Nuclear models and neutrino cross sections

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Merits and faults of the effective theory Random Phase Approximations are discussed in the perspective of its use in the prediction of neutrino-nucleus cross sections.

The Random Phase Approximation (RPA) is an effective theory aiming to describe the excitation of many-body systems. In nuclear physics the RPA has been applied with sucess over a wide range of excitation energies. In Fig. 1 we show inclusive (e,e') and (ν,ν') cross sections on ^{16}O target nucleus as a function of the nuclear excitation energy. In both cases the incoming energy of the lepton has been fixed at 1 GeV and the scattering angle at 30^o. In the figure three different excitation regions are emphasized. At few MeV of excitation energy there are discrete states, from 15 to 30 MeV there is the giant resonance excitation, and at hundred of MeV there is the quasi-elastic peak.

The RPA describes the nuclear excited states as a linear combination of particle-hole (ph) and hole-particle (hp) excitations

$$|\Psi_n>=\sum_{ph}(X_{ph}a^+_p a_h + Y_{ph}a^+_h a_p)|\Psi_o> \ . \qquad (1)$$

Aim of the theory is the evaluation of the X_{ph} and Y_{ph} amplitudes for each excited state $|\Psi_n>$. This is done by solving the secular equations

$$\left(\begin{array}{cc} A & B \\ B^* & A^* \end{array}\right)\left(\begin{array}{c} X \\ Y \end{array}\right) = (E_n - E_o)\left(\begin{array}{c} X \\ -Y \end{array}\right) ,$$

where the coefficients of the matrix are expressed in terms of single particle energies and wave functions as:

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h)\,\delta_{ph,p'h'} + \\ <ph'|V^{eff}|hp'> - <ph'|V^{eff}|p'h> ,$$

and

$$B_{ph,p'h'} =<pp'|V^{eff}|hh'> - <pp'|V^{eff}|h'h> \ .$$

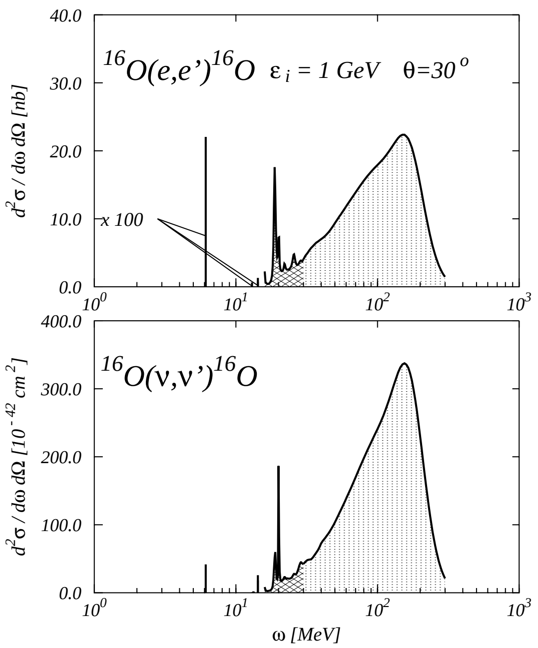

Figure 1. Electron (upper panel) and neutrino (lower panel) cross sections.

The set of single particle wave functions and energies is one of the inputs of the theory. The results presented in this report have been obtained by using single particle wave funtions and energies generated by a Woods-Saxon potential whose parameters have been fixed to reproduce the energies of the levels close to the Fermi surface and rms charge radii [1].

	^{12}C	^{12}N	^{12}B
LM1	17.2	20.2	14.3
LM2	18.8	21.7	15.9
PP	16.7	19.6	13.8
NuInt05	15.1	18.04	12.2
exp	15.1	17.3	13.4

Table 1
Energies, in MeV, of the isospin triplet 1^+ excited states referred to the ^{12}C ground state.

