross sections. It could also be applied to more sophisticated MF models. However, for quasi elastic scattering, our calculation takes into account, in a simple way, the main physical ingredients since in this energy region collective nuclear excitations and finite size effects are not important [15,18,19,21].

We have shown that the re-interaction spreads the cross section strength at both higher and lower energies, with respect to the pure MF result. This produces a general lowering effect of about 15% in the cross section.

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VI. APPENDIX

In this appendix we briefly recall the link between response function, Green's function and cross sections. The commonly used approximations adopted in the derivation of the neutrino or antineutrino cross section on nucleus are: first order pertubation theory, impulse approximation and ultra-relativistic limit (i.e. the terms containig the leptons rest masses are neglected). With these assumptions the cross section can be written, in the lab frame, as [16]:

$$
\begin{aligned}
\frac{d^2\sigma}{d\Omega_l dE_l} = \frac{g^2 E_l^2}{2\pi^2}\frac{4\pi}{2J_i+1}\Big\{ & cos^2\frac{\theta}{2}\Big[\sum_{J=0}^{\infty} | < J_f||\mathcal{M}_J - \frac{\omega}{|\mathbf{q}|}\mathcal{L}_J||J_i > |^2\Big] \\
& +\Big(\frac{q^2}{2\mathbf{q}^2}cos^2\frac{\theta}{2} + sin^2\frac{\theta}{2}\Big) \\
& \times\Big[\sum_{J=1}^{\infty}\Big(| < J_f||\mathcal{J}_J^{el}||J_i > |^2 + | < J_f||\mathcal{J}_J^{mag}||J_i > |^2\Big)\Big] \\
& \mp sin\frac{\theta}{2}\frac{1}{|\mathbf{q}|}\Big(q^2 cos^2\frac{\theta}{2} + \mathbf{q}^2 sin^2\frac{\theta}{2}\Big)^{\frac{1}{2}} \\
& \Big[\sum_{J=1}^{\infty} 2Re < J_f||\mathcal{J}_J^{mag}||J_i >< J_f||\mathcal{J}_J^{el}||J_i >^* \Big]\Big\} \ ,
\end{aligned} \tag{28}
$$

where the $\mp$ should be used for neutrino or antineutrino scattering respectively. In the above expression we have indicated with g the universal weak coupling constant, with E_l the energy of the emitted lepton, with $q \equiv (\omega, \mathbf{q})$ the four-momentum transfer, with θ the angle between incoming and outgoing leptons, with $|J_i >$ and $|J_f >$ the initial and final states of the nuclear system characterized by their total angular momenta J. The quantities $\mathcal{M}_J$, $\mathcal{L}_J$, $\mathcal{J}_J^{mag}$, and $\mathcal{J}_J^{el}$ are multipole expanded operators obtained by separating the hadronic tensor in charge, longitudinal and transverse electric and magnetic operators. As discussed above, these operators are expressed in terms of one–body operators within the nuclear many–body Hilbert space.

In the expression (28) of the cross section the leptonic variables are separated from the hadronic ones. The information about the hadronic part is fully contained in the reduced matrix elements of the transition amplitude from the nuclear ground state $|J_i >$ to the excited state $|J_f >$. In eq. (28) the sums over all the possible values of J have been included since we are interested in an energy range above the nucleon emission threshold. The nuclear states form a complete basis since they are eigenstates of the nuclear hamiltonian H, therefore it is possible to relate the transition matrix elements to the linear response function as it is usually defined in many–body theories [17]:

$$
S(|\mathbf{q}|,\omega) = \sum_f < f|\mathcal{O}(\mathbf{q})|i >< f|\mathcal{O}(\mathbf{q})|i >^\dagger \delta(E_f - \omega) \tag{29}
$$

$$
= \frac{1}{\pi} Im < i|\mathcal{O}^\dagger(\mathbf{q})G(\omega)\mathcal{O}(\mathbf{q})|i > . \tag{30}
$$

$G(\omega)$ is the many–body Green function:

$$
G(\omega) = \frac{1}{H-\omega-i\eta} - \frac{1}{H+\omega+i\eta}, \tag{31}
$$

where we have indicated with H the full many-body hamiltonian. In the above equations $|i >$ and $|f >$ indicates the initial and final nuclear states, E_f the eigenvalue of $|f >$, and $\mathcal{O}(\mathbf{q})$ a generic many–body operator.