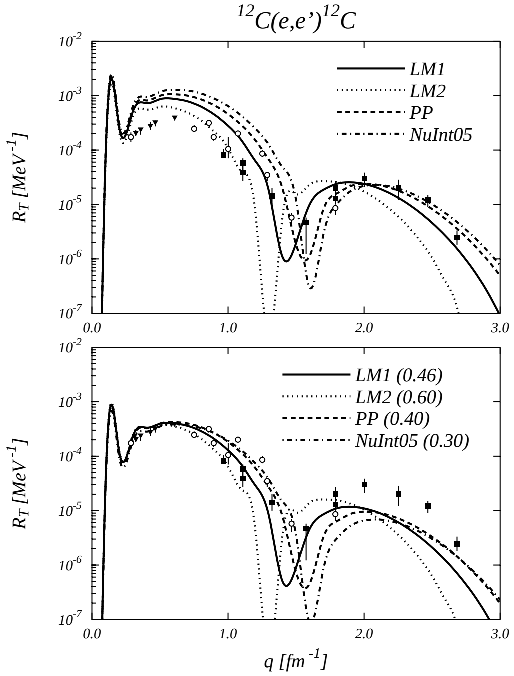

Figure 2. Magnetic form factors of the 1^+ state in ^{12}C calculated with the various interactions and compared with the experimental data. In the panel (b) the results of (a) have been multiplied by the quenching factors given in the figure in order to reproduce the data in the peak.

The other input of theory is the effective interaction V^{eff}. This is not the vacuum nucleon-nucleon interaction, but it is an effective interaction in the medium, behaving well at short internucleon distances. We would like to point out the sensitivity of the various RPA results on the choice of the effective interaction. For this reason we used various effective interactions which have the same dignity from first principles point of view.

We used two zero-range interactions whose parameters have been fixed to reproduce muonic atom polarization phase shift in the ^{208}Pb region (LM1) [2], and spin responses in ^{12}C [3].

We also used the polarization potential (PP) [4] which is a finite-range interaction whose parameters have been tuned to reproduce some properties of nuclear matter. In addition we defined a new interaction of zero-range type (NuInt05) whose parameters have been fixed to reproduce the energy of the isovector 1^+ state in ^{12}C at 15.11 MeV.

In Tab. 1 we compare the energies of the isovector 1^+ states in both charge conserving and charge exchange reactions with the experimental values. The two main issues we want to discuss in the present report are already emerging from these results: the sensitivity of the RPA results to the residual interaction and the need of an explicit treatement of degrees of freedom beyond one-particle one-hole excitations. The uncertainty on the energies calculated with the different interactions is of about 2 MeV. Even though the NuInt05 interaction reproduces the excitation energy of the charge conserving 1^+ state, it is unable to reproduce the energies of the charge exchange excitation. This indicates the limits of the prediction power of the RPA theory. In the evaluation of some observable, the use of effective, and phenomenological, nucleon-nucleon interactions cannot substitute the explicit treatment of many-particle many-hole excitations.

The above discussed issues are shown even better in Fig. 2 where the magnetic form factors of the 1^+ state in ^{12}C calculated with the various interactions are compared to the data measured in inelastic electron scattering experiments [5]. There is a large spreading of the theoretical curves. A common feature is that all the results overestimate the data. This is a well known problem of the RPA in the description of the magnetic