We are interested in nuclear excitation energies whose values are well above the nucleon emission threshold i.e. in reactions disintegrating the nucleus. Generally the nuclear final state can be very complicated, for example, it can be composed by a set of unbound nucleons or various nucleon clusters. We make the simplifying assumption that the nuclear final state is formed by one particle in the continuum and a residual nucleus composed by $A-1$ nucleons. Even within this assumption the evaluation of the nuclear transition amplitudes, or of the nuclear responses, is rather difficult if one attempts to solve the nuclear many–body problem using the full interacting hamiltonian H.

A successful model used to obtain a simplified solution of the full problem is the MF model, where the nucleons are supposed to move in an average potential independently from each other. This means that in the above equations the hamiltonian H is substituted with a MF hamiltonian H_0 which is formed by a sum of single particle hamiltonians whose eigenvectors form a basis of single particle states.

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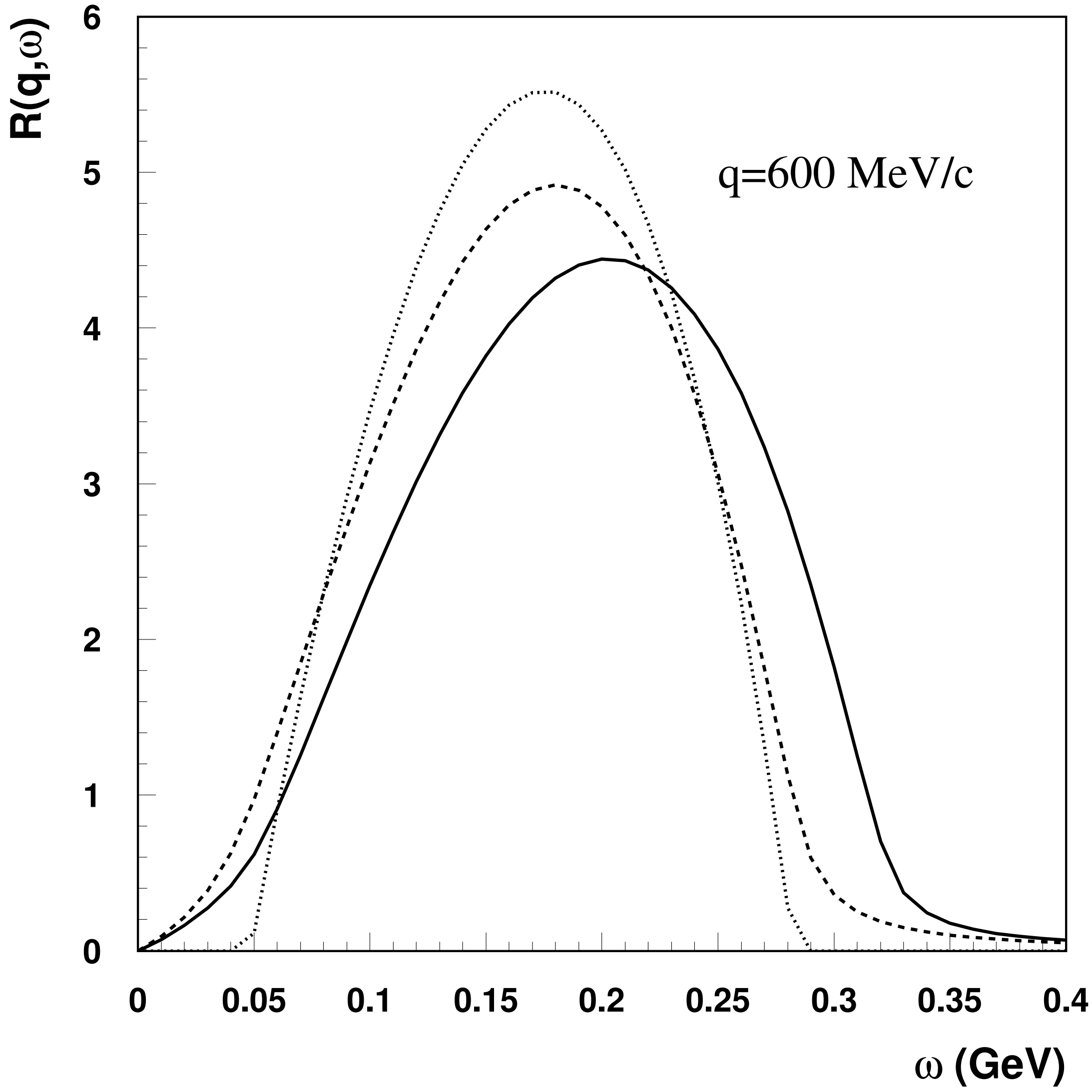

FIG. 1. Dependence of $R(|\mathbf{q}|, \omega)$ on the transferred energy ω for ν_μ - nucleus scattering. The dotted line shows the result of a bare FG calculation, the dashed one has been obtained with $M^* = M$, and the solid line includes all the considered FSI effects (see text). The responses per nucleon on isoscalar target (N=Z) are represented.

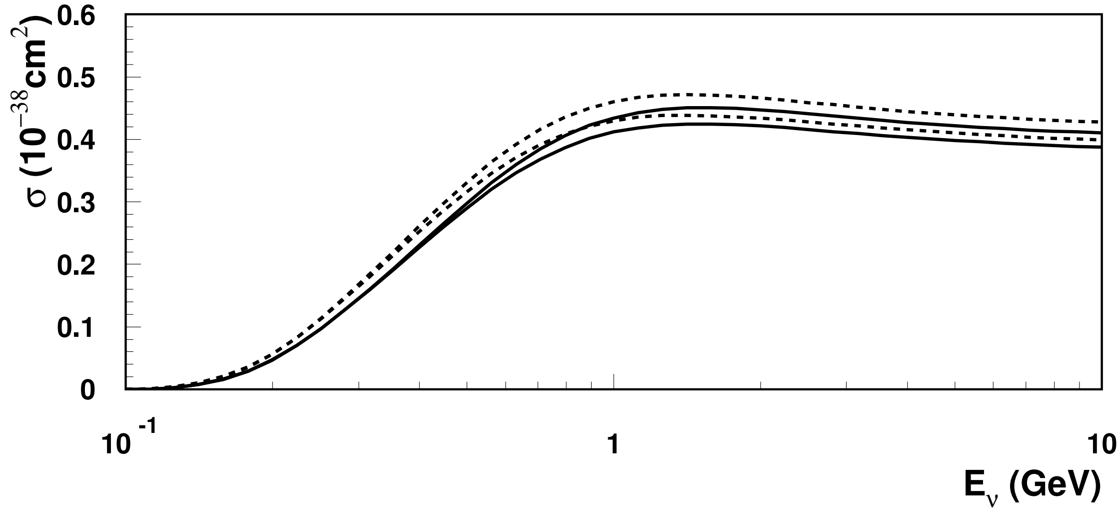

FIG. 2. Sensitivity of the total ν_μ - nucleus cross section to the parametrization of $\Gamma(\omega)$ and to M^*/M (see text). In this figure, and in the following ones, the cross sections per nucleon on isoscalar target are shown.

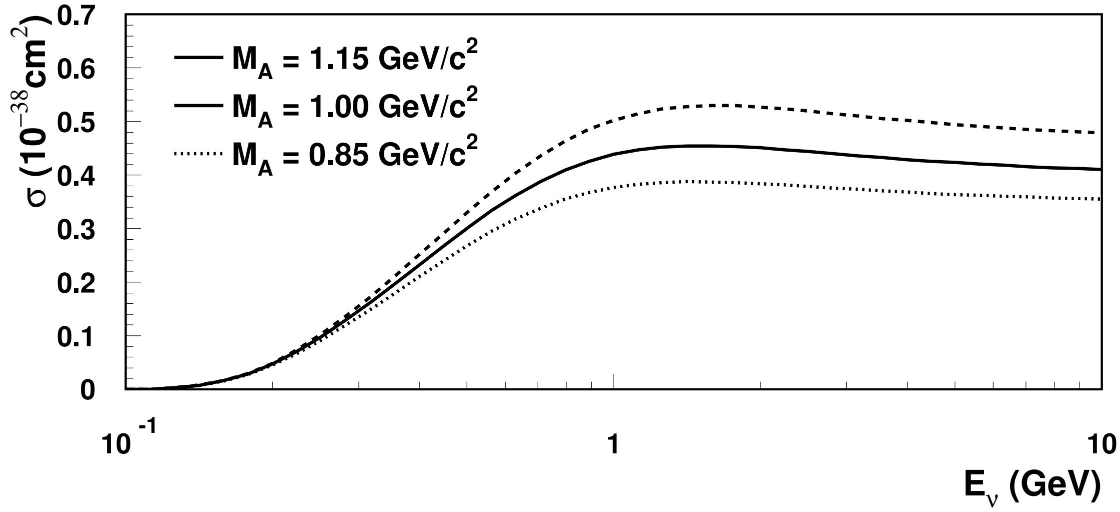

FIG. 3. Sensitivity of the total ν_μ - nucleus cross section to the value of the axial mass M_A (see text).