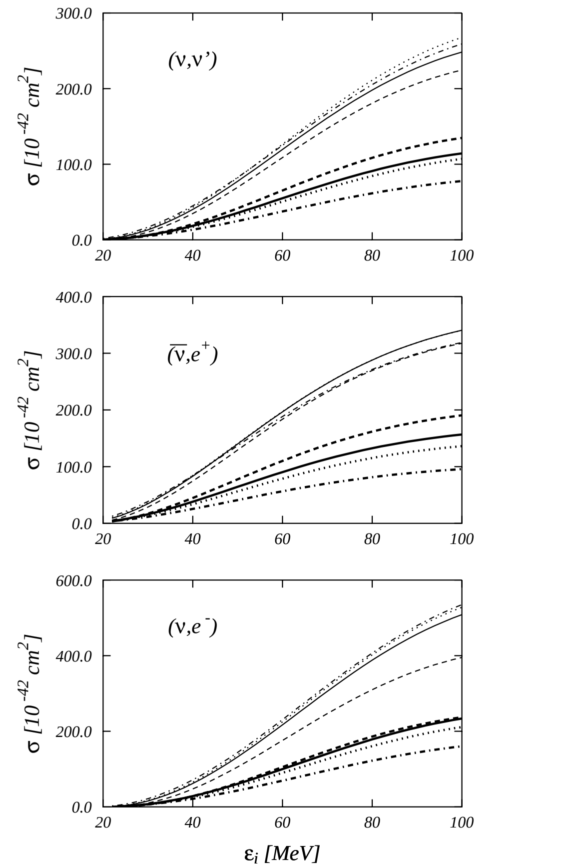

Figure 3. Total neutrino cross sections exciting the 1^+ states of Tab. 1 as a function of the neutrino energy.

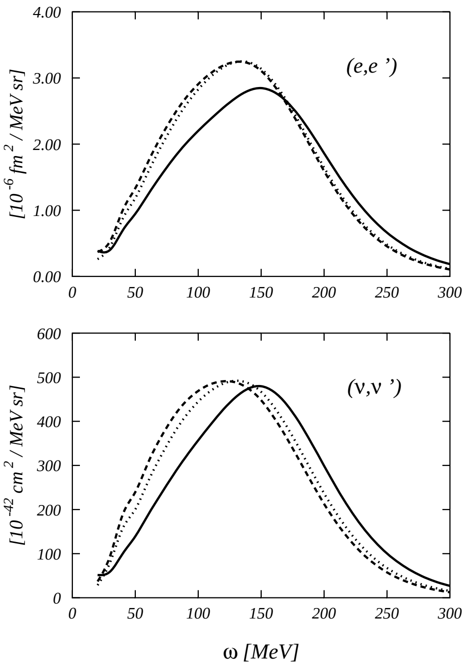

Figure 4. Electron and neutrino cross sections in the quasi-elastic region. The energy of the leptons is 1 GeV and the scattering angle 30^o. The full lines have been obtained with RPA calculations using the LM1 interaction, the dotted lines with the PP interaction. The dashed lines show the mean-field results.

form factors of medium and heavy nuclei [6,7]. It has been argued that the explicit inclusion of many-particles many-holes excitations can solve the problem [8]. In a very crude and phenomenological approach the various curves are multiplied by a factor to reproduce the data in the maximum of the form factor. The values of these quenching factors are given in the panel (b) of Fig. 2, where the renormalized results are shown. Obviously, the spreading between the various curves is reduced.

The consequences of these theoretical uncertainties on the neutrino cross sections are presented in Fig. 3 where the total neutrino cross sections for the excitation of the three 1^+ states of Tab. 1 are shown as a function of the neutrino energies. The thinner upper lines are the bare RPA results, while the lower curves, the thicker ones, have been obtained by multiplying the RPA results with the quenching factors fixed in Fig. 2.

While in the electron excitation the use of quenching factors reduced the spreading of the results, in the neutrino case this reduction is absent. On the opposite the spreading has increased. This is a further indication of the fact that electrons and neutrinos excite the same states in different manners. In the case of electron the excitation is induced by a vector current while neutrinos excitations are dominated by the axial vector current [1].