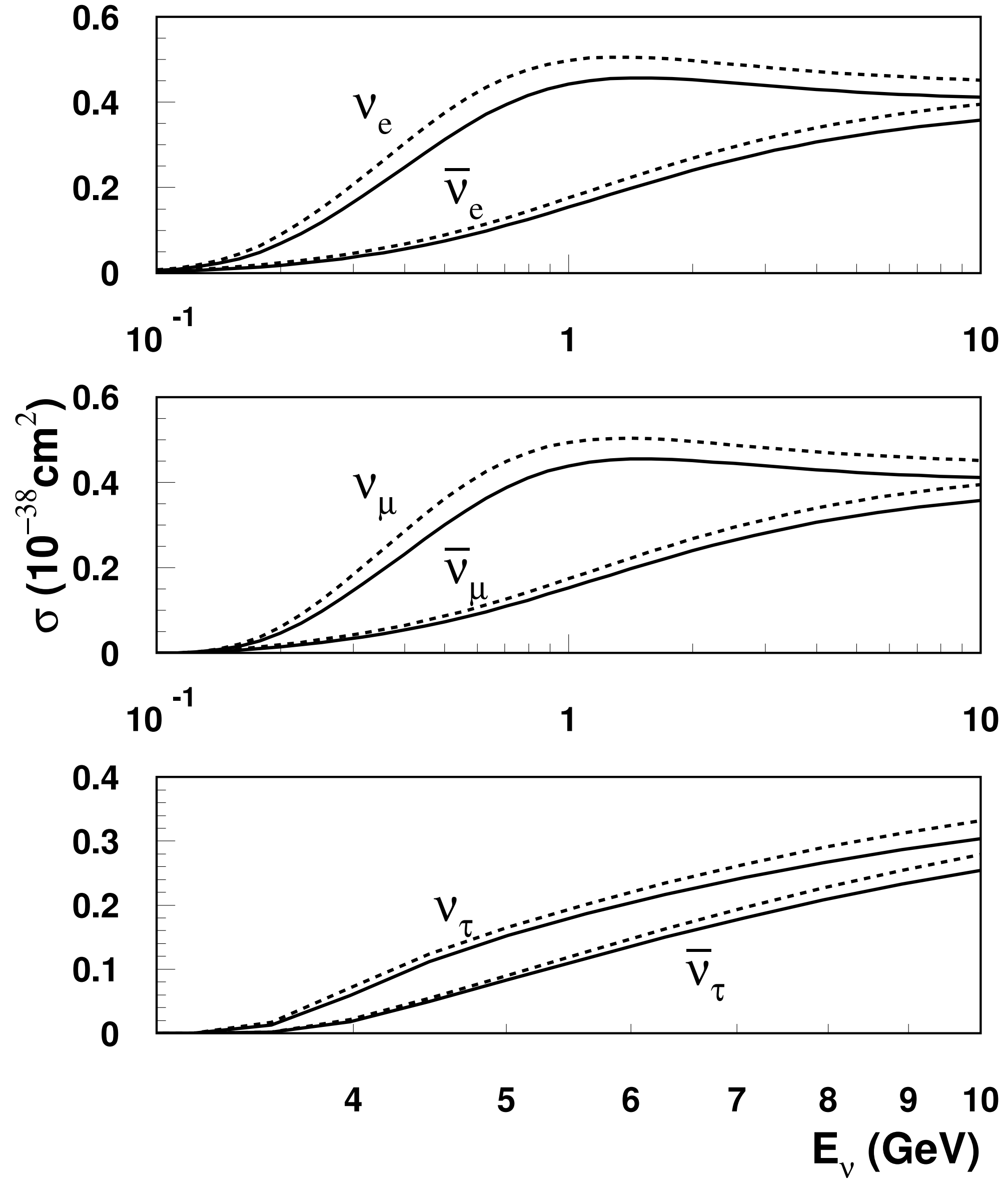

FIG. 4. Cross sections for quasi elastic ν - nucleus and $\bar{\nu}$ - nucleus scattering for all flavours. The dashed lines show the result of a bare FG calculation, while solid lines include FSI effects.