The role of the effective interactions and of the many-particles many-holes degrees of freedom in

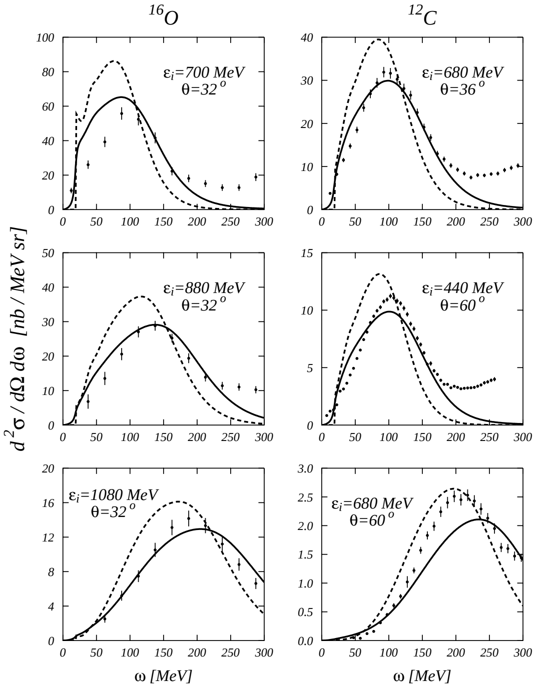

Figure 5. Electron scattering cross sections in the quasi-elastic region. The mean-field results are shown by the dashed lines. The inclusion of the FSI produces the full lines.

the giant resonance region, has been thourogly investigated in [1] and we report here the main results. The uncertainty on the total cross section is large for neutrinos of 20-40 MeV. These uncertainties have heavy consequences on the cross sections of low enwrgy neutrinos such as supernovae neutrinos and neutrinos coming from muon decay at rest. When the neutrino energy is above the 50 MeV, the results are rather independent from the nucleon-nucleon interaction and indicate that the inclusion many-particles many-holes excitations reduces the RPA cross sections by a 10-15% factor.

We show in Fig. 4 an example of RPA effects in the quasi-elastic region. Full and dotted lines have been calculated in the continuum RPA framework by using the LM1 and the PP interaction respectively. These results are compared with those obtained with a mean-field model, i.e. by setting V^{eff} to zero. The use of zero-range interaction overestimates the RPA effects. They are negligible when a finite-range interaction is used. In the quasi-elastic peak the the probe can resolve distances of about 0.5 fm, therefore zero-range interactions are not any more reliable.

Our calculations show that, in the quasi-elastic region, the RPA effects are rather small. However, many-body effects beyond RPA are not negligible, as it is shown in Fig. 5 where electron scattering cross sections calculated within the mean-field model are compared to data. In the quasi-elastic region, these complicated many-body effects, which in the RPA language are described as many-particles many-holes excitations, are usually called Final State Interactions (FSI).

Our treatment of the FSI assumes that they do not dependent on the angular momentum ad the parity of the excitation. Under this assumption it is possible to correct RPA, or mean-field, responses for the presence of FSI by foldig them with a Lorentz function [11]. The parameters of this function are fixed by hadron scattering data [12]. We have shown that the inclusion of the FSI reduces the quasi-elastic total neutrino cross sections by a 10-15% [13] factor.

In summary, the effective theory RPA allows us to investigate spectroscopic and dynamical properties of the nuclear excitations. RPA calculations are necessary to produce giant resonances and collective low-lying states. The RPA results are however strongly dependent upon the effective nucleon-nucleon interaction used. Interactions equivalent from the spectroscopic point of view can produce very different excited states (). The interactions we have used give rather similar results for charge conserving natural parity excitations. The situation for unnatural parity and charge-exchange excitation is quite uncertain. In these cases it emerges the necessity of including degrees of freedom beyond the RPA assumptions since their effects cannot be simulated by readjusting the parameters of the effective interaction. For neutrinos of energy smaller than 100 MeV the neutrino-nucleus cross sections are still very model dependent.

In the quasi-elastic region, which is dominated by single particle dynamics, our results indicate that RPA effects are not relevant. However the comparison with electron scattering data shows the need of considering FSI, whose effects lowers the total neutrino cross sections by a 10-15% factor.

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