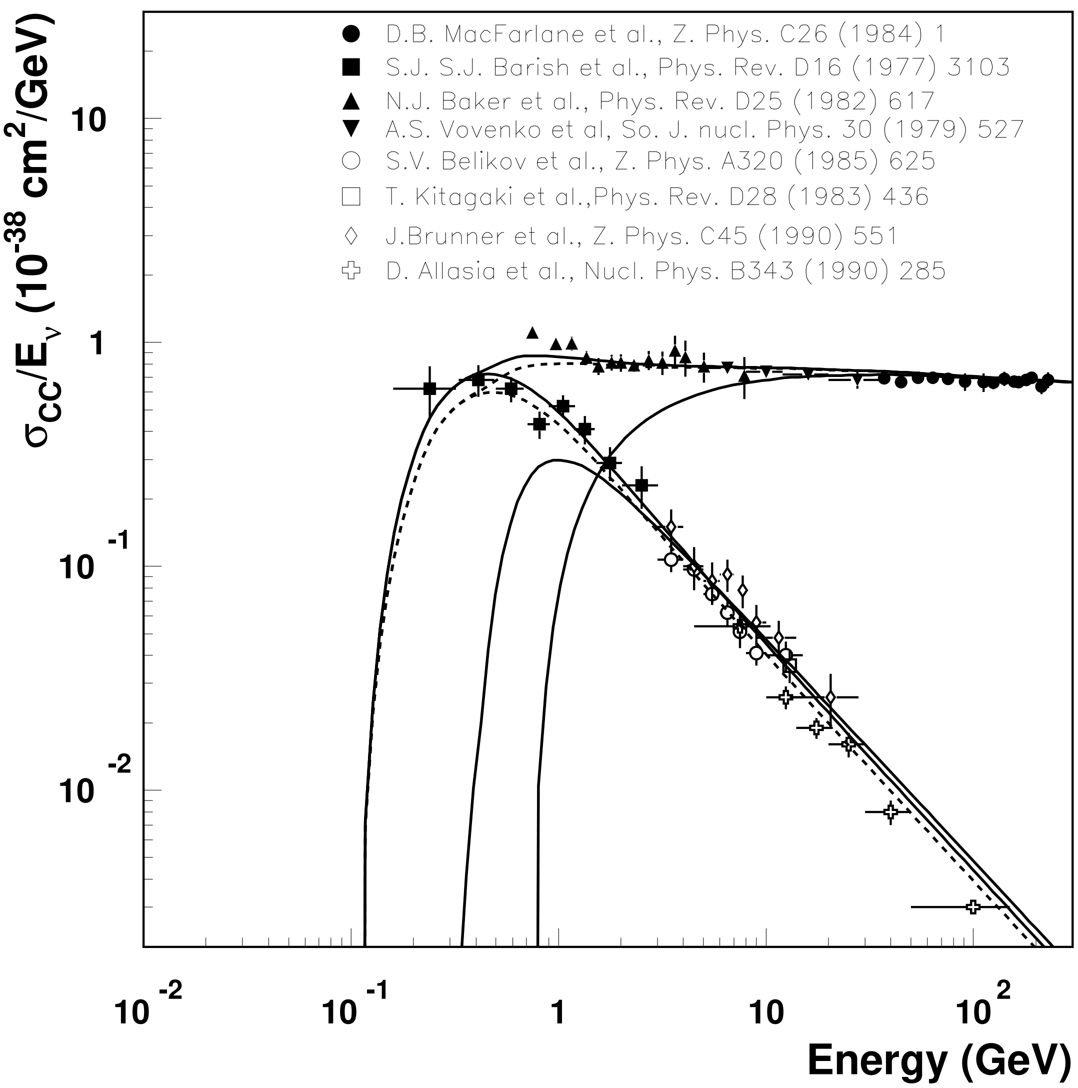

FIG. 5. Cross section per nucleon (isoscalar target) for quasi elastic ν_μ - nucleus scattering compared with experimental data. The three contributions of the *quasi elastic*, *nucleon resonances* and *deep inelastic scattering*, as calculated in the framework of the FG model [28], are shown with solid lines. The dashed line shows the result of the quasi-elastic cross section calculated in this work including also FSI effects. The total cross section, computed with the two different results in the quasi elastic region, is also reported. The 15% lowering of the quasi elastic peak, due to Final State Interactions is clearly visible